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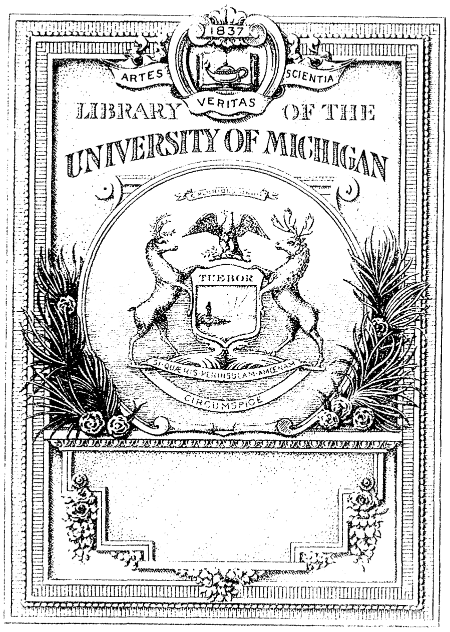

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THE ELEMENTS OF

PLANE AND SPHERICAL TRIGONOMETRY

BY

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PREFATORY NOTE

The authors have endeavored to present the essential parts of trigonometry in a form adapted to practical applications. They have had in mind particularly the requirements of schools of technology. In high-school courses it is recommended that the chapters and sections marked with a star be omitted.

Most of the problems and numerical exercises are new. The answers have been purposely omitted in some cases in the solution of triangles, in the belief that the student should be given considerable practice in the use of the check formulas.

In the preparation of the book the authors have freely consulted standard works on the subject. They are also indebted to Dr. H. Y. Benedict of the University of Texas, and to Dr. W. H. Bruce of the Denton Normal Institute, Denton, Texas, for many valuable suggestions.

THOMAS U. TAYLOR,
CHARLES PURYEAR.

October, 1902.

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PLANE TRIGONOMETRY

CHAPTER I

TRIGONOMETRIC FUNCTIONS OF AN ACUTE ANGLE

1. Definition. The six parts of a triangle are the three sides and the three angles. In a plane triangle, if any three parts be given, provided at least one of them be a side, the remaining parts can be computed. Finding the unknown parts is called *solving* the triangle.

Plane trigonometry is, primarily, that branch of mathematics which has for its object the solution of plane triangles. In the extended sense in which the term is now used, it includes also the investigation of the relations of angular magnitudes in general.

2. Functions. Suppose $y = 5x^2 - 2x + 3$. Then it is evident that the value of y depends upon that of x. Thus, if $x = 3$, $y = 45 - 6 + 3 = 42$; if $x = 1$, $y = 6$; if $x = 0$, $y = 3$. When one quantity is so connected with another that the value of the one depends upon the value of the other, the one is said to be a *function* of the other. In the above equation, y is a function of x.

3. Trigonometric Functions of an Acute Angle. Let MAN (Fig. 1) be any acute angle. From any point, B, in one of the sides draw BC perpendicular to the other side, forming the right triangle ACB. Let $\angle MAN = A$. By taking

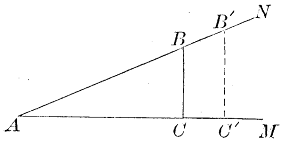

FIG. 1

the sides of this triangle two and two, we can form the six ratios $\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AC}{BC}, \frac{AB}{AC}, \frac{AB}{BC}$. The values of these ratios do not depend upon the *lengths* of the sides of the triangle in which they occur; for, if any other point in AN, as B', be taken, the triangle $AC'B'$ is similar to the triangle ACB, and we have $\frac{B'C'}{AB'} = \frac{BC}{AB}, \frac{AC'}{AB'} = \frac{AC}{AB}$, etc., since homologous sides of similar triangles are proportional. But if A be made to vary, these ratios *will* change in value, since the triangles corresponding to different values of A are not similar to the triangle ACB considered above. These six ratios depend, therefore, for their values upon the value of A. They are, accordingly, *functions* of A and are called the *trigonometric functions* of the angle A.

The trigonometric functions form an important class of functions, and an exhaustive study of them is essential in higher mathematics.

4. Names of the Trigonometric Functions. Let ACB (Fig. 2) be a right triangle. The trigonometric functions of either of its acute angles are named as follows:

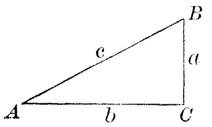

Fig. 2

The *sine* is the ratio of the opposite side to the hypotenuse;

The *cosine* is the ratio of the adjacent side to the hypotenuse;

The *tangent* is the ratio of the opposite side to the adjacent side;

The *cotangent* is the ratio of the adjacent side to the opposite side;

The *secant* is the ratio of the hypotenuse to the adjacent side;

The *cosecant* is the ratio of the hypotenuse to the opposite side.

Applying these definitions to the angle A, we have

sine of the angle A is $\dfrac{BC}{AB}$;
cosine " " " " $\dfrac{AC}{AB}$;
tangent " " " " $\dfrac{BC}{AC}$;
cotangent " " " " $\dfrac{AC}{BC}$;
secant " " " " $\dfrac{AB}{AC}$;
cosecant " " " " $\dfrac{AB}{BC}$.

"Sine of the angle A," "cosine of the angle A" are usually written *sin A, cos A*; similar abbreviations being used for the other functions.

Then, denoting the sides of the triangle by a, b, c, as indicated in the figure, the definitions for the angle A take the form

$$\begin{array}{ll} \sin A = \dfrac{a}{c}, & \cot A = \dfrac{b}{a}, \\ \cos A = \dfrac{b}{c}, & \sec A = \dfrac{c}{b}, \\ \tan A = \dfrac{a}{b}, & \csc A = \dfrac{c}{a}. \end{array}$$

It should be noted that each of these functions, being the ratio of the length of one line to that of another, is a *number*.

Questions. Is the sine of A greater or less than 1? How is it with each of the other functions?

Two other functions are defined as follows: The *versed sine* of an angle equals unity minus its cosine; the *coversed sine* is unity minus the sine. Thus,

$$\operatorname{versin} A = 1 - \frac{b}{c}, \qquad \operatorname{coversin} A = 1 - \frac{a}{c}.$$

Applying the above definitions to the angle B, we have

$$\sin B = \frac{b}{c}, \qquad \sec B = \frac{c}{a},$$

$$\cos B = \frac{a}{c}, \qquad \csc B = \frac{c}{b},$$

$$\tan B = \frac{b}{a}, \qquad \text{versin } B = 1 - \frac{a}{c},$$

$$\cot B = \frac{a}{b}, \qquad \text{coversin } B = 1 - \frac{b}{c}.$$

The angles A and B are *complements* of each other, and by comparing their functions we see that

$$\sin A = \cos B, \qquad \cos A = \sin B, \text{ etc.}$$

That is, *the sine of an angle equals the cosine of its complement; the cosine of an angle equals the sine of its complement;* with similar relations for the other functions.

Let the student state them.

All these relations are comprised in the general statement: Any function of an acute angle equals the *cofunction* of its complement.

5. Relations between the Functions. Formulas. From the definitions in Sec. 4 we see that sin A and csc A are reciprocals of each other, or

$$\sin A = \frac{1}{\csc A}, \qquad \csc A = \frac{1}{\sin A}. \qquad (1)$$

Also, cos A and sec A are reciprocals, or

$$\cos A = \frac{1}{\sec A}, \qquad \sec A = \frac{1}{\cos A}. \qquad (2)$$

Also, $$\tan A = \frac{1}{\cot A}, \qquad \cot A = \frac{1}{\tan A}. \qquad (3)$$

EXERCISES

1. If the sine of an angle is $\frac{2}{5}$, what is its cosecant?
2. If $\tan A = \frac{3}{7}$, what is the value of $\cot A$?
3. Given $\sec A = \frac{5}{3}$, required $\cos A$.

6. From the definitions in Sec. 4 we have, also,

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} = \tan A.$$

$$\therefore \tan A = \frac{\sin A}{\cos A}. \tag{4}$$

$$\frac{\cos A}{\sin A} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{a} = \cot A.$$

$$\therefore \cot A = \frac{\cos A}{\sin A}. \tag{5}$$

Formula (4) may be read: *the sine of an angle divided by its cosine equals its tangent;* or, briefly, *sine divided by cosine equals tangent.*

Express formula (5) in a similar way.

7. From the right triangle ACB (Fig. 2) we have, by geometry,

$$a^2 + b^2 = c^2.$$

Dividing each member of this equation by c^2, we have

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1,$$

or

$$(\sin A)^2 + (\cos A)^2 = 1,$$

or, as it is usually written,

$$\sin^2 A + \cos^2 A = 1. \tag{6}$$

Again, dividing each member of the above equation by b^2, we have

$$\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2,$$

or

$$\mathbf{1 + \tan^2 A = \sec^2 A.} \qquad (7)$$

In the same equation divide by a^2 and prove, also,

$$\mathbf{1 + \cot^2 A = \csc^2 A.} \qquad (8)$$

As we have seen, $\sin A$ is a number; $\sin^2 A$ means the square of that number.

Formula (6) may be read: *the square of the sine of an angle plus the square of its cosine equals 1;* or, briefly, *sine square plus cosine square equals 1.*

Express formulas (7) and (8) in a similar way.
Formulas (1) to (8) should be memorized.

EXERCISES

1. Given $\cos A = \frac{12}{13}$, find $\sin A$ by formula (6).
2. If $\sin A = \frac{3}{5}$, find $\tan A$ by formulas (6) and (4); use your result to find $\sec A$ by formula (7).
3. If $\sec A = \frac{17}{8}$, find $\tan A$.
4. Write down formulas (1) to (8) from memory and give the proof for each.

8. Functions of 45°, 60°, 30°. The numerical values of the functions of 45°, 60°, 30° can readily be found.

To find the functions of 45°, construct a right angle, C, and, adopting any convenient unit, set off $CA = CB = 1$.

Then, $A = 45°$, $AB = \sqrt{2}$. Why?

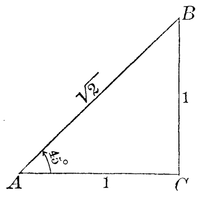

FIG. 3

Therefore,

$$\sin 45° = \frac{1}{\sqrt{2}}, \qquad \cos 45° = \frac{1}{\sqrt{2}},$$

$$\tan 45° = \frac{1}{1} = 1, \qquad \sec 45° = \frac{\sqrt{2}}{1} = \sqrt{2},$$

$$\cot 45° = 1, \qquad \csc 45° = \sqrt{2}.$$

To find the functions of 60° and 30°, construct an equilateral triangle, ABC (Fig. 4), having each side equal to 2. Then each of its angles equals 60°. Draw BD perpendicular to AC. By geometry $AD = 1$, and $\overline{BD}^2 = \overline{AB}^2 - \overline{AD}^2$, from which $BD = \sqrt{3}$. Then, from the right triangle ADB, since

$$\angle A = 60°,$$

we have

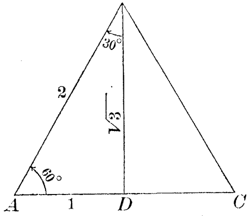

FIG. 4

$$\sin 60° = \frac{\sqrt{3}}{2}, \qquad \cot 60° = \frac{1}{\sqrt{3}},$$

$$\cos 60° = \frac{1}{2}, \qquad \sec 60° = 2,$$

$$\tan 60° = \sqrt{3}, \qquad \csc 60° = \frac{2}{\sqrt{3}}.$$

Again, BD bisects the angle ABC [why?], giving

$$\angle ABD = 30°,$$

and we have

$$\sin 30° = \frac{1}{2}, \qquad \cot 30° = \sqrt{3},$$

$$\cos 30° = \frac{\sqrt{3}}{2}, \qquad \sec 30° = \frac{2}{\sqrt{3}},$$

$$\tan 30° = \frac{1}{\sqrt{3}}, \qquad \csc 30° = 2.$$

The student should familiarize himself with the right triangles in Figs. 3 and 4 and thus be able readily to recall the functions of the angles considered.

EXERCISES

1. Deduce the values of the functions of 45° by using a right triangle having each leg equal to a.

2. Construct an equilateral triangle having each side equal to a, draw the perpendicular as in Fig. 4, and find the functions of 60° and 30°.

3. Verify formulas (1) to (8) for the angles 45°, 60°, 30°. Thus, making $A = 60°$, (6) becomes $\sin^2 60° + \cos^2 60° = 1$.

Substituting the values found in this section,

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1, \quad \text{or} \quad \frac{3}{4} + \frac{1}{4} = 1,$$

which verifies formula (6) for the angle 60°.

9. If one of the functions of an acute angle be given, the angle can be constructed, and the remaining functions can be found. For example, let it be required to construct the acute angle which has $\frac{3}{5}$ for its sine. Construct the right triangle ACB (Fig. 5), having $BC = 3$, $AB = 5$.

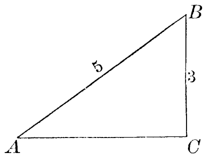

Fig. 5

How is this done?

Then $\angle A$ is the required angle, for $\sin A = \frac{3}{5}$.

To find the remaining functions, we have $\overline{AC}^2 = 25 - 9$, or $AC = 4$. Therefore,

$$\cos A = \tfrac{4}{5}, \quad \tan A = \tfrac{3}{4}, \quad \cot A = \tfrac{4}{3}, \quad \sec A = \tfrac{5}{4}, \quad \csc A = \tfrac{5}{3}.$$

The same results could have been obtained by using for BC and AB any two numbers proportional to 3 and 5.

Try 6 and 10.

EXERCISES

Construct the angle A and find the remaining functions, having given:

1. $\cos A = \frac{5}{13}$. 2. $\csc A = \frac{17}{8}$. 3. $\tan A = \frac{m}{n}$. 4. $\cot A = \frac{5}{12}$.

MISCELLANEOUS EXERCISES

1. Prove $\dfrac{\sec x}{\csc x} = \tan x$.

2. Prove $\dfrac{1 + \tan x}{\sec x} = \sin x + \cos x$.

3. Prove $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$.

4. Which is greater, $\sin 42°$ or $\cos 42°$? $\tan 54°$ or $\cot 54°$?

5. Given $\sec A = \frac{5}{3}$, find the remaining functions by formulas (1) to (8).

6. Given $\tan A = \frac{8}{15}$, find the remaining functions by formulas (1) to (8).

7. Given $\tan A = t = \dfrac{t}{1}$, find the remaining functions.

8. Each side of an equilateral triangle is 14. Find the altitude by using one of the functions of 60°.

9. The hypotenuse of a right triangle is 26 and one angle is 30°. Find the length of each leg, correct to four places of decimals.

10. In a right triangle one angle is 60° and the opposite side is 8. Find the other leg and the hypotenuse.

CHAPTER II

LOGARITHMS

10. If $a^x = n$, the *exponent* x is called the *logarithm* of n to the base a.

So in the equation $7^3 = 343$, the exponent 3 is the logarithm of 343 to the base 7.

Definition. With reference to any base, the *logarithm of a number is the exponent of the power to which the base must be raised to produce the given number.*

To ask, What is the logarithm of 1296 to the base 6? is the same as to ask, To what power must the base 6 be raised in order to produce 1296?

The equation $a^x = n$ is said to be in the *exponential* form; such an equation can always be written in the equivalent *logarithmic* form $\log_a n = x$, which is read: *logarithm of* n *to the base* a *equals* x.

Any positive number except 1 may be used as the base of a system of logarithms.

EXERCISES

1. The base being 4, what are the logarithms of the following numbers? 64, 1024, 4096, 2.

2. What are the values of the following? $\log_5 625$, $\log_3 81$, $\log_7 49$, $\log_9 729$.

3. Express the following equations in the logarithmic form: $8^3 = 512$, $2^5 = 32$, $10^3 = 1000$, $10^{-3} = 0.001$, $10^1 = 10$, $10^0 = 1$, $10^{3.52453} = 3353$, $c^y = p$, $c^{x-y} = q$.

4. Express the following in the exponential form:

$$\log_6 216 = 6, \qquad \log_9 81 = 2, \qquad \log_b m = y,$$

$$\log_a pq = y + z, \qquad \log_{10} 10000 = 4, \qquad \log_{10} 326 = 2.51322.$$

11. Laws of Exponents. Properties of Logarithms. Since *logarithms are exponents,* it may be well to recall the laws of exponents as given in algebra.

We have, for all values of a, x, y, etc.,

$$a^x \cdot a^y = a^{x+y}, \qquad \text{thus} \qquad a^3 \cdot a^4 = a^7;$$

$$a^x \div a^y = a^{x-y}, \qquad \text{"} \qquad a^8 \div a^3 = a^5;$$

$$(a^x)^p = a^{px}, \qquad \text{"} \qquad (a^3)^4 = a^{12};$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}, \qquad \text{"} \qquad a^{\frac{1}{3}} = \sqrt[3]{a};$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p, \quad \text{thus} \quad a^{\frac{3}{4}} = \sqrt[4]{a^3} = (\sqrt[4]{a})^3;$$

$$a^{-s} = \frac{1}{a^s}, \qquad \text{thus} \qquad a^{-5} = \frac{1}{a^5};$$

and, for any value of a except 0,

$$a^0 = 1, \qquad \text{thus} \qquad 13^0 = 1.$$

From these laws are derived the following properties of logarithms:

(a) *The logarithm of the product of two numbers equals the logarithm of the first plus the logarithm of the second.*

For example, $\log 33 = \log 3 + \log 11.$

Proof. Let a be the base and m and n the two numbers, and let

$$\log_a m = x,$$

and

$$\log_a n = y.$$

Then, by Sec. 10, $a^x = m.$

By multiplication,

$$\frac{a^y = n}{a^{x+y} = mn}.$$

Therefore, by Sec. 10, $\log_a mn = x + y.$

For x and y substitute their values, and we have

$$\log_a mn = \log_a m + \log_a n.$$

Corollary. The logarithm of the continued product of any number of factors equals the sum of their logarithms.

Thus, let $m = cd.$

Then, $\log cdn = \log mn = \log m + \log n = \log (cd) + \log n$
$= \log c + \log d + \log n.$

(b) *The logarithm of a fraction equals the logarithm of the numerator minus the logarithm of the denominator.*

Thus, $\log \frac{17}{5} = \log 17 - \log 5.$

Proof. Denote the fraction by $\frac{m}{n}$ and let

$$\log_a m = x,$$

and $$\log_a n = y.$$

Then, $$a^x = m,$$

and $$a^y = n.$$

By division, $$a^{x-y} = \frac{m}{n}.$$

Therefore, by Sec. 10, $\log_a \frac{m}{n} = x - y,$

or $$\log_a \frac{m}{n} = \log_a m - \log_a n.$$

(c) *The logarithm of a power of a number equals the logarithm of the number multiplied by the exponent of the power.*

Thus, $\log 23^5 = 5 \log 23.$

Proof. Let n be the number and p the exponent of the power, and let

$$\log_a n = x.$$

Then, $$a^x = n.$$

Raising each member of this equation to the power p,

$$a^{px} = n^p.$$

Therefore, $$\log_a n^p = px,$$

or, substituting for x its value,

$$\log_a n^p = p \log_a n.$$

(d) *The logarithm of a root of a number equals the logarithm of the number divided by the index of the root.*

Thus, $$\log \sqrt[7]{3963} = \frac{\log 3963}{7}.$$

Proof. Let n be the number and r the index of the root.

Then, $$\log_a \sqrt[r]{n} = \log_a n^{\frac{1}{r}} = \frac{1}{r} \log_a n = \frac{\log_a n}{r}.$$

(e) Since $a^1 = a$, the logarithm of the base is unity.

(f) Since $a^0 = 1$, the logarithm of 1 is 0.

(g) If $a > 1$, $a^{-x} = \frac{1}{a^x} = \frac{1}{\infty} = 0$. $\therefore \log 0 = -\infty$ in any system in which the base is greater than 1.

EXERCISES

Given $\log 2 = 0.30103$, $\log 3 = 0.47712$, $\log 7 = 0.84510$, find the logarithms of the following numbers:

6, 18, 32, 192, $\frac{162}{7}$, $\frac{343}{96}$, 7^6, 36^5, $\sqrt{126}$, $\sqrt[3]{686}$, $\sqrt[17]{486}$.

12. Common System of Logarithms. The common system of logarithms is that in which the base is 10. When no base is indicated, it is understood to be 10. It is easy to find the common logarithms of numbers which are integral powers of 10.

Thus, $10^0 = 1$. $\therefore \log 1 = 0$.

$10^1 = 10$. $\therefore \log 10 = 1$.

$10^2 = 100$. $\therefore \log 100 = 2$.

$10^3 = 1000$. $\therefore \log 1000 = 3$.

$10^4 = 10000$. $\therefore \log 10000 = 4$.

Again, $10^{-1} = \frac{1}{10} = 0.1$. $\therefore \log 0.1 = -1$.

$10^{-2} = \frac{1}{100} = 0.01$. $\therefore \log 0.01 = -2$.

$10^{-3} = \frac{1}{1000} = 0.001$. $\therefore \log 0.001 = -3$.

$10^{-4} = \frac{1}{10000} = 0.0001$. $\therefore \log 0.0001 = -4$.

Consider next the number 257. It lies between 100 and 1000. The logarithm of 257, therefore, lies between 2 and 3. By methods explained in algebra it is found to be, approximately, 2.40993. Similarly, the logarithm of any positive number consists of a whole number plus a decimal. The whole number is called the *characteristic*, and the decimal part is called the *mantissa*.

Table I in the Appendix gives the *mantissas* of the logarithms of numbers from 1 to 10000. It is not necessary to give the *characteristics* in the table, for they may be found by inspection, as follows: Consider any positive number that has, say, three digits to the left of the decimal point, 457.62 for example. It lies between 100 and 1000; its logarithm, therefore, is greater than 2 and less than 3. Hence, $\log 457.62 = 2 +$ a decimal, and the characteristic is 2.

So any number that has four digits to the left of the decimal point, as 5687.037, lies between 1000 and 10,000; its logarithm is therefore greater than 3 and less than 4. Hence, $\log 5687.037 = 3 +$ a decimal, and the characteristic is 3.

For numbers greater than 1, therefore, we see that *the characteristic is positive and 1 less than the number of digits to the left of the decimal point.*

Any positive number less than 1 may be expressed, exactly or approximately, as a decimal, and the characteristic of its logarithm determined as follows: Consider the decimal 0.0068. It is greater than 0.001 and less than 0.01. The logarithm of 0.001 is -3, and the logarithm of 0.01 is -2. Hence, the logarithm of 0.0068 is greater than -3 and less than -2; *i.e.*,

$$\log 0.0068 = -3 \textit{ plus } \text{a decimal},$$

or

$$\log 0.0068 = -2 \textit{ minus } \text{a decimal}.$$

It will be found convenient to adopt the first form, keeping the *decimal* part of the logarithm always *positive*.

Hence, the characteristic of $\log 0.0068$ is -3.

Similarly, 0.0007 is greater than 0.0001 and less than 0.001; and since

$$\log 0.0001 = -4,$$

and

$$\log 0.001 = -3,$$

it follows that log 0.0007 is greater than -4 and less than -3. Hence,

$$\log 0.0007 = -4 \text{ plus a decimal};$$

the characteristic is therefore -4.

We see, then, that if a number is entirely decimal, *the characteristic is negative, and numerically 1 greater than the number of ciphers immediately following the decimal point.*

EXERCISE

Determine the characteristic of the logarithm of each of the following numbers: 368, 42, 4687, 5238.64, 23.1, 13.42, 6.342, 0.0063, 0.0008, 0.81, 0.43.

Next, suppose that the mantissa of the logarithm of 2681 is given to be .42830; by the rule the characteristic is 3.

$$\therefore \log 2681 = 3.42830.$$

Suppose the mantissa of the logarithm of 0.0634 is given to be .80209; by the rule the characteristic is -2. Then we have, keeping the mantissa positive,

$$\log 0.0634 = -2 + .80209,$$

or, more compactly,

$$\log 0.0634 = \bar{2}.80209,$$

where the minus sign applies only to the characteristic.

Instead of the negative characteristics, -1, -2, -3, etc., it is often more convenient to write their equals, $9-10$, $8-10$, $7-10$, etc. Thus,

$$\begin{aligned} \log 0.634 &= \bar{1}.80209 = 9.80209 - 10, \\ \log 0.0634 &= \bar{2}.80209 = 8.80209 - 10, \\ \log 0.00634 &= \bar{3}.80209 = 7.80209 - 10. \end{aligned}$$

By inspection of these results it is seen that if a number is entirely decimal, the characteristic of its logarithm *may be found by subtracting the number of ciphers immediately following the decimal point from 9 and annexing* -10 *to the result.*

13. Logarithms of Numbers having the Same Sequence of Digits. *If two numbers have the same sequence of digits*, i.e., *if they differ only in the position of the decimal point, their logarithms have the same mantissas.*

Thus, log 26.834 and log 2683.4 have the same mantissa; for

$$2683.4 = 26.834 \times 100.$$

Therefore, by Sec. 11, (a),

$$\log 2683.4 = \log 26.834 + \log 100 = \log 26.834 + 2;$$

and adding the whole number 2 to the logarithm of 26.834 does not affect its mantissa.

In general, if two numbers differ only in the position of the decimal point, the greater is equal to the less multiplied by 10, 100, 1000, or some other integral power of 10. The logarithm of the greater, therefore, equals the logarithm of the less increased by 1, 2, 3, or some other whole number, and the addition of a whole number to the logarithm of the less does not affect its mantissa.

The statement that the mantissas are the same for all numbers differing only in the position of the decimal point is true only when the decimal part of every logarithm is kept positive. For, if we consider the number .26834, which has the same sequence of figures as 2683.4, we have

$$\log .26834 = \bar{1}.42868 = -1 + .42868 = -.57132.$$

In the last form the decimal part of the logarithm has not been kept positive, and does *not* agree with the decimal part of log 2683.4. The operations of adding and subtracting logarithms are also made simpler by keeping the decimal part positive.

EXERCISES

1. Given $\log 326.82 = 2.51431$, write down the logarithms of the following numbers: 32682, 32.682, 3.2682, 0.32682, 3268.2, 0.00032682, 326820, 3268200.

2. Write down three numbers, each greater than 1, and three numbers, each less than 1, such that their logarithms have the same mantissas as the logarithm of 5468.

3. Make the decimal part positive in the following logarithms: -2.36823, -1.21656, -3.42857. Thus,

$$\begin{aligned} -2.36823 &= -2 - .36823 = -2 - 1 + 1 - .36823 \\ &= -3 + .63177 = \bar{3}.63177. \end{aligned}$$

4. Make the decimal part negative in the following: $\bar{2}.18734$, $\bar{1}.21683$, $\bar{3}.46827$. Thus,

$$\bar{2}.18734 = -2 + .18734 = -1.81266.$$

5. Determine the base in each of the following cases:

$$\log_a \tfrac{1}{3} = \bar{1}.25;\ \log_b \tfrac{1}{8} = \bar{1}.25;\ \log_c \tfrac{1}{32} = -1.25.$$

MISCELLANEOUS EXERCISES

Read the explanation of Table I in the Appendix, and then use logarithms to perform the operations indicated in the following exercises.

1. Find the value of $\dfrac{1368.34 \times 267}{835}$.

Solution. Let $$x = \frac{1368.34 \times 267}{835}.$$

Then, by Sec. 11, (a) and (b),

$$\log x = \log 1368.34 + \log 267 - \log 835.$$

$$\begin{aligned} \log 1368.34 &= 3.13620 \\ \log 267 &= \underline{2.42651} \\ &\ 5.56271 \\ \log 835 &= \underline{2.92169} \\ \log x &= 2.64102 \\ x &= 437.54. \end{aligned}$$

2. Find the value of $\dfrac{1760 \times \sqrt[15]{86834}}{19.13 \times 27^{\frac{3}{4}}}$.

Solution. Let $x = \dfrac{1760 \times \sqrt[15]{86834}}{19.13 \times 27^{\frac{3}{4}}}$.

By Sec. 11, $\log x = \log 1760 + \dfrac{\log 86834}{15} - (\log 19.13 + \frac{3}{4}\log 27)$.

$$\begin{array}{rl|rl}
\log 1760 = & 3.24551 & & \\
\frac{1}{15}\log 86834 = & \underline{0.32925} & & \\
 & 3.57476 & \log 19.13 = & 1.28171 \\
 & \underline{2.35523} & \frac{3}{4}\log 27 = & \underline{1.07352} \\
\log x = & 1.21953 & & 2.35523 \\
x = & 16.578. & &
\end{array}$$

3. Find the value of x from the equation $x = \frac{183}{3687}$.

Solution. We have $\log x = \log 183 - \log 3687.$

The subtrahend in this case being greater than the minuend, we add 10 to the logarithm of 183 and subtract 10 from it, in order to avoid a negative mantissa, writing

$$\begin{array}{rl}
\log 183 = & 12.26245 - 10 \\
\log 3687 = & \underline{3.56667} \\
\log x = & 8.69578 - 10 \\
x = & 0.04963.
\end{array}$$

4. Find the value of the 7th root of 0.08683.

Solution. Let $x = \sqrt[7]{0.08683}.$

Then, $\log x = \dfrac{\log 0.08683}{7}.$

$$\begin{array}{rl}
\log 0.08683 = & 8.93867 - 10 \\
 & \underline{60 \qquad\qquad 60} \\
7\,) & \underline{68.93867 - 70} \\
\log x = & 9.84838 - 10 \\
x = & 0.7053.
\end{array}$$

Here we add and subtract 60 in order that the quotient obtained by dividing the negative part of log 0.08683 by 7 may be 10.

5. $\frac{263 \times 178}{341}$. *Ans.* 137.29.

6. $\frac{37816}{231 \times 17}$. *Ans.* 9.6298.

7. $\frac{0.82134 \times 276}{0.69473}$. *Ans.* 326.29.

8. $0.01368 \times (1478)^{\frac{3}{2}}$. *Ans.* 777.32.

9. $\sqrt[5]{0.06834}$. *Ans.* 0.5847.

10. $\sqrt[17]{168342}$. *Ans.* 2.0296.

11. $\frac{256.8 \times (19)^{\frac{5}{3}} \times \sqrt{1283}}{1863 \times 23.12}$. *Ans.* 2.8891.

12. $\frac{12.359 \times 0.17954}{0.00238}$. *Ans.* 932.33.

13. $\frac{(8.17)^3 \times (0.236)^2 \times 536}{\sqrt[5]{129347}}$. *Ans.* 1546.3.

14. $\frac{26843.9 \times 0.0063}{187 \times 68492}$. *Ans.* 0.13204.

15. $186 \times 27924 \times 0.00123$. *Ans.* 6388.6.

16. $\frac{12963}{1.3 \times 1.8 \times 2.1}$. *Ans.* 2638.1.

17. $\frac{183}{12970}$. *Ans.* 0.014109.

18. $\frac{237}{46.1 \times 963}$. *Ans.* 0.0053385.

19. $\sqrt[7]{0.0063} \times (52.61)^{\frac{5}{3}}$. *Ans.* 358.15.

CHAPTER III

SOLUTION OF RIGHT TRIANGLES

14. A right triangle can be solved if, besides the right angle, any two parts be given, provided one of them be a side.

Let the right triangle be ACB (Fig. 6). We have

$$\sin A = \frac{a}{c}, \qquad \cos A = \frac{b}{c}, \qquad \tan A = \frac{a}{b}.$$

Since $A + B = 90°$, if either A or B be one of the given parts, the other acute angle is at once known.

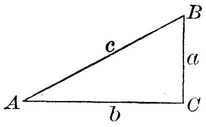

FIG. 6

We may, therefore, in any case select from the three equations above one which contains only one unknown quantity and solve for that unknown. Then select another which contains only one unknown quantity and solve as before.

Before proceeding farther the student should read the explanation of the Tables of Trigonometric Functions in the Appendix.

EXAMPLES

1. Given $A = 38°$, $c = 187$, to find B, a, b.

Solution.
$$B = 90° - A = 52°.$$

$$\sin A = \frac{a}{c},$$

or
$$\frac{a}{187} = \sin 38°.$$

$$\therefore a = 187 \sin 38°.$$

$$\therefore \log a = \log 187 + \log \sin 38°.$$

$$\begin{aligned} \log 187 &= 2.27184 \\ \log \sin 38^\circ &= \underline{9.78934 - 10} \\ \log a &= 2.06118 \\ a &= 115.127. \end{aligned}$$

We have also
$$\frac{b}{187} = \cos 38^\circ,$$
or
$$\begin{aligned} b &= 187 \cos 38^\circ. \\ \log b &= \log 187 + \log \cos 38^\circ. \\ \log 187 &= 2.27184 \\ \log \cos 38^\circ &= \underline{9.89653 - 10} \\ \log b &= 2.16837 \\ b &= 147.36. \end{aligned}$$

The accuracy of the work should be tested by solving by other formulas, or by seeing whether the parts of the triangle as found satisfy other relations which must subsist if the solution is correct. Any formula involving one or more of the required parts not used in the solution may be used as a *check formula*. Thus, having used in the solution two of the three equations above, the third may be used as a check formula. Then a and b ought to satisfy the relation $\tan A = \frac{a}{b}$, from which

$$\begin{aligned} \log \tan A &= \log a - \log b. \\ \log a &= 12.06118 - 10 \\ \log b &= \underline{\ 2.16837\qquad\ } \\ \therefore \log \tan A &= \ 9.89281 - 10 \end{aligned}$$

and from the table we find

$$\log \tan A = \log \tan 38^\circ = 9.89281 - 10.$$

The two results agree.

Another check formula is $a^2 = c^2 - b^2$,

or
$$a^2 = (c + b)(c - b).$$

Applying logarithms,
$$\begin{aligned} 2 \log a &= \log (c + b) + \log (c - b) \\ &= \log 334.357 + \log 39.643. \end{aligned}$$

$$\begin{aligned} \log 334.357 &= 2.51421 \\ \log 39.643 &= \underline{1.59816} \\ 2 \log a &= 4.12237 \\ \log a &= 2.06118. \end{aligned}$$

This agrees with the value of $\log a$ as determined above. In some cases there may be a slight discrepancy, due to the fact that the values given in the tables are only approximately correct; correct, that is, only as far as five places of decimals.

2. Given $A = 67°$, $b = 148$, to find B, a, c.

$$B = 90° - A = 23°,$$

$$\frac{a}{b} = \tan A,$$

or

$$\frac{a}{148} = \tan 67°.$$

$$\therefore \log a = \log 148 + \log \tan 67°.$$

$$\begin{aligned} \log 148 &= 2.17026 \\ \log \tan 67° &= \underline{0.37215} \\ \log a &= 2.54241 \\ a &= 348.67. \end{aligned}$$

To find c,

$$\cos A = \frac{b}{c},$$

or

$$\cos 67° = \frac{148}{c}.$$

$$\begin{aligned} \log \cos 67° &= \log 148 - \log c. \\ \log c &= \log 148 + \log \cos 67°. \end{aligned}$$

$$\begin{aligned} \log 148 &= 12.17026 - 10 \\ \log \cos 67° &= \underline{\ \ 9.59188 - 10} \\ \log c &= \ \ 2.57838 \\ c &= 378.77. \end{aligned}$$

Checks. $\sin A = \frac{a}{c}$, $a^2 = (c + b)(c - b)$.

Apply these tests.

3. Given $a = 327$, $c = 578$, to find A, B, b.

$$\sin A = \frac{a}{c},$$

or

$$\sin A = \tfrac{327}{578}.$$

$$\log \sin A = \log 327 - \log 578.$$

$$\begin{aligned} \log 327 &= 12.51455 - 10 \\ \log 578 &= \underline{\ \ 2.76193\qquad\ } \\ \log \sin A &= \ \ 9.75262 - 10 \\ A &= 34° \, 27' \, 13''. \\ B &= 90° - A = 55° \, 32' \, 47''. \end{aligned}$$

To find b, we have

$$\frac{b}{c} = \cos A.$$

$$\therefore \log b = \log c + \log \cos A.$$

$$\begin{aligned} \log 578 &= 2.76193 \\ \log \cos 34^\circ\, 27'\, 13'' &= \underline{9.91623 - 10} \\ \log b &= 2.67816 \\ b &= 476.61. \end{aligned}$$

Checks. $\tan A = \frac{a}{b}$, $b^2 = (c + a)(c - a)$, or $b^2 = 905 \times 251$.

$$\begin{aligned} \therefore 2 \log b &= \log 905 + \log 251. \\ \log 905 &= 2.95665 \\ \log 251 &= \underline{2.39967} \\ 2 \log b &= 5.35632 \end{aligned}$$

or
$$\log b = 2.67816, \text{ as above.}$$

Apply the other test.

4. Given $a = 26$, $b = 37$, to find A, B, c.

We have
$$\tan A = \frac{a}{b} = \frac{26}{37}.$$

$$\begin{aligned} \therefore \log \tan A &= \log 26 - \log 37. \\ \log 26 &= 11.41497 - 10 \\ \log 37 &= \underline{\ \ 1.56820\qquad\ } \\ \log \tan A &= \ \ 9.84677 - 10 \\ A &= 35^\circ\, 5'\, 44''. \\ B &= 90^\circ - A = 54^\circ\, 54'\, 16''. \end{aligned}$$

To find c, we have
$$\frac{a}{c} = \sin A.$$

$$\therefore \log a - \log c = \log \sin A,$$

or
$$\log c = \log a - \log \sin A.$$

$$\begin{aligned} \log 26 &= 11.41497 - 10 \\ \log \sin 35^\circ\, 5'\, 44'' &= \underline{\ \ 9.75962 - 10} \\ \log c &= \ \ 1.65535 \\ \therefore c &= 45.22. \end{aligned}$$

Check. $\cos A = \frac{b}{c}$.

The student will find it convenient to remember the following relations obtained from the formulas at the beginning of this section:

$$a = c \sin A,\ \ b = c \cos A,\ \ a = b \tan A.$$

15. The solution may be effected in any case, also, without the use of logarithms.

EXAMPLE

Given $A = 32°$, $c = 200$, to find a, b, B.

We have
$$\frac{a}{c} = \sin A,$$

or
$$\frac{a}{200} = \sin 32°.$$
$$\therefore a = 200 \times \sin 32°.$$

From the Table of Sines, we find
$$\sin 32° = 0.5299.$$
$$\therefore a = 0.5299 \times 200,$$

or
$$a = 105.98.$$

Also,
$$b = 200 \times \cos 32° = 200 \times 0.8480,$$

or
$$b = 169.60.$$
$$B = 90° - A = 58°.$$

EXERCISES

Solve the following right triangles, checking the results when the answers are not given:

1. Given $a = 43$, $c = 58$, to find A, B, b.
   Ans. $A = 47° 50' 55''$, $B = 42° 9' 5''$, $b = 38.924$.
2. Given $a = 31$, $b = 98$, to find A, B, c.
   Ans. $A = 17° 33' 12''$, $B = 62° 16' 48''$, $c = 102.79$.
3. Given $A = 30° 12'$, $c = 288.6$.
   Ans. $a = 145.173$, $b = 249.43$.
4. Given $a = 146.83$, $c = 197.14$.
   Ans. $A = 48° 8' 33''$, $b = 131.55$.
5. Given $a = 0.1442$, $b = 0.1647$.
   Ans. $A = 41° 12' 14''$, $c = 0.21891$.
6. Given $A = 68° 23'$, $a = 223.95$.
   Ans. $b = 88.744$, $c = 240.78$.
7. Given $b = 168.92$, $c = 289.64$.
   Ans. $B = 35° 40' 33''$, $a = 235.28$.

8. Given $B = 64° 38'$, $b = 38.21$.
Ans. $c = 42.287$, $a = 18.116$.

9. Given $a = 42$, $c = 145$.
Ans. $A = 16° 50' 14''$, $b = 138.78$.

10. Given $b = 12$, $c = 13$.
Ans. $A = 67° 22' 48''$, $a = 5$.

11. Given $b = 35$, $c = 37$.
12. Given $A = 63° 12'$, $c = 128$.
13. Given $B = 47° 17'$, $a = 23.48$.
14. Given $A = 27° 36'$, $b = 468$.
15. Given $a = 123$, $b = 178$.

16. Definitions. In Fig. 7, let A and B be any two points of which B is at the greater height above the sea level. In the vertical plane through A and B draw the horizontal lines AC, BD. Then, the angle CAB is called the *angle of elevation* of B with reference to A; and the angle DBA is called the *angle of depression* of A with reference to B.

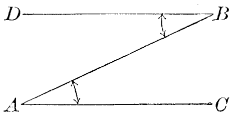

FIG. 7

The angle of elevation of a heavenly body is called its *altitude*.

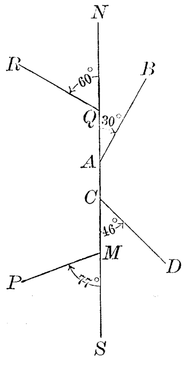

FIG. 8

The acute angle which a line on the earth's surface makes with a meridian (*i.e.*, a north-and-south line) drawn through its initial point determines the *bearing* of the line. Thus, in Fig. 8, NS is a meridian; the bearing of AB is N. 30° E., which is read *north 30° east;* the bearing of CD is S. 46° E.; of MP, S. 77° W.; of QR, N. 60° W.

Heights and Distances. Many practical problems in which heights and distances are to be found may be solved by the use of right triangles if we regard the earth's surface as a plane. Examples will be found at the end of this chapter.

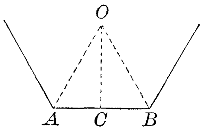

FIG. 9

17. Area of a Regular Polygon. Let AB (Fig. 9) be a side of a regular polygon of n sides; O the center and r $(= OC)$ the radius of the inscribed circle, S the area of the polygon; to find the value of S in terms of n and r.

Now, $\angle AOB = \dfrac{360°}{n}$. $\therefore \angle AOC = \dfrac{180°}{n}$.

Also, area of $\triangle AOB = OC \times AC = r \times r \tan \dfrac{180°}{n}$;
and as there are n such triangles, we have

$$S = nr^2 \tan \frac{180°}{n}.$$

EXERCISES

1. Express S in terms of a and n, where $a = AB$.
2. Find the perimeter in terms of r and n.

Find in terms of r the areas of the following regular polygons:

3. Triangle.
4. Square.
5. Hexagon.
6. Octagon.
7. Dodecagon.

MISCELLANEOUS EXERCISES

1. The bounding lines of a tennis court include a rectangle 78 feet by 36 feet. Find the angle made by a diagonal and the shorter side. *Ans.* 65° 13′ 29″.

2. Find the length of the shadow cast by a vertical rod 12 feet long when the altitude of the sun is 78° 42′. *Ans.* 2.397 feet.

3. What is the bearing of the line along which a man must go in order to reach a point 4 miles east and 9 miles north of his starting point? *Ans.* N. 23° 57′ 46″ E.

4. What is the radius of a circle in which a chord 100 feet long subtends an angle of 1° at the center? *Ans.* 5729.71 feet.

5. What is the radius of a circle when tangents AI, BI, 300 feet long, form an angle $AIB = 126°$? *Ans.* 588.78 feet.

Solve without logarithms.

6. The radius of a circle is 2864.93 feet. A chord, AB, and a tangent, AC, are each 100 feet. Find the length of BC. *Ans.* 1.747 feet.

7. The radius of a circle is 24.96 feet. Find the perimeter of the regular inscribed pentagon. *Ans.* 146.61 feet.

8. What is the angle of elevation of the top of a tower 146 feet high from a point 325 feet from its foot? *Ans.* 24° 11′ 26″.

Solve without logarithms.

9. A train started at 10 : 15 a.m. and proceeded on a straight track at a uniform rate of 34 miles per hour. At 11 : 45 a.m. it had reached a point 26 miles further west than the starting point. Find the bearing of the track. *Ans.* N. 30° 39′ 3″ W., or S. 30° 39′ 3″ W.

10. From a point 80 feet from the base of a tower the angle of elevation of an object in one of the windows is 50°, and the angle of elevation of the top of the tower is 58° 10′. Find the distance from the object to the top of the tower. *Ans.* 33.512 feet.

Solve without logarithms.

11. The radius of a circle is 50 feet. Find the area of the regular inscribed pentagon. *Ans.* 5944.1 square feet.

12. A regular octagon is inscribed in a circle whose radius is 49.92 feet. Find the length of the side and of the apothem.
Ans. Side, 38.21 feet; apothem, 46.12 feet.

13. Find the area of the octagon in Example 12.
Ans. 7048.33 square feet.

14. A chord 16 inches long subtends at the center of a circle an angle of 46° 24′. Find the side of the inscribed square. *Ans.* 28.72 inches.

15. At a point 50 feet from the base of a tower the angle of elevation of the top was found, by the use of a transit, to be 68° 18′. If the height of the instrument was 4.75 feet, find the height of the tower.
Ans. 130.39 feet.

16. The radius of a circle is 1910.1 feet. Tangents at the points A and B meet at D. If $AB = 432$ feet, find the angle DAB.
Ans. 6° 29′ 35″.

17. The radius of a circle is 1146.28 feet. Tangents at A and B meet at T. If $AB = 800$ feet, find the angle TAB and the length of AT.
Ans. $\angle TAB = 20° 25' 25''$; $AT = 426.84$.

18. The angle of elevation of a point, P, from a point, B', due west of P was found to be B, and from a point, A', due south of B' it was found to be A. If $A'B' = c$, show that the height of P was

$$\frac{c}{\sqrt{\cot^2 A - \cot^2 B}}.$$

CHAPTER IV

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

18. Coördinates. Let XX', YY' (Fig. 10) be two lines intersecting at right angles at O; P, any point in their plane. Draw PM perpendicular to XX'. Then, OM and MP are called the *coördinates* of the point P; OM is called the *abscissa*, and MP the *ordinate.* XX' is called the *x-axis*, YY' the *y-axis*. Their intersection, O, is called the *origin* of coördinates. The following conventions have been adopted: *abscissas* measured to the right of the y-axis are positive; those to the left, negative: *ordinates* measured upward from the x-axis are positive; downward, negative. Abscissas are usually denoted by the letter x, ordinates by y. With these conventions, any point in a plane can be located with reference to two rectangular axes in the plane if its coördinates are given. Thus, the equations $x = 3$, $y = 4$ determine the point whose abscissa is 3 and whose ordinate is 4. This point is referred to as the point $(3, 4)$, or, in general, as the point (x, y), the abscissa being written first.

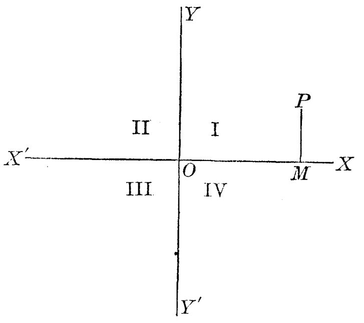

FIG. 10

The line joining P with the origin is called the *distance* of P, and *is always considered positive.* The axes divide the angular magnitude about the origin into four *quadrants*, designated as the first, second, third, fourth, as indicated by the numerals in Fig. 5.

EXERCISES

1. Adopt a convenient scale and construct the following points: $x = 4, y = 5$; $x = -3, y = 2$; $x = 2, y = -7$.

2. Construct the following points: $(3, 5)$, $(-2, -3)$, $(3, 0)$, $(0, -4)$, $(-3, 7)$.

3. What is the ordinate of any point on the x-axis? the abscissa of any point on the y-axis? What are the coördinates of the origin? Write down the coördinates of three points on the x-axis.

4. Find the *distance* for each of these points: $(3, 4)$, $(-6, 8)$, $(17, -8)$.

19. Angles of any Magnitude. In Chapter I only acute angles were considered, and the definitions of the trigonometric functions there given have no meaning except with reference to acute angles. We shall now show how angles of any magnitude may be generated, and shall distinguish between positive and negative angles and give definitions of the trigonometric functions that will apply to any angle.

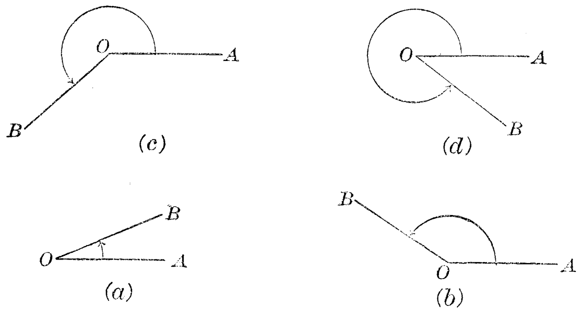

FIG. 11

If a line, OB (Fig. 11), coinciding at first with OA, be made to rotate about O as a pivot, as indicated by the arrow, until it comes into the position OB, it is said to describe, or generate, or turn through, the angle *AOB*. *OA* is called the *initial* line, *OB* the *terminal* line, of the angle. If *OB* revolves until it comes again into its original position, *OA*, the angle described is called a *perigon*. Half a perigon is called a *straight angle;* one three-hundred-and-sixtieth of a perigon is called a *degree*. If *OB*, after describing the angle *AOB*, be made to turn in the same way still further through one or more perigons, it will come back to the same position, *OB*. In this way may be generated angles of any magnitude. Thus, if the angle indicated by arrow 1 (Fig. 12) be 40°, the angle indicated by arrow 2 is 40° + 360°, or 400°, the magnitude of the angle in any case depending upon the amount of turning.

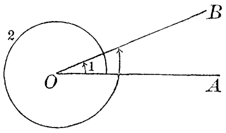

FIG. 12

20. Negative Angles. In Figs. 11 and 12 we have supposed *OB* to rotate in a manner opposite to the hands of a clock (counter-clockwise). Angles may be generated also by causing *OB* to turn clockwise, as indicated by the arrow in Fig. 13. Here we adopt a convention similar to that in Sec. 18, *viz.:* angles generated by a *counter-clockwise* rotation are positive; angles generated by a *clockwise* rotation are negative.

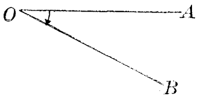

FIG. 13

Questions. How many degrees are there in the negative angle described by the minute hand of a clock in five minutes? in three hours and twenty-five minutes? How long does it take the minute hand to describe the angle −210°? What negative angles give the same position of the terminal line as the positive angle 150°?

21. General Definitions of the Trigonometric Functions. Let the initial line of the angle XOB (Fig. 14) lie along the x-axis on the positive side and let P be any point in the terminal line; denote the angle XOB by A, and the abscissa, ordinate, and distance of P by x, y, r respectively. Then, for every position of OB, the functions of the angle A are defined as follows:

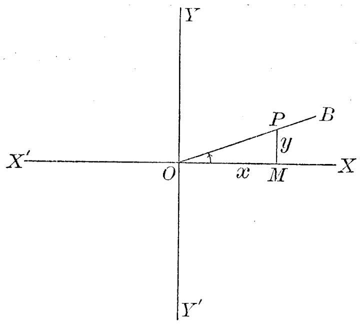

FIG. 14

1. sine of A equals $\frac{\text{ordinate}}{\text{distance}}$; written $\sin A = \frac{y}{r}\cdot$

2. cosine “ “ “ $\frac{\text{abscissa}}{\text{distance}}$; “ $\cos A = \frac{x}{r}\cdot$

3. tangent “ “ “ $\frac{\text{ordinate}}{\text{abscissa}}$; “ $\tan A = \frac{y}{x}\cdot$

4. cotangent “ “ “ $\frac{\text{abscissa}}{\text{ordinate}}$; “ $\cot A = \frac{x}{y}\cdot$

5. secant “ “ “ $\frac{\text{distance}}{\text{abscissa}}$; “ $\sec A = \frac{r}{x}\cdot$

6. cosecant “ “ “ $\frac{\text{distance}}{\text{ordinate}}$; “ $\csc A = \frac{r}{y}\cdot$

The words *ordinate*, *abscissa*, *distance*, as used above, mean the ordinate, abscissa, and distance of *any point* P on the terminal line of the angle considered.

Let the abscissa of P contain a units of length, the ordinate b units, and the distance r units; then, if OB falls in the first quadrant, as in Fig. 14, $x = +a$, $y = +b$, and from the definitions we have

$$\sin A = \frac{\text{ordinate}}{\text{distance}} = \frac{b}{r}, \qquad \cot A = \frac{\text{abscissa}}{\text{ordinate}} = \frac{a}{b},$$

$$\cos A = \frac{\text{abscissa}}{\text{distance}} = \frac{a}{r}, \qquad \sec A = \frac{\text{distance}}{\text{abscissa}} = \frac{r}{a},$$

$$\tan A = \frac{\text{ordinate}}{\text{abscissa}} = \frac{b}{a}, \qquad \csc A = \frac{\text{distance}}{\text{ordinate}} = \frac{r}{b}.$$

If the terminal line falls in the second quadrant (Fig. 15), $x = -a$, $y = +b$, and we have, remembering that the distance is always positive,

$$\sin A = \frac{b}{r}, \qquad \cot A = -\frac{a}{b},$$

$$\cos A = \frac{-a}{r} = -\frac{a}{r}, \qquad \sec A = -\frac{r}{a},$$

$$\tan A = -\frac{b}{a}, \qquad \csc A = \frac{r}{b}.$$

For an angle in the third quadrant, *i.e.*, an angle whose terminal line lies in the third quadrant,

$$x = -a, \qquad y = -b.$$

For an angle in the fourth quadrant,

$$x = +a, \qquad y = -b.$$

Let the student construct a figure for each of these cases and write down the values of the functions in terms of a and b.

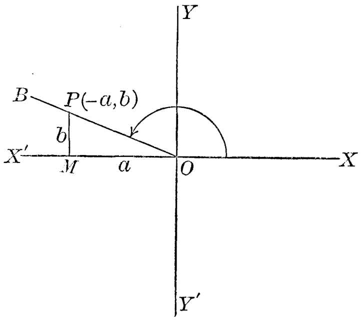

Fig. 15

When the angle is in the first quadrant, it is readily seen that the definitions given above agree with those previously given in Sec. 4. The latter definitions are general, and include those of Sec. 4 as a special case.

22. The formulas of Sec. 4 are true also for any angle. Thus, from 1 and 2 of Sec. 21,

$$\sin A = \frac{1}{\csc A}, \quad \csc A = \frac{1}{\sin A}; \tag{1}$$

from 2 and 5, $$\cos A = \frac{1}{\sec A}, \quad \sec A = \frac{1}{\cos A}; \tag{2}$$

from 3 and 4, $$\tan A = \frac{1}{\cot A}, \quad \cot A = \frac{1}{\tan a}. \tag{3}$$

Also, for any position of the revolving line,

$$\frac{\sin A}{\cos A} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x} = \tan A. \tag{4}$$

Similarly, prove formula (5).

Again, in whatever quadrant the terminal line may lie, the abscissa, ordinate, and distance of P form a right triangle, giving

$$x^2 + y^2 = r^2.$$

Dividing each member of this equation by r^2,

$$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1, \quad \text{or} \quad \sin^2 A + \cos^2 A = 1. \tag{6}$$

Prove formulas (7) and (8) by dividing in turn by x^2 and y^2.

23. Algebraic Signs of the Functions. Let x and y be the coördinates of any point in the terminal line of an angle. Then, as in Sec. 13, it will be seen that if the angle is between 0° and 90°, *i.e.*, if the terminal line falls in the first quadrant, x and y are both positive; and since r is always positive, all the functions of the angle are positive.

If the angle is between 90° and 180°, the terminal line falls in the second quadrant, and x is negative and y positive. Therefore, the sine and the cosecant are positive, while all the other functions are negative.

If the angle is between 180° and 270°, the terminal line is in the third quadrant, and x and y are both negative; so that the sine, cosine, secant, and cosecant are negative, while the tangent and cotangent are positive.

If the angle is between 270° and 360°, the terminal line falls in the fourth quadrant, and x is positive and y negative; so that the sine, cosecant, tangent, and cotangent are negative, while the cosine and secant are positive.

The student should construct the figure to illustrate each case, as in Sec. 13.

These results may be arranged in a tabular form as below:

QUADRANT	SIN	COS	TAN	COT	SEC	CSC
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

24. Functions of 0°, 90°, 180°, 270°, 360°. Changes in the Magnitude of the Functions. Let the variable angle A (Fig. 16)

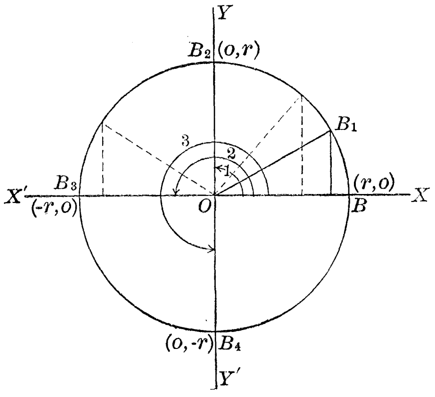

FIG. 16

be generated by a line, OB, of fixed length, r, lying at first along OX and revolving counter-clockwise.

The student should draw OB in several positions in each of the quadrants.

1. *Functions of 0°.* When $A = 0°$, OB lies along the x-axis, and the coördinates of B are $x = r$, $y = 0$. Hence,

$$\sin 0° = \frac{\text{ordinate}}{\text{distance}} = \frac{0}{r} = 0, \qquad \cot 0° = \frac{\text{abscissa}}{\text{ordinate}} = \frac{r}{0} = \infty,$$

$$\cos 0° = \frac{\text{abscissa}}{\text{distance}} = \frac{r}{r} = 1, \qquad \sec 0° = \frac{\text{distance}}{\text{abscissa}} = \frac{r}{r} = 1,$$

$$\tan 0° = \frac{\text{ordinate}}{\text{abscissa}} = \frac{0}{r} = 0, \qquad \csc 0° = \frac{\text{distance}}{\text{ordinate}} = \frac{r}{0} = \infty.$$

2. As A increases, y, the ordinate of B, increases; its abscissa, x, decreases, r remaining constant. Hence, in the first quadrant, $\sin A$, $\frac{y}{r}$, increases but remains less than 1; $\cos A$, $\frac{x}{r}$, decreases and remains less than 1; $\tan A$, $\frac{y}{x}$, increases, being 0 when $A = 0°$, then a number less than 1, then 1 (when $A = 45°$), then a number greater than 1, which can have any value, however great, since x gets nearer and nearer to 0 as A approaches 90°.

3. *Functions of 90°.* When OB has turned through 90° (arrow 1), coming into the position OB_2, $x = 0$, $y = r$, and we have

$$\sin 90° = \frac{y}{r} = \frac{r}{r} = 1, \qquad \cot 90° = \frac{x}{y} = \frac{0}{r} = 0,$$

$$\cos 90° = \frac{x}{r} = \frac{0}{r} = 0, \qquad \sec 90° = \frac{r}{x} = \frac{r}{0} = \infty,$$

$${}^{*}\tan 90° = \frac{y}{x} = \frac{r}{0} = \infty, \qquad \csc 90° = \frac{r}{y} = \frac{r}{r} = 1.$$

---

[*] As shown in 2, if A, being first less than 90°, be made to approach 90°, $\tan A$ is *positive* and gets greater and greater, and can be made greater than any assigned number, however great; this is expressed by writing $\tan 90° = +\infty$. But if A be greater than 90°, and be made to approach 90° by a clockwise

4. As A passes through the value 90°, x becomes negative and increases* numerically while y is positive and decreases. Hence, in the second quadrant,

sin A, $\frac{y}{r}$, is positive and decreases;
cos A, $\frac{x}{r}$, is negative and increases;
tan A is negative and decreases;
cot A is negative and increases;
sec A is negative and decreases;
csc A is positive and increases.

5. *Functions of 180°.* When OB has turned through 180° (arrow 2), $x = -r$, $y = 0$. Hence,

$$\sin 180° = \frac{0}{r} = 0, \qquad \cot 180° = \frac{-r}{0} = \infty,$$

$$\cos 180° = \frac{-r}{r} = -1, \qquad \sec 180° = \frac{r}{-r} = -1,$$

$$\tan 180° = \frac{0}{-r} = 0, \qquad \csc 180° = \frac{r}{0} = \infty.$$

6. When A passes through 180°, x remains negative and decreases, y becomes negative and increases. Hence, in the third quadrant,

sin A, $\frac{y}{r}$, is negative and increases;
cos A, $\frac{x}{r}$, is negative and decreases;
tan A, $\frac{y}{x}$, is positive and increases.

---

rotation, tan A is *negative*, and can be made numerically greater than any assigned number, however great; this is expressed by writing $\tan 90° = -\infty$. Hence, when it is not known whether A becomes 90° from having been less than 90° or from having been greater than 90°, there is an ambiguity in the sign of tan 90°, and we must write $\tan 90° = \pm\infty$. Similarly, whenever the symbol ∞ occurs in this discussion it should have the double sign.

[*] In this discussion the words *increase* and *decrease* refer to *numerical* increase and decrease.

7. When $A = 270°$, $x = 0$, $y = -r$. Hence,

$$\sin 270° = \frac{-r}{r} = -1, \qquad \cot 270° = \frac{0}{-r} = 0,$$

$$\cos 270° = \frac{0}{r} = 0, \qquad \sec 270° = \frac{r}{0} = \infty,$$

$$\tan 270° = \frac{-r}{0} = \infty, \qquad \csc 270° = \frac{r}{-r} = -1.$$

8. When A passes through 270°, x becomes positive and increases, while y is negative and decreases. Hence,

$\sin A$, $\frac{y}{r}$, is negative and decreases;

$\cos A$, $\frac{x}{r}$, is positive and increases;

$\tan A$, $\frac{y}{x}$, is negative and decreases.

When $A = 360°$, OB has returned to its original position and the functions of 360° are the same as those of 0°.

In the same manner, the changes in the values of $\cot A$, $\sec A$, $\csc A$ may be traced for the several quadrants.

Questions. Which of the functions increase as A increases from 0° to 90°? Which decrease? As A increases from 90° to 180° which of the functions increase and which decrease? What is the greatest value of $\sin A$? of $\cos A$? the least values?

EXERCISE

Trace the changes in the value of $\sin A$ as A varies from 0° to 360°. Trace also the changes in the values of $\cos A$ and $\tan A$ separately.

25. Given one Function, to find the Others. Suppose we have given $\cot A = \frac{60}{11}$. Since cotangent equals abscissa divided by ordinate, let us construct the point B (Fig. 17) whose coördinates are 60 and 11. Draw OB; then the angle XOB has its cotangent equal to $\frac{60}{11}$. But $\frac{60}{11} = \frac{-60}{-11}$, and by constructing the point $B'(-60, -11)$ we determine the angle XOB', which also has $\frac{60}{11}$ for its cotangent. Similarly, corresponding to a given value of any one function, there are always two positive angles, each less than 360°.

Since $OB = OB' = \sqrt{60^2 + 11^2} = 61$, all the functions of XOB and XOB' can be written down. Thus, denoting XOB and XOB' by A and A' respectively,

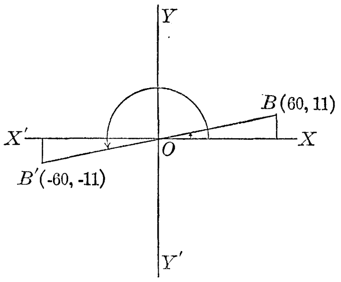

FIG. 17

$\sin A = \frac{11}{61}$,
$\cos A = \frac{60}{61}$,
$\tan A = \frac{11}{60}$,
$\cot A = \frac{60}{11}$,

$\sec A = \frac{61}{60}$,
$\csc A = \frac{61}{11}$,
$\sin A' = -\frac{11}{61}$,
$\cos A' = -\frac{60}{61}$,

$\tan A' = \frac{11}{60}$,
$\cot A' = \frac{60}{11}$,
$\sec A' = -\frac{61}{60}$,
$\csc A' = -\frac{61}{11}$.

EXERCISES

Construct the two positive angles less than 360° corresponding to the given function in each of the following cases. Find also the remaining functions of each angle.

1. $\sin A = \frac{3}{5}$.
2. $\csc A = -\frac{17}{8}$.
3. $\tan A = \frac{9}{40}$.
4. $\cos A = -\frac{24}{25}$.
5. $\sin A = \frac{1}{2 \cdot \sqrt{2}}$.
6. $\tan A = \frac{\sqrt{5}}{2}$.

26. If OB (Fig. 15) be turned, counter-clockwise or clockwise, through 360° or any multiple of 360°, it will again occupy the position shown in the figure. Hence, *if an angle be increased or diminished by 360° or any multiple of 360°, its functions are unchanged.*

EXERCISES

1. $\tan 496^\circ = \tan(360^\circ + 196^\circ) = \tan 196^\circ$.
2. $\sin 1263^\circ = \sin(3 \times 360^\circ + 183^\circ) = \sin 183^\circ$.
3. Prove $\cos 762^\circ = \cos 42^\circ$.
4. Express $\sec 780^\circ\ 13'$ as the secant of an acute angle.
5. Name three angles that have the same functions as $46^\circ\ 24'$.

27. Functions of Negative Angles. In each of the accompanying figures let the line OC turn counter-clockwise, generating an angle $XOC = A$, indicated by arrow 1. Let OC' turn clockwise through an angle equal in magnitude to XOC, indicated by arrow 2. Then $\angle XOC' = -A$.

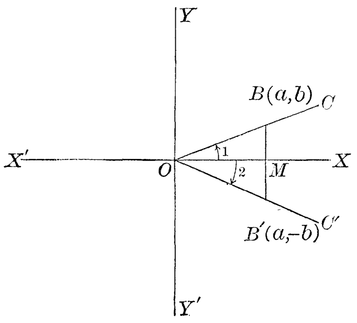

FIG. 18

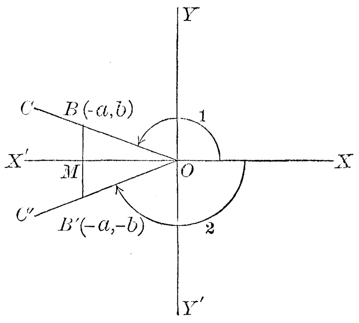

FIG. 19

From any point, B, in OC, draw BB' perpendicular to XX', meeting OC' in B'. In the right triangles MOB, MOB' the side OM is common, and the acute angles MOB, MOB' are equal in magnitude. Hence, the triangles are congruent (*i.e.*, one could be made to coincide with the other). Let the number of linear units in OM, MB be a and b respectively, and let $OB = OB' = r$. Then, giving to a and b their proper signs according to the quadrants in which OB and OB' lie, the coördinates of B and B' will be as shown in the several figures. Confining our attention to the case in which OB is in the second quadrant, OB' in the third (Fig. 19), we have

$$\sin(-A) = -\frac{b}{r} = -\sin A, \qquad \cot(-A) = \frac{a}{b} = -\cot A,$$

$$\cos(-A) = -\frac{a}{r} = \cos A, \qquad \sec(-A) = -\frac{r}{a} = \sec A,$$

$$\tan(-A) = \frac{-b}{-a} = \frac{b}{a} = -\tan A, \qquad \csc(-A) = -\frac{r}{b} = -\csc A.$$

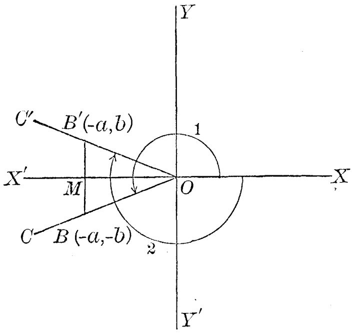

FIG. 20

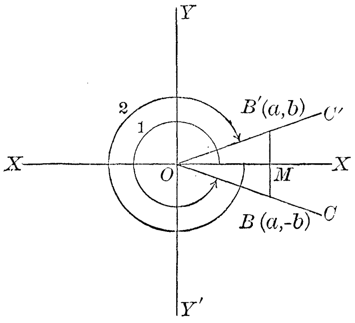

FIG. 21

The same relations are true for all cases. Let the student prove them for each figure.

28. Functions of 90° + A in Terms of Functions of A. In each of the accompanying figures let the angle XOC, marked by arrow 1, be denoted by A. If OC' be drawn perpendicular to OC, the angle XOC', marked by arrow 2, will be $90° + A$. Take $OB = OB' = r$ and draw BM, $B'M'$ perpendicular to XX'. In the right triangles OMB, $OM'B'$, $OB = OB'$ and the acute angles BOM and $OB'M'$ are equal, having their sides perpendicular each to each. Hence, the triangles are congruent. Denoting the absolute values of OM, MB by a and b, as in

Sec. 27, the coördinates of B and B' will be as shown in the several figures. Then, for the case where A falls in the third quadrant and $90 + A$ in the fourth (Fig. 24), we have

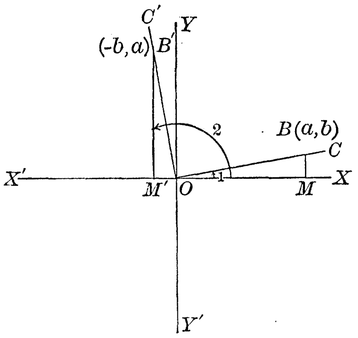

FIG. 22

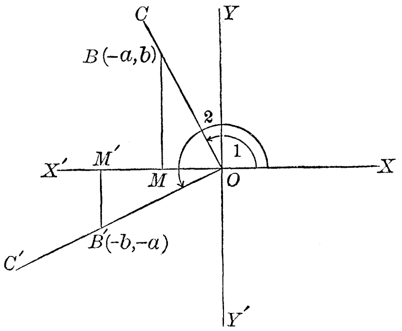

FIG. 26

$$\sin(90^\circ + A) = -\frac{a}{r} = \cos A, \qquad \cot(90^\circ + A) = -\frac{b}{a} = -\tan A,$$

$$\cos(90^\circ + A) = \frac{b}{r} = -\sin A, \qquad \sec(90^\circ + A) = \frac{r}{b} = -\csc A,$$

$$\tan(90^\circ + A) = -\frac{a}{b} = -\cot A, \qquad \csc(90^\circ + A) = -\frac{r}{a} = \sec A.$$

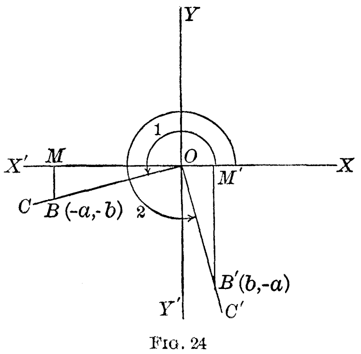

FIG. 24

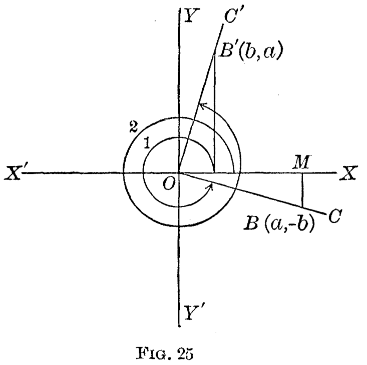

FIG. 25

EXERCISE

Prove the above relations for each of the other cases.

From these results it follows that if an angle be diminished by 90° the sine becomes cosine, cosine becomes negative sine, tangent becomes negative cotangent, cotangent becomes negative tangent, secant becomes negative cosecant, cosecant becomes secant.

29. Functions of 90° — A. The figures of the preceding section may be used to express the functions of $90° - A$ in terms of functions of A. In each of those figures let $\angle XOC' = A$, then $\angle XOC = A - 90°$. Then we have, as in the last article,

$$\sin(A - 90°) = -\cos A, \qquad \cot(A - 90°) = -\tan A,$$
$$\cos(A - 90°) = \sin A, \qquad \sec(A - 90°) = \csc A,$$
$$\tan(A - 90°) = -\cot A, \qquad \csc(A - 90°) = -\sec A.$$

By Sec. 27, if the sign of an angle be changed, the sign of each of its functions, except the cosine and the secant, will be changed. Hence,

$$\sin(90° - A) = \cos A, \qquad \cot(90° - A) = \tan A,$$
$$\cos(90° - A) = \sin A, \qquad \sec(90° - A) = \csc A,$$
$$\tan(90° - A) = \cot A, \qquad \csc(90° - A) = \sec A.$$

These relations were proved in Sec. 4 for the case where A is acute.

30. Functions of 135°, 225°, 315°. Let XOC (Fig. 26) be an angle of 135°. Then $\angle COX' = 45°$. Take $OM = 1$, and draw MB perpendicular to XX'. Then $MB = 1$, $OB = \sqrt{2}$, and the coördinates of B are $(-1, 1)$; and we have

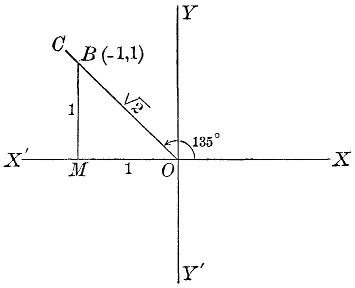

FIG. 26

$$\sin 135° = \frac{1}{\sqrt{2}}, \qquad \cos 135° = -\frac{1}{\sqrt{2}},$$

$$\tan 135° = -1, \qquad \sec 135° = -\sqrt{2},$$
$$\cot 135° = -1, \qquad \csc 135° = \sqrt{2}.$$

For 225° OC falls in the third quadrant, and for 315° it falls in the fourth; in each case take $OM = 1$ and construct the triangle OMB as above and find the coördinates of B.

31. Functions of 120°, 150°, 210°, 240°, 300°, 330°. The functions of each of these angles are closely related to those of 30° and 60°, as will be seen from the figures. We shall construct the figure and find the functions of 240°. Let XOC (Fig. 27) be an angle of 240°. Then the acute angle $X'OC$ formed by its terminal line and the X-axis is 60°.

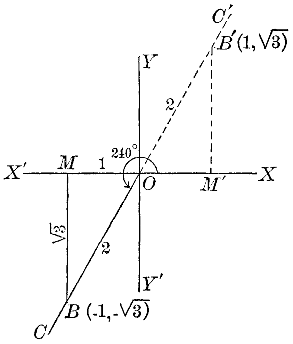

FIG. 27

Construct, then, an angle of 60° in the first quadrant and set off on its terminal line a distance $OB' = 2$, and draw the perpendicular $B'M'$. Then the triangle $OB'M'$ equals the triangle ADB of Fig. 4. Why?

Hence, the coördinates of B' are $1, \sqrt{3}$.

Now, setting off $OB = 2$ and drawing the perpendicular BM, the triangle OBM equals the triangle $OB'M'$, and we have, for the coördinates of B, $-1, -\sqrt{3}$. Hence,

$$\sin 240° = -\frac{\sqrt{3}}{2}, \qquad \cot 240° = \sqrt{3},$$

$$\cos 240° = -\frac{1}{2}, \qquad \sec 240° = -2,$$

$$\tan 240° = \frac{1}{\sqrt{3}}, \qquad \csc 240° = -\frac{2}{\sqrt{3}}.$$

For each of the other angles named above construct first the given angle, then in the first quadrant an angle equal to the acute angle formed by the terminal line of the given angle and the X-axis, and proceed as above.

In the following table are collected the functions of the angles named in the caption of this section, and for 45°, 30°, 60° previously found. The student should be able readily to construct the figure for each case and to obtain the results from it.

ANGLE	SIN	COS	TAN	COT	SEC	CSC
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
135°	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
225°	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
315°	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	-2	$-\frac{2}{\sqrt{3}}$
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	2	$-\frac{2}{\sqrt{3}}$
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2

Should it be required to express any of the above functions as a decimal, it is best first to *rationalize* the denominator when it contains a radical. Thus,

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.41421+}{2} = 0.70710+.$$

EXERCISES

1. Write down from the above table all the functions in which the denominator is irrational, and rationalize as above.

2. Express decimally to five places sin 120°, sec 30°, cot 240°.

32. By means of formulas (1) to (8) any one of the functions can be expressed in terms of any one of the others. Thus, to express the tangent in terms of each of the other functions:

By formula (3), $\tan = \dfrac{1}{\cot}$,

which gives tangent in terms of cotangent.

By formula (7), $\tan^2 = \sec^2 - 1$,

or $\tan = \sqrt{\sec^2 - 1}$.

By formula (8), $\cot = \sqrt{\csc^2 - 1}$.

$$\therefore \tan = \frac{1}{\cot} = \frac{1}{\sqrt{\csc^2 - 1}}.$$

By formula (4), $\tan = \dfrac{\sin}{\cos}$.

But, by formula (6), $\cos = \sqrt{1 - \sin^2}$.

$$\therefore \tan = \frac{\sin}{\sqrt{1 - \sin^2}}.$$

By formula (6) also, $\sin = \sqrt{1 - \cos^2}$.

$$\therefore \tan = \frac{\sqrt{1 - \cos^2}}{\cos}.$$

Hence, we have the following values for tangent:

$$\tan = \frac{\sin}{\sqrt{1 - \sin^2}} = \frac{\sqrt{1 - \cos^2}}{\cos} = \frac{1}{\cot} = \sqrt{\sec^2 - 1} = \frac{1}{\sqrt{\csc^2 - 1}}.$$

Let the student write down formulas (1) to (8) and by means of them express each function in terms of each of the others separately.

MISCELLANEOUS EXERCISES

Prove the following relations :

1. $\tan x + \cot x = \sec x \csc x$.

2. $(\tan x + \cot x)^2 = \sec^2 x + \csc^2 x$.

3. $\cos^3 x \tan x + \sin^2 x \cos x \tan x = \sin x$.

4. $(1 + \tan x)(1 + \cot x) = 2 + \sec x \csc x$.

5. $(\tan x - \sin x)\csc^3 x = \dfrac{\sec x}{1 + \cos x}$.

6. $\dfrac{\cot x - \tan x}{\cos x - \sin x} = \dfrac{\cos x + \sin x}{\cos x \sin x}$.

7. $(\sin x \cos y + \cos x \sin y)^2 + (\cos x \cos y - \sin x \sin y)^2 = 1$.

8. $\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$.

9. $\cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$.

Determine x from the following equations, x being an angle between $0°$ and $90°$:

10. $\sin 6x = \frac{1}{2}$. [One solution is given by the equation $6x = 30°$, another by the equation $6x = 150°$; obtain the others by adding $360°$ successively to these angles.]

11. $\sin 3x = \cos x$. *Ans.* $x = 22\frac{1}{2}°$, or $45°$.

12. $\tan 6x = \cot 3x$. *Ans.* $x = 10°$.

13. $5\tan^2 x + 3\sec^2 x = 11$. *Ans.* $x = 45°$.

14. $\text{versin}^2 x = \frac{3}{2}$. *Ans.* $x = 60°$.

15. $\tan 5x = 1$. *Ans.* $x = 9°$, or $81°$.

16. $\sin 5x = \dfrac{\sqrt{3}}{2}$. *Ans.* $x = 12°$, or $84°$.

CHAPTER V

ADDITION AND SUBTRACTION FORMULAS

33. **To prove the relation**

$$\sin(x + y) = \sin x \cos y + \cos x \sin y,$$

or, *the sine of the sum of two angles equals the sine of the first times the cosine of the second, plus the cosine of the first times the sine of the second.*

We shall first prove this and the succeeding propositions for the case where x and y are each less than 90°, and their sum also less than 90°. In Fig. 28, let $\angle LON = x$, $\angle NOM = y$, then $\angle LOM = x + y$. From any point, C, in the *terminal* line of $x + y$ draw CA and CD perpendicular to OL and ON respectively; DB and DE perpendicular to OL and AC respectively. The acute angles LON and DCE have their sides perpendicular each to each.

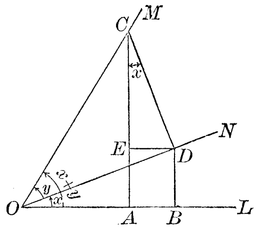

Fig. 28

$$\therefore \angle DCE = x.$$

Now, $$\sin(x + y) = \frac{AC}{OC} = \frac{BD + EC}{OC} = \frac{BD}{OC} + \frac{EC}{OC}.$$

But $$\frac{BD}{OC} = \frac{BD}{OD} \times \frac{OD}{OC} = \sin x \cos y,$$

and $$\frac{EC}{OC} = \frac{EC}{CD} \times \frac{CD}{OC} = \cos x \sin y.$$

Substituting these values above, we have

$$\mathbf{\sin(x + y) = \sin x \cos y + \cos x \sin y.} \tag{9}$$

In deciding upon the artifice by which $\frac{BD}{OC}$ is to be transformed into $\sin x \cos y$, we may proceed as follows:

First write $$\frac{BD}{OC} = \frac{BD}{\ldots} \times \frac{\cdots}{OC}.$$

Then, under BD, write the *hypotenuse* of the right triangle of which BD is a leg, that is, OD; then, of course, OD must also be written over OC, giving

$$\frac{BD}{OC} = \frac{BD}{OD} \times \frac{OD}{OC}.$$

So in each of the similar cases below.

34. **To prove the relation**

$$\cos(x + y) = \cos x \cos y - \sin x \sin y,$$

or, *the cosine of the sum of two angles equals the cosine of the first times the cosine of the second, minus the sine of the first times the sine of the second.*

From Fig. 28,

$$\cos(x + y) = \frac{OA}{OC} = \frac{OB - ED}{OC} = \frac{OB}{OC} - \frac{ED}{OC}.$$

But $$\frac{OB}{OC} = \frac{OB}{OD} \times \frac{OD}{OC} = \cos x \cos y,$$

and $$\frac{ED}{OC} = \frac{ED}{CD} \times \frac{CD}{OC} = \sin x \sin y.$$

Substituting these values above,

$$\mathbf{\cos(x + y) = \cos x \cos y - \sin x \sin y.} \qquad (10)$$

35. **To prove the relation**

$$\sin(x - y) = \sin x \cos y - \cos x \sin y,$$

or, *the sine of the difference of two angles equals the sine of the first times the cosine of the second, minus the cosine of the first times the sine of the second.*

In Fig. 29, let $\angle LON = x$, $\angle NOM = y$.

Then, $\angle LOM = x - y$.

From any point, C, in the terminal line of $x - y$ draw CA, CD perpendicular to OL and ON respectively, as above.

Draw also DB, CE perpendicular respectively to OL, DB. The acute angles LON, CDE are equal. Why?

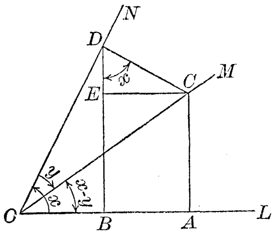

FIG. 29

$\therefore \angle CDE = x$; and we have

$$\sin(x-y) = \frac{AC}{OC} = \frac{BD - ED}{OC} = \frac{BD}{OC} - \frac{ED}{OC}.$$

Now, $$\frac{BD}{OC} = \frac{BD}{OD} \times \frac{OD}{OC} = \sin x \cos y,$$

and $$\frac{ED}{OC} = \frac{ED}{CD} \times \frac{CD}{OC} = \cos x \sin y.$$

Hence, $$\mathbf{\sin(x-y) = \sin x \cos y - \cos x \sin y.} \qquad (11)$$

36. To prove the relation

$$\cos(x-y) = \cos x \cos y + \sin x \sin y,$$

or, *the cosine of the difference of two angles equals the cosine of the first times the cosine of the second, plus the sine of the first times the sine of the second.*

From Fig. 29,

$$\cos(x-y) = \frac{OA}{OC} = \frac{OB + EC}{OC} = \frac{OB}{OC} + \frac{EC}{OC}.$$

Now, $$\frac{OB}{OC} = \frac{OB}{OD} \times \frac{OD}{OC} = \cos x \cos y,$$

and $$\frac{EC}{OC} = \frac{EC}{CD} \times \frac{CD}{OC} = \sin x \sin y.$$

$$\therefore \mathbf{\cos(x-y) = \cos x \cos y + \sin x \sin y.} \qquad (12)$$

In formulas (9), (10), (11), (12), it should be noted that for the *sine* the algebraic sign between the two terms on the right is the same as that between x and y on the left; for the *cosine* the algebraic signs are different in the two members.

Formulas (9) and (10) are called *addition formulas*, (11) and (12) *subtraction formulas*.

37. General Proof of the Addition and Subtraction Formulas. By paying attention to algebraic signs, we may apply the proof given in Secs. 33, 34 to the case where x and y are acute angles whose sum is between 90° and 180°. In this case the terminal line of the angle $x + y$ falls in the second quadrant.

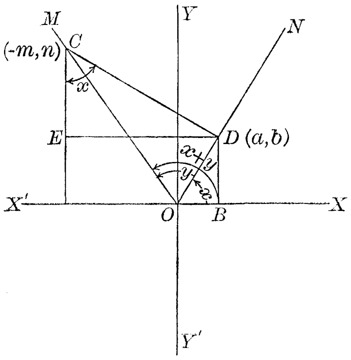

FIG. 30

In Fig. 30, let $\angle XON = x$, $\angle NOM = y$.

Then, $\angle XOM = x + y$.

From any point, C, in the terminal line of $x + y$ draw the perpendiculars CA and CD to XX' and ON respectively; also DE, DB perpendicular respectively to CA, XX'.

Then, $\angle DCE = x$.

Let the coördinates of C be $(-m, n)$, of D (a, b), and the *number* of linear units in ED and OC be l and r respectively.

Now, $\sin(x+y) = \frac{n}{r} = \frac{AC}{OC} = \frac{BD+EC}{OC} = \frac{BD}{OC} + \frac{EC}{OC}$.

But $$\frac{BD}{OC} = \frac{BD}{OD} \times \frac{OD}{OC} = \sin x \cos y,$$

and $$\frac{EC}{OC} = \frac{EC}{CD} \times \frac{CD}{OC} = \cos x \sin y.$$

$$\therefore \sin(x+y) = \sin x \cos y + \cos x \sin y. \qquad \text{(a)}$$

Also, $$\cos(x+y) = \frac{-m}{r}.$$

But, from the figure, $l = m + a$.

$$\therefore -m = a - l.$$

$$\therefore \cos(x+y) = \frac{-m}{r} = \frac{a-l}{r} = \frac{a}{r} - \frac{l}{r} = \frac{OB}{OC} - \frac{ED}{OC}.$$

But $$\frac{OB}{OC} = \frac{OB}{OD} \times \frac{OD}{DC} = \cos x \cos y,$$

and $$\frac{ED}{OC} = \frac{ED}{CD} \times \frac{CD}{OC} = \sin x \sin y.$$

$$\therefore \cos(x+y) = \cos x \cos y - \sin x \sin y. \qquad \text{(b)}$$

From (a) and (b) it follows that the addition formulas are true when x and y are each less than 90° and their sum greater than 90°. The proof is extended so as to include all possible cases, as follows:

1. Let one of the component angles, say x, be increased by 90°, and let $90 + x = A$.

Then, $$\begin{aligned} \sin(A+y) &= \sin(90+x+y) \\ &= \cos(x+y) && \text{by Sec. 20} \\ &= \cos x \cos y - \sin x \sin y. \end{aligned}$$

But again, by Sec. 28,

$$\sin A = \cos x,$$
$$\cos A = -\sin x.$$

Substituting these values for $\cos x$ and $-\sin x$, we have

$$\sin(A + y) = \sin A \cos y + \cos A \sin y. \qquad \text{(c)}$$

Similarly,
$$\begin{aligned}\cos(A + y) &= \cos(90 + x + y)\\ &= -\sin(x + y)\\ &= -\sin x \cos y - \cos x \sin y;\end{aligned}$$

or
$$\cos(A + y) = \cos A \cos y - \sin A \sin y. \qquad \text{(d)}$$

From (c) and (d) it follows that the addition formulas are true after one of the acute angles x or y has been increased by 90°.

2. Next let y *also* be increased by 90°, and let

$$90° + y = B.$$

Then,
$$\begin{aligned}\sin(A + B) &= \sin(A + 90° + y)\\ &= \cos(A + y)\\ &= \cos A \cos y - \sin A \sin y. \quad \text{By (d)}\end{aligned}$$

But
$$\sin B = \cos y,$$

and
$$\cos B = -\sin y.$$

$$\therefore \sin(A + B) = \sin A \cos B + \cos A \sin B. \qquad \text{(e)}$$

Similarly,
$$\begin{aligned}\cos(A + B) &= \cos(A + 90° + y)\\ &= -\sin(A + y)\\ &= -\sin A \cos y - \cos A \sin y, \quad \text{by (c)}\end{aligned}$$

or
$$\cos(A + B) = \cos A \cos B - \sin A \sin B. \qquad \text{(f)}$$

From (e) and (f) it follows that the addition formulas are true after both of the component angles have been increased by 90°. By successive applications of the same course of reasoning it may be shown that they are true after one or both of the component angles have been increased by any number of times 90°.

[Thus, for the next step, add 90° to A, and let $90° + A = A'$, and proceed as before.]

They are true, therefore, for any *positive* values of the component angles.

In exactly the same way the subtraction formulas (11) and (12) may be extended so as to include all positive values of x and y.

Now let one of the component angles, say y, be *negative*. Let $y = -B$ where B is positive; then,

$$\sin(x+y) = \sin(x-B) = \sin x \cos B - \cos x \sin B,$$

by formula (11).

$$\therefore \sin(x+y) = \sin x \cos y + \cos x \sin y,$$

since $\cos B = \cos y$ and $-\sin B = \sin y$.

Therefore, formula (9) is true when one of the angles is negative.

Next let both x and y be negative and let $x = -A$, $y = -B$, where A and B are positive.

Then, $\sin(x+y) = \sin(-A-B) = -\sin(A+B)$

$$= -\sin A \cos B - \cos A \sin B.$$

$$\therefore \sin(x+y) = \sin x \cos y + \cos x \sin y.$$

Therefore, formula (9) is true when both the component angles are negative.

In the same way formulas (10), (11), (12) may be extended so as to include the case where one or both the component angles are negative. The addition and subtraction formulas are therefore true universally.

Let the student give the proof of formulas (10), (11), (12): first, when x is positive and $y = -B$; second, when $x = -A$, $y = -B$.

38. To prove the relation

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}. \qquad (13)$$

We have

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\dfrac{\sin x \cos y}{\cos x \cos y} + \dfrac{\cos x \sin y}{\cos x \cos y}}{\dfrac{\cos x \cos y}{\cos x \cos y} - \dfrac{\sin x \sin y}{\cos x \cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

We may proceed in the same way and deduce a formula for $\tan(x-y)$; or we may substitute $-y$ for y in formula (13). Thus,

$$\tan(x-y)=\tan[x+(-y)]=\frac{\tan x+\tan(-y)}{1-\tan x\tan(-y)},$$

or

$$\mathbf{\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}}. \qquad (14)$$

Express formulas (13) and (14) in words.

Similarly, let the student prove

$$\cot(x+y)=\frac{\cot x\cot y-1}{\cot y+\cot x},$$

and

$$\cot(x-y)=\frac{\cot x\cot y+1}{\cot y-\cot x}.$$

39. **To prove the relations**

$$\mathbf{\sin A+\sin B=2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)}, \qquad (15)$$

$$\mathbf{\sin A-\sin B=2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)}, \qquad (16)$$

$$\mathbf{\cos A+\cos B=2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)}, \qquad (17)$$

$$\mathbf{\cos A-\cos B=-2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)}. \qquad (18)$$

Let $A=x+y, \quad B=x-y.$

Then, $x=\frac{1}{2}(A+B), \quad y=\frac{1}{2}(A-B).$

By the addition and subtraction formulas,

$$\sin A=\sin x\cos y+\cos x\sin y,$$
$$\sin B=\sin x\cos y-\cos x\sin y.$$

Adding,

$$\sin A+\sin B=2\sin x\cos y$$
$$=2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B).$$

Subtracting,

$$\sin A-\sin B=2\cos x\sin y$$
$$=2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B).$$

Similarly, $\cos A = \cos x \cos y - \sin x \sin y,$

$\cos B = \cos x \cos y + \sin x \sin y.$

Adding, $\cos A + \cos B = 2 \cos x \cos y$

$= 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B).$

Subtract and prove formula (18).

Formula (15) is equivalent to the general proposition: *the sine of one angle plus the sine of another equals twice the sine of half their sum, times the cosine of half their difference.*

Express formulas (16), (17), and (18) similarly. From formulas (15) and (16), by division, prove

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}. \tag{19}$$

40. Functions of Twice any Angle. In formula (9) let $y = x$, and we have

$$\sin (x + x) = \sin x \cos x + \cos x \sin x,$$

or
$$\sin 2x = 2 \sin x \cos x. \tag{20}$$

Let $y = x$ in formula (10), and we have

$$\cos (x + x) = \cos x \cos x - \sin x \sin x,$$

or
$$\cos 2x = \cos^2 x - \sin^2 x. \tag{21}$$

Since $\sin^2 x + \cos^2 x = 1$, we have also

$$\cos 2x = 1 - \sin^2 x - \sin^2 x,$$

or
$$\cos 2x = 1 - 2 \sin^2 x; \tag{22}$$

and
$$\cos 2x = \cos^2 x - (1 - \cos^2 x),$$

or
$$\cos 2x = 2 \cos^2 x - 1. \tag{23}$$

Formula (20) may be read: *the sine of twice any angle equals twice the sine of the angle times the cosine of the angle.*

Express formulas (21), (22), and (23) similarly.

Put $y = x$ in formula (13) and prove

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}. \tag{24}$$

41. In formulas (22) and (23), let $2x = A$, then $x = \frac{1}{2}A$, and we have

$$\cos A = 1 - 2\sin^2 \tfrac{1}{2} A,$$

$$\mathbf{2\sin^2 \tfrac{1}{2} A = 1 - \cos A;} \tag{25}$$

$$\cos A = 2\cos^2 \tfrac{1}{2} A - 1,$$

or

$$\mathbf{2\cos^2 \tfrac{1}{2} A = 1 + \cos A.} \tag{26}$$

From formulas (25) and (26), if $\cos A$ be given, $\sin \frac{1}{2}A$ and $\cos \frac{1}{2}A$ can be found.

From formulas (25) and (26), we have, by division and reduction,

$$\mathbf{\tan \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{1 + \cos A}}.} \tag{27}$$

EXERCISES

1. Given $\cos 30° = \dfrac{\sqrt{3}}{2}$, find $\sin 15°$ and $\cos 15°$.

Putting $A = 30°$ in formula (25),

$$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2} = \frac{2 - \sqrt{3}}{2}.$$

$$\therefore \sin 15° = \tfrac{1}{2}\sqrt{2 - \sqrt{3}}.$$

Similarly, from formula (26),

$$2\cos^2 15° = 1 + \frac{\sqrt{3}}{2},$$

$$\cos 15° = \tfrac{1}{2}\sqrt{2 + \sqrt{3}}.$$

2. Given $\cos 36° = \dfrac{\sqrt{5} + 1}{4}$, prove $\cos 18° = \sqrt{\dfrac{5 + \sqrt{5}}{8}}$. Find also $\sin 18°$.

3. Find sine, cosine, and tangent of 22° 30′.

42. The addition and subtraction formulas may be used to express the sine and cosine of an angle in terms of functions of its supplement. Thus,

$$\sin(180° - A) = \sin 180° \cos A - \cos 180° \sin A.$$

Since $\sin 180° = 0,$

and $\cos 180° = -1,$

this becomes $\mathbf{\sin(180° - A) = \sin A.}$ (28)

Similarly, $\mathbf{\cos(180° - A) = -\cos A.}$ (29)

Since A is the supplement of $180° - A$, we have the important relations: *the sine of an angle equals the sine of its supplement; the cosine of an angle equals the negative cosine of its supplement.*

Similar relations can be proved for the other functions.

Thus, $\sec(180° - A) = \dfrac{1}{\cos(180° - A)} = \dfrac{1}{-\cos A} = -\sec A.$

Similarly,

$$\csc(180° - A) = \csc A,$$

$$\tan(180° - A) = \frac{\sin(180° - A)}{\cos(180° - A)} = \frac{\sin A}{-\cos A} = -\tan A,$$

$$\cot(180° - A) = -\cot A.$$

Similarly,

$$\sin(270° - A) = -\cos A, \text{ etc.}$$

The addition and subtraction formulas may be used also to *recall* the values of $\sin(90° + A)$ and $\cos(90° + A)$ in terms of functions of A as given in Sec. 28.

Thus, $\sin(90° + A) = \sin 90° \cos A + \cos 90° \sin A = \cos A,$

since $\sin 90° = 1$ and $\cos 90° = 0.$

Also, $\cos(90° + A) = \cos 90° \cos A - \sin 90° \sin A = -\sin A.$

EXERCISES

1. Express the functions of $180° - A$ in terms of functions of A by means of a figure, as in Sec. 28, taking A in the first quadrant.

2. Find an angle that has the same sine as (a) 150°; (b) 123° 22′ 36″; (c) 42°.

3. If $\cos 18° = 0.95106$, find $\cos 162°$.

MISCELLANEOUS EXERCISES

Prove the following relations :

1. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\cot y + \cot x}{\cot y - \cot x}$.

2. $\dfrac{\sin(x-y)}{\cos(x+y)} = \dfrac{\cot y - \cot x}{\cot y \cot x - 1}$.

3. $\dfrac{\cot x + 1}{\cot x - 1} = \sec 2x + \tan 2x$.

4. $\cot(x+y) = \dfrac{1 - \tan x \tan y}{\tan x + \tan y}$.

5. $\sin \frac{1}{2} A = \sqrt{\dfrac{1 - \cos A}{2}}$.

6. $\cos \frac{1}{2} A = \sqrt{\dfrac{1 + \cos A}{2}}$.

7. $\tan \frac{1}{2} A = \dfrac{1 - \cos A}{\sin A}$.

8. $\dfrac{\sin A}{\text{versin } A} = \cot \frac{1}{2} A$.

9. $\tan \frac{1}{2} A + \cot \frac{1}{2} A = 2 \csc A$.

10. $\sin \frac{1}{2} A = \sqrt{\dfrac{\text{versin } A}{2}}$.

11. $\dfrac{2 \tan \frac{1}{2} A}{1 + \tan^2 \frac{1}{2} A} = \sin A$.

12. $\dfrac{1 - \tan^2 \frac{1}{2} A}{1 + \tan^2 \frac{1}{2} A} = \cos A$.

13. $\dfrac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} = \tan(x+y)$.

14. $\dfrac{\cos 2x + \cos 2y}{\cos 2x - \cos 2y} = -\cot(x+y)\cot(x-y)$.

15. $\dfrac{\sin 7x + \sin x}{\cos 7x + \cos x} = \tan 4x$.

16. Prove $\dfrac{\cos 52^\circ\, 30' + \cos 7^\circ\, 30'}{\sin 52^\circ\, 30' + \sin 7^\circ\, 30'} = \sqrt{3}$.

17. Prove $\sin 3x = 3 \sin x - 4 \sin^3 x$.

Write $\sin 3x = \sin(2x + x)$; apply formulas (9), (20), (22).

18. Express $\cos 3x$ in terms of $\cos x$.

19. Prove $\tan 4x = \dfrac{4 \tan x - 4 \tan^3 x}{1 - 6 \tan^2 x + \tan^4 x}$.

20. Prove $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C$
$+ \cos A \cos B \sin C - \sin A \sin B \sin C$.

Put $x = A + B$, $y = c$ in formula (9).

21. Prove $\cos(A + B + C) = \cos A \cos B \cos C - \sin A \sin B \cos C$
$- \sin A \cos B \sin C - \cos A \sin B \sin C$.

22. Use the results of Exercises 20 and 21 to prove

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan A \tan C - \tan B \tan C}.$$

23. If A, B, C are the angles of a triangle, prove, by using the result of Exercise 22, that

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

24. Express $\sin 3x$ in terms of $\sin x$ by putting $A = B = C = x$ in Exercise 20.

25. Express $\cos 3x$ in terms of $\cos x$ by using the result of Exercise 21.

CHAPTER VI

PROPERTIES OF TRIANGLES; SOLUTION OF OBLIQUE TRIANGLES

43. In the solution of oblique triangles four cases present themselves:

1. *Given a side and two angles.*
2. *Given two sides and their included angle.*
3. *Given two sides and an angle opposite one of them.*
4. *Given the three sides.*

The solution is effected, as in the case of right triangles, by means of relations between the sides and the angles. Formulas embodying such relations will now be established.

44. Law of Sines. *In any triangle the sides are proportional to the sines of the opposite angles.*

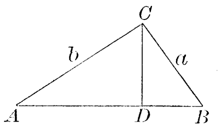

FIG. 31

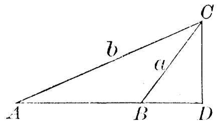

FIG. 32

Proof. In the triangle ABC draw the perpendicular CD. Then, if all the angles are acute, as in Fig. 31, we have

$$\sin A = \frac{CD}{b},$$

$$\sin B = \frac{CD}{a}.$$

$\therefore$ by division,
$$\frac{\sin A}{\sin B} = \frac{\frac{CD}{b}}{\frac{CD}{a}} = \frac{a}{b}.$$

If one of the angles, as B (Fig. 32), be obtuse, then, since the sine of an angle equals the sine of its supplement,

$$\sin B = \sin CBD = \frac{CD}{a},$$

while

$$\sin A = \frac{CD}{b}, \text{ as before.}$$

$$\therefore \frac{\sin A}{\sin B} = \frac{a}{b}.$$

In like manner,

$$\frac{\sin A}{\sin C} = \frac{a}{c},$$

$$\frac{\sin B}{\sin C} = \frac{b}{c}.$$

These results may be written

$$\frac{\mathbf{a}}{\mathbf{\sin A}} = \frac{\mathbf{b}}{\mathbf{\sin B}} = \frac{\mathbf{c}}{\mathbf{\sin C}}. \qquad (30)$$

45. Case 1. *Given one side and two angles.*

Given $a = 263$, $A = 81°$, $B = 52°$, to find b, c, C.

The law of sines and the relation $A + B + C = 180°$ suffice for the solution.

We have at once $C = 180° - (A + B) = 47°$.

To find b, apply the law of sines, preferably writing the unknown, b, first. Thus,

$$\frac{b}{a} = \frac{\sin B}{\sin A}.$$

$$\therefore \log b - \log a = \log \sin B - \log \sin A,$$

or

$$\log b = \log a + \log \sin B - \log \sin A.$$

$$\begin{aligned}
\log 263 &= 2.41996 \\
\log \sin 52° &= \underline{9.89653 - 10} \\
&\ 12.31649 - 10 \\
\log \sin 81° &= \underline{9.99462 - 10} \\
\log b &= 2.32187 \\
b &= 209.83.
\end{aligned}$$

Similarly, to find c, $\dfrac{c}{a} = \dfrac{\sin C}{\sin A}$.

$$\begin{aligned}
\log c &= \log a + \log \sin C - \log \sin A.\\
\log 263 &= 2.41996\\
\log \sin 47^\circ &= \underline{9.86413 - 10}\\
& 12.28409 - 10\\
\log \sin 81^\circ &= \underline{9.99462 - 10}\\
\log c &= 2.28947\\
c &= 194.75.
\end{aligned}$$

Check. $\dfrac{c}{b} = \dfrac{\sin C}{\sin B}$, from which

$$\log c - \log b = \log \sin C - \log \sin B.$$

$$\begin{array}{rl|rl}
\log c = & 12.28947 - 10 & \log \sin C = & 19.86413 - 20\\
\log b = & \underline{2.32187} & \log \sin B = & \underline{9.89653 - 10}\\
& 9.96760 - 10 & & 9.96760 - 10
\end{array}$$

In all exercises in this chapter in which the answers are not given the student should *check* his results. In doing this he should use the law of sines and also the law of tangents (Sec. 46), which affords a better check; when the three sides are given the check $A + B + C = 180^\circ$ will suffice.

EXERCISES

Solve the following triangles:

1. Given $A = 32^\circ\ 19'$, $B = 76^\circ\ 42'$, $a = 268.94$, to find C, b, c. *Ans.* $C = 70^\circ\ 59'$, $b = 489.57$, $c = 475.61$.

2. Given $B = 51^\circ\ 47'$, $C = 63^\circ\ 32'$, $b = 20.64$, to find A, a, c. *Ans.* $A = 64^\circ\ 41'$, $a = 23.748$, $c = 23.517$.

3. Given $A = 52^\circ\ 24'\ 36''$, $C = 74^\circ\ 12'\ 16''$, $a = 2468.12$, to find B, b, c. *Ans.* $B = 53^\circ\ 23'\ 8''$, $b = 2500.1$, $c = 2997.1$.

4. Given $A = 114^\circ\ 23'$, $B = 33^\circ\ 41'$, $a = 323$, to find C, b, c. *Ans.* $b = 196.68$, $c = 187.58$.

5. Given $A = 98^\circ\ 20'$, $B = 44^\circ\ 9'$, $a = 964.5$, to find C, b, c. *Ans.* $b = 679.83$, $c = 593.64$.

6. Given $A = 54^\circ\ 14'$, $B = 44^\circ\ 27'$, $a = 1264$, to find C, b, c. *Ans.* $b = 1090.9$, $c = 1539.9$.

7. Given $B = 14° 26' 38''$, $C = 67° 13' 12''$, $b = 23.64$, to find A, a, c. *Ans.* $a = 93.776$, $c = 87.384$.

8. Given $B = 46° 48'$, $C = 84° 23'$, $b = 174$, to find A, a, c. *Ans.* $a = 179.64$, $c = 237.55$.

9. Given $A = 67° 54'$, $B = 34° 52'$, $b = 435.67$, to find C, a, c. *Ans.* $a = 706.28$, $c = 743.45$.

10. Given $B = 32° 12'$, $C = 108° 36'$, $a = 467$, to find A, b, c. *Ans.* $b = 374.80$, $c = 685.73$.

11. Given $A = 43° 22'$, $C = 34° 26'$, $a = 346.24$, to find B, b, c.

12. Given $B = 74° 35'$, $C = 46° 23'$, $c = 246.13$, to find A, a, b.

13. Given $B = 52°$, $C = 98°$, $c = 95.46$, to find A, a, b.

14. Given $A = 23° 18'$, $B = 106°$, $a = 34.7$, to find C, b, c.

15. Given $B = 42°$, $C = 116°$, $a = 564$, to find A, b, c.

46. In Case 2 we need also

The Law of Tangents. *In any triangle the sum of any two sides is to their difference as the tangent of one half the sum of the angles opposite those sides is to the tangent of one half their difference.* Or, in symbols,

$$\frac{\mathbf{a} + \mathbf{b}}{\mathbf{a} - \mathbf{b}} = \frac{\tan \frac{1}{2}(\mathbf{A} + \mathbf{B})}{\tan \frac{1}{2}(\mathbf{A} - \mathbf{B})}. \tag{31}$$

To prove this, we have, by the law of sines,

$$\frac{a}{b} = \frac{\sin A}{\sin B};$$

by composition and division,

$$\frac{a + b}{a - b} = \frac{\sin A + \sin B}{\sin A - \sin B}.$$

$\therefore$ by formula (19), $\dfrac{a + b}{a - b} = \dfrac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$

Similarly, $\dfrac{a+c}{a-c}=\dfrac{\tan\frac{1}{2}(A+C)}{\tan\frac{1}{2}(A-C)}$, etc.

47. **Case 2.** *Given two sides and the angle included by them.* The method of solution is shown in the following example.

Given $b = 360$, $c = 483$, $A = 62°$, to find a, B, C.

By the law of tangents, $\dfrac{c+b}{c-b}=\dfrac{\tan\frac{1}{2}(C+B)}{\tan\frac{1}{2}(C-B)}$.

From the given parts,

$c+b=843,\ c-b=123,\ C+B=180°-62°=118°;\ \frac{1}{2}(C+B)=59°.$

Substituting these values in the above equation,

$$\frac{843}{123}=\frac{\tan 59°}{\tan\frac{1}{2}(C-B)}.$$

$$\therefore \log 843-\log 123=\log\tan 59°-\log\tan\tfrac{1}{2}(C-B),$$

or $$\log\tan\tfrac{1}{2}(C-B)=\log\tan 59°+\log 123-\log 843.$$

$$\begin{aligned}
\log\tan 59° &= 10.22123-10\\
\log 123 &= \underline{\ \ 2.08991\qquad\ }\\
&\quad\ 12.31114-10\\
\log 843 &= \underline{\ \ 2.92583\qquad\ }\\
\log\tan\tfrac{1}{2}(C-B) &= \ \ 9.38531-10\\
\tfrac{1}{2}(C-B) &= 13°\ 38'\ 57''.
\end{aligned}$$

But $\frac{1}{2}(C+B)=59°$.

Adding, $C=72°\ 38'\ 57''$.

Subtracting, $B=45°\ 21'\ \ 3''$.

We now find a by the law of sines;

$$\frac{a}{c}=\frac{\sin A}{\sin C}.$$

$$\log a=\log c+\log\sin A-\log\sin C,$$

or $$\log a=\log 483+\log\sin 62°-\log\sin 72°\ 38'\ 57''.$$

$$\begin{aligned}
\log 483 &= \ \ 2.68395\\
\log\sin 62° &= \underline{\ \ 9.94593-10}\\
&\quad\ 12.62988-10\\
\log\sin 72°\ 38'\ 57'' &= \underline{\ \ 9.97978-10}\\
\log a &= \ \ 2.65010\\
a &= 446.79.
\end{aligned}$$

Check. $\dfrac{a}{b}=\dfrac{\sin A}{\sin B}$.

Apply this test.

EXERCISES

Solve the following triangles:

1. Given $A = 58° 16'$, $b = 656$, $c = 834$, to find B, C, a.
   Ans. $B = 48° 46' 9''$, $C = 72° 57' 51''$, $a = 741.85$.

2. Given $A = 78° 12'$, $b = 146$, $c = 174$, to find B, C, a.
   Ans. $B = 44° 45' 17''$, $C = 57° 2' 43''$, $a = 202.98$.

3. Given $B = 53° 18'$, $a = 640$, $c = 364$, to find A, C, b.
   Ans. $A = 92° 3' 46''$, $C = 34° 38' 14''$, $b = 513.46$.

4. Given $C = 67° 29'$, $a = 27$, $b = 22$, to find A, B, c.
   Ans. $A = 63° 39' 1''$, $B = 46° 53' 59''$, $c = 28.213$.

5. Given $C = 54° 26'$, $a = 123$, $b = 93$, to find A, B, c.
   Ans. $A = 77° 53' 46''$, $B = 47° 40' 14''$, $c = 102.33$.

6. Given $A = 56°$, $b = 324$, $c = 456$, to find B, C, a.
   Ans. $B = 44° 20' 41''$, $C = 79° 39' 19''$, $a = 3842.9$.

7. Given $A = 56° 10'$, $b = 345.6$, $c = 564.8$, to find B, C, a.
   Ans. $B = 37° 37' 47''$, $C = 86° 12' 13''$, $a = 470.18$.

8. Given $a = 294$, $b = 114$, $C = 108° 24'$, to find A, B, c.
   Ans. $A = 53° 27' 1''$, $B = 18° 8' 59''$, $c = 347.27$.

9. Given $a = 32.64$, $b = 46.82$, $C = 44° 12'$, to find A, B, c.
   Ans. $A = 44° 10' 32''$, $B = 91° 37' 28''$, $c = 32.654$.

10. Given $b = 3463.8$, $a = 1896.4$, $C = 45° 28'$, to find A, B, c.
    Ans. $A = 32° 21' 20''$, $B = 102° 10' 40''$, $c = 2525.9$.

11. Given $a = 256$, $c = 438$, $B = 66° 24'$, to find A, C, b.

12. Given $b = 374$, $c = 562$, $A = 104° 32'$, to find B, C, a.

13. Given $a = 235.4$, $b = 392.4$, $C = 53° 18'$, to find A, B, c.

14. Given $c = 822$, $b = 654$, $A = 57° 32'$, to find B, C, a.

15. Given $A = 54° 20'$, $b = 638$, $c = 362$, to find B, C, a.

48. Case 3. *Given two sides and an angle opposite one of them.*

Let the given parts be a, b, A.

Let us first undertake to construct the triangle geometrically. At any point, as A in the line PQ (Fig. 33), construct an angle QAR equal to the given angle A. On AR take AC equal to the given side b; with C as a center and a radius equal to the other given side a, describe an arc. Then, if the side a is less than the perpendicular CD, the arc will not cut PQ and the construction will be impossible. If the side a is just equal to CD, the arc will touch PQ at D and the right triangle ADC fulfills the given conditions. If a is greater than CD but less than CA (b), the arc will cut PQ in two points, B_1 and B_2, and by joining each of them to C we form two triangles AB_1C and AB_2C, both of which satisfy the given conditions if the angle A be acute. Here, then, there are two solutions.

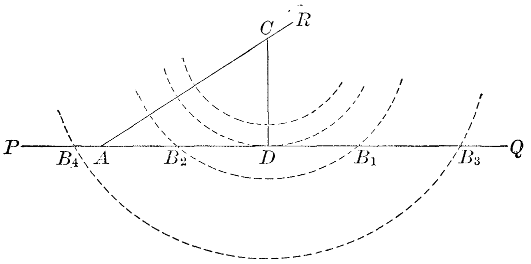

FIG. 33

Note that the angles AB_1C and AB_2C are supplementary. Why?

If a is greater than CA, the arc will cut PQ in two points, but on opposite sides of A. By joining B_3 with C we form a triangle, AB_3C, which fulfills the given conditions; but the triangle AB_4C, formed by joining B_4 with C, does not; for the angle CAB_4 is not equal to the given angle A. Hence, in Case 3, which is called the *ambiguous case*, the problem may be impossible, or it may have one solution, or it may have two solutions. *Two solutions are possible only when the side opposite the given angle is the less of the two given sides.*

The student should draw a separate figure for each case.

We shall now illustrate the trigonometrical solution by examples.

1. Given $A = 54°$, $a = 64$, $b = 53$, to find B, C, c.

By the law of sines, $\frac{\sin B}{b} = \frac{\sin A}{a}$. $\therefore \sin B = \frac{53 \times \sin 54°}{64}$.

This determines the value of $\sin B$. But any angle and its supplement have the same sine. We must, therefore, accept, as values of B, not only the acute angle found in the table corresponding to the computed value of $\sin B$, but also its supplement, unless a reason can be assigned for rejecting one of them. Applying logarithms to the equation above, we have

$$\log \sin B = \log 53 + \log \sin 54° - \log 64.$$

$$\begin{array}{rl}
\log 53 = & 1.72428 \\
\log \sin 54° = & \underline{9.90796 - 10} \\
 & 11.63224 - 10 \\
\log 64 = & \underline{1.80618 \qquad\quad} \\
\log \sin B = & 9.82606 - 10
\end{array}$$

The corresponding angle in the table is 42° 3′ 56″. Its supplement is 137° 56′ 4″. This value must be rejected; for, in this example, b is less than a and, therefore, by geometry, B must be less than A.

Hence, we have $B = 42° 3' 56''$.

To find C, we have $C = 180° - (A + B) = 83° 56' 4''$.

To find c, we have $\frac{c}{a} = \frac{\sin C}{\sin A}$.

$$\log c = \log a + \log \sin C - \log \sin A.$$

$$\begin{array}{rl}
\log 64 = & 1.80618 \\
\log \sin 83° 56' 4'' = & \underline{9.99756 \qquad\quad} \\
 & 11.80374 - 10 \\
\log \sin 54° = & \underline{9.90796 - 10} \\
\log c = & \underline{1.89578 \qquad\quad} \\
c = & 78.665.
\end{array}$$

Check. $\frac{c}{b} = \frac{\sin C}{\sin B}$.

Apply this test.

2. Let $a = 172$, $b = 213$, $A = 48°$.

To find B, we have $\dfrac{\sin B}{\sin A} = \dfrac{b}{a}$.

$$\therefore \log \sin B = \log b + \log \sin A - \log a.$$

$$\begin{array}{rl} \log 213 = & 2.32838 \\ \log \sin 48° = & \underline{9.87107 - 10} \\ & 12.19945 - 10 \\ \log 172 = & \underline{2.23553} \\ \log \sin B = & 9.96392 - 10 \end{array}$$

The corresponding angle in the table is 66° 58′; its supplement is 113° 2′. Both these angles fulfill the condition $B > A$, and we have two solutions. Denoting the two values of B by B_1 and B_2,

$$B_1 = 66° \, 58', \qquad B_2 = 113° \, 2'.$$

In the triangle AB_1C (Fig. 34) denote the angle ACB_1 by C_1 and the side AB_1 by c_1. Then,

$$C_1 = 180° - (A + B_1) = 65° \, 2'.$$

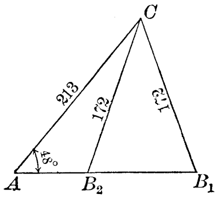

FIG. 34

To find c_1, we have $\dfrac{c_1}{a} = \dfrac{\sin C_1}{\sin A}$.

$$\therefore \log c_1 = \log a + \log \sin C_1 - \log \sin A.$$

$$\begin{array}{rl} \log 172 = & 2.23553 \\ \log \sin 65° \, 2' = & \underline{9.95739 - 10} \\ & 12.19292 - 10 \\ \log \sin 48° = & \underline{9.87107 - 10} \\ \log c_1 = & 2.32185 \\ c_1 = & 209.82. \end{array}$$

Similarly, in the triangle AB_2C_1 denote the angle ACB_2 by C_2 and the side AB_2 by c_2. Then,

$$C_2 = 180° - (A + B_2) = 18° \, 58';$$

and to find c_2, we have $\dfrac{c_2}{a} = \dfrac{\sin C_2}{\sin A}$.

$$\log c_2 = \log a + \log \sin C_2 - \log \sin A.$$

$$\begin{array}{rl} \log 172 = & 2.23553 \\ \log \sin 18° \, 58' = & \underline{9.51191 - 10} \\ & 11.74744 - 10 \\ \log \sin 48° = & \underline{9.87107 - 10} \\ \log c_2 = & 1.87637 \\ c_2 = & 75.227. \end{array}$$

Hence, we have for the required parts the two sets of values:

1. $B_1 = 66° 58'$, $C_1 = 65° 2'$, $c_1 = 209.82$.
2. $B_2 = 113° 2'$, $C_2 = 18° 58'$, $c_2 = 75.227$.

EXERCISES

Solve the following triangles:

1. Given $a = 540$, $b = 486$, $A = 32° 14'$, to find B, C, c.
Ans. $B = 28° 49' 57''$, $C = 118° 46' 3''$, $c = 883.4$.

2. Given $a = 481.34$, $b = 358.26$, $A = 36° 24'$, to find B, C, c. *Ans.* $B = 26° 12' 38''$, $C = 117° 23' 22''$, $c = 720.22$.

3. Given $a = 28$, $c = 15$, $A = 104°$, to find B, C, b.
Ans. $B = 44° 40' 54''$, $C = 31° 19' 6''$, $b = 20.291$.

4. Given $a = 22.5$, $b = 40.4$, $A = 42° 14'$, to find B, C, c.
Ans. 1. $B_1 = 41° 33' 52''$, $C_1 = 107° 2' 8''$, $c_1 = 516.29$.
2. $B_2 = 138° 26' 8''$, $C_2 = 10° 9' 52''$, $c_2 = 95.293$.

5. Given $a = 88$, $b = 136$, $A = 32°$, to find B, C, c.
Ans. $B = 54° 58' 53''$, $C = 93° 1' 7''$, $c = 165.83$.

6. Given $b = 480$, $c = 361$, $C = 46° 4'$, to find A, B, a.
Ans. 1. $B_1 = 73° 14' 30''$, $A_1 = 60° 41' 30''$, $a_1 = 437.13$.
2. $B_2 = 106° 45' 30''$, $A_2 = 27° 10' 30''$, $a_2 = 228.94$.

7. Given $b = 356$, $c = 716$, $B = 43°$, to find A, C, a.

8. Given $a = 342$, $b = 368$, $B = 18° 12'$, to find A, C, c.
Ans. $A = 16° 52' 26''$, $C = 144° 55' 34''$, $c = 677.07$.

9. Given $a = 41.4$, $b = 52.8$, $A = 40° 19'$, to find B, C, c.
Ans. 1. $B_1 = 55° 36' 20''$, $C_1 = 84° 4' 40''$, $c = 63.646$.
2. $B_2 = 124° 23' 40''$, $C_2 = 15° 17' 20''$, $c = 16.872$.

10. Given $a = 461$, $c = 513$, $C = 39° 56'$, to find A, B, b.
Ans. $A = 35° 13' 37''$, $B = 104° 50' 23''$, $b = 772.56$.

11. Given $a = 32.4$, $b = 23.1$, $A = 36° 17'$, to find B, C, c.

12. Given $b = 186$, $c = 127$, $B = 63° 12'$, to find A, C, a.

13. Given $A = 42° 14'$, $a = 52.5$, $b = 40.4$, to find B, C, c.

14. Given $B = 46° 28'$, $b = 241$, $c = 324$, to find A, C, a.

15. Given $C = 38° 15'$, $c = 48$, $b = 36$, to find A, B, a.

Without actually solving the following triangles, ascertain how many solutions are possible in each case:

16. Given $A = 36° 40'$, $a = 276$, $b = 251$.

17. Given $B = 41° 13'$, $b = 18$, $a = 43$.

18. Given $C = 43°$, $a = 27$, $b = 36$.

19. Given $A = 56°$, $a = 203$, $b = 222$.

20. Given $B = 33° 14'$, $b = 213$, $c = 165$.

49. We shall now derive formulas for finding the angles of a triangle when the sides are given, in order to solve Case 4.

Law of Cosines. *In any triangle the square of one side equals the sum of the squares of the other two sides minus twice the continued product of those sides and the cosine of their included angle,* or, in symbols, for the side a,

$$\mathbf{a^2 = b^2 + c^2 - 2\,bc \cos A.} \qquad (32)$$

To prove this, consider first the case where A is acute (Fig. 35). Draw the perpendicular BD. Then, $DC = b - AD$. Squaring, and adding $\overline{BD}^2$ to each member,

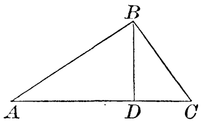

FIG. 35

$$\overline{DC}^2 + \overline{BD}^2 = b^2 - 2\,b \cdot AD + \overline{AD}^2 + \overline{BD}^2.$$

For $\overline{DC}^2 + \overline{BD}^2$ substitute a^2, and for $\overline{AD}^2 + \overline{BD}^2$ substitute c^2, and we have

$$a^2 = b^2 - 2\,b \cdot AD + c^2.$$

But
$$\frac{AD}{c} = \cos A.$$

$$\therefore AD = c \cos A.$$

Substituting this value for AD,

$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

If A is obtuse (Fig. 36), we have

$$CD = b + AD.$$

Squaring, and adding $\overline{BD}^2$ to each member,

$$\overline{CD}^2 + \overline{BD}^2 = b^2 + 2\,b \cdot AD + \overline{AD}^2 + \overline{BD}^2,$$

or $$a^2 = b^2 + c^2 + 2\,b \cdot AD.$$

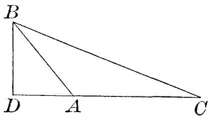

FIG. 36

But $\dfrac{AD}{c} = \cos BAD = -\cos A$, since the cosine of an angle equals the negative cosine of its supplement.

$$\therefore AD = -c \cos A.$$

Substituting above, $a^2 = b^2 + c^2 - 2\,bc \cos A.$

Similarly, $b^2 = a^2 + c^2 - 2\,ac \cos B.$

$c^2 = a^2 + b^2 - 2\,ab \cos C.$

50. From the law of cosines we can find the cosine of any angle of a triangle in terms of the sides. Thus, solving the above equations, we have

$$\mathbf{\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}}. \qquad (33)$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2\,ac}.$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab}.$$

These formulas can be used to find the angles when the sides are given.

EXAMPLE

Given $a = 8$, $b = 12$, $c = 15$, to find A.

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc} = \frac{144 + 225 - 64}{2 \times 12 \times 15} = \frac{305}{360} = 0.84722.$$

$$\therefore A = 32° \, 5'.$$

51. The formulas of the preceding article are not adapted to logarithmic computation [why?], and we proceed to derive formulas for the functions of the half angles, in which logarithms may be advantageously employed.

To express cos $\frac{1}{2}$ A in terms of a, b, c.

By formula (26),

$$2\cos^2\tfrac{1}{2}A = 1 + \cos A.$$

For $\cos A$ substitute its value as given in Sec. 50, and we have

$$\begin{aligned}2\cos^2\tfrac{1}{2}A &= 1 + \frac{b^2+c^2-a^2}{2\,bc}\\ &= \frac{2\,bc+b^2+c^2-a^2}{2\,bc}\\ &= \frac{(b+c)^2-a^2}{2\,bc}\\ &= \frac{(b+c+a)(b+c-a)}{2\,bc}.\end{aligned}$$

Now, let $\quad b+c+a=2\,s.$

Then, subtracting $2\,a$ from each member,

$$b+c-a=2\,(s-a).$$

$$\therefore 2\cos^2\tfrac{1}{2}A = \frac{4\,s\,(s-a)}{2\,bc}.$$

$$\therefore \cos\tfrac{1}{2}\mathbf{A} = \sqrt{\frac{\mathbf{s\,(s-a)}}{\mathbf{bc}}}. \tag{34}$$

Since $\frac{1}{2}A < 90°$, $\cos\frac{1}{2}A$ is positive, and the positive sign is to be used with the radical.

In like manner, $\quad \cos\frac{1}{2}B = \sqrt{\dfrac{s\,(s-b)}{ac}}.$

$$\cos\tfrac{1}{2}C = \sqrt{\frac{s\,(s-c)}{ab}}.$$

52. To express sin $\frac{1}{2}$ A in terms of a, b, c. By formula (25),

$$
\begin{aligned}
2\sin^2 \tfrac{1}{2}A &= 1 - \cos A \\
&= 1 - \frac{b^2 + c^2 - a^2}{2\,bc} \\
&= \frac{2\,bc - b^2 - c^2 + a^2}{2\,bc} \\
&= \frac{a^2 - (b^2 - 2\,bc + c^2)}{2\,bc} \\
&= \frac{a^2 - (b - c)^2}{2\,bc} \\
&= \frac{(a + b - c)(a - b + c)}{2\,bc} \\
&= \frac{2(s - c) \times 2(s - b)}{2\,bc}.
\end{aligned}
$$

$$\therefore \sin \tfrac{1}{2}\mathbf{A} = \sqrt{\frac{(\mathbf{s} - \mathbf{b})(\mathbf{s} - \mathbf{c})}{\mathbf{bc}}}. \qquad (35)$$

Similarly,

$$\sin \tfrac{1}{2}B = \sqrt{\frac{(s - a)(s - c)}{ac}}.$$

$$\sin \tfrac{1}{2}C = \sqrt{\frac{(s - a)(s - b)}{ab}}.$$

53. To express tan $\frac{1}{2}$ A in terms of a, b, c. From formulas (35) and (34), we have, by division,

$$\frac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} = \sqrt{\frac{(s - b)(s - c)}{bc} \div \frac{s(s - a)}{bc}}.$$

$$\therefore \tan \tfrac{1}{2}\mathbf{A} = \sqrt{\frac{(\mathbf{s} - \mathbf{b})(\mathbf{s} - \mathbf{c})}{\mathbf{s}(\mathbf{s} - \mathbf{a})}}. \qquad (36)$$

Similarly,

$$\tan \tfrac{1}{2}B = \sqrt{\frac{(s - a)(s - c)}{s(s - b)}}.$$

$$\tan \tfrac{1}{2}C = \sqrt{\frac{(s - a)(s - b)}{s(s - c)}}.$$

54. We are now prepared to solve

Case 4. *Given the three sides of a triangle, to find the three angles.*

We may find the values of the half angles by using the formulas for the sine, the cosine, or the tangent, but a modification of the tangent formula furnishes the simplest solution.

Thus, from formula (36),

$$\tan^2 \tfrac{1}{2} A = \frac{(s-b)(s-c)}{s(s-a)}.$$

Multiplying numerator and denominator by $s - a$,

$$\tan^2 \tfrac{1}{2} A = \frac{(s-a)(s-b)(s-c)}{s(s-a)^2},$$

or

$$\tan \tfrac{1}{2} A = \frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Denoting $\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$ by r, we have

$$\mathbf{\tan \tfrac{1}{2} A = \frac{r}{s-a}.} \qquad (37)$$

In like manner,

$$\tan \tfrac{1}{2} B = \frac{r}{s-b}.$$

$$\tan \tfrac{1}{2} C = \frac{r}{s-c}.$$

EXAMPLE

Given $a = 126$, $b = 322.6$, $c = 253$, to find A, B, C.

We have

$$\begin{aligned} a &= 126 \\ b &= 322.6 \\ c &= \underline{253} \\ 2s &= 701.6 \\ s &= 350.8. \\ s-a &= 224.8. \\ s-b &= 28.2. \\ s-c &= 97.8. \end{aligned}$$

$$\log r = \tfrac{1}{2}\left[\log(s-a) + \log(s-b) + \log(s-c) - \log s\right].$$

$$\begin{aligned}
\log(s-a) &= 2.35180\\
\log(s-b) &= 1.45025\\
\log(s-c) &= \underline{1.99034}\\
&\ 5.79239\\
\log s &= \underline{2.54506}\\
&\ 2\,\overline{)3.24733}\\
\log r &= 1.62367
\end{aligned}$$

Then,

$$\begin{aligned}
\tan\tfrac{1}{2}A &= \frac{r}{s-a}.\\
\log\tan\tfrac{1}{2}A &= \log r - \log(s-a).\\
\log r &= 11.62367 - 10\\
\log(s-a) &= \underline{\ 2.35180\qquad\ }\\
\log\tan\tfrac{1}{2}A &= \ \ 9.27187 - 10\\
\tfrac{1}{2}A &= 10^\circ\ 35'\ 33''.\\
A &= 21^\circ\ 11'\ 6''.
\end{aligned}$$

$$\begin{aligned}
\tan\tfrac{1}{2}B &= \frac{r}{s-b}.\\
\log r &= 1.62367\\
\log(s-b) &= \underline{1.45025}\\
\log\tan\tfrac{1}{2}B &= 0.17342\\
\tfrac{1}{2}B &= 56^\circ\ 8'\ 49''.\\
B &= 112^\circ\ 17'\ 38''.
\end{aligned}$$

$$\begin{aligned}
\tan\tfrac{1}{2}C &= \frac{r}{s-c}.\\
\log r &= 11.62367 - 10\\
\log(s-c) &= \underline{\ 1.99034\qquad\ }\\
\log\tan\tfrac{1}{2}C &= \ \ 9.63333 - 10\\
\tfrac{1}{2}C &= 23^\circ\ 15'\ 39''.\\
C &= 46^\circ\ 31'\ 18''.
\end{aligned}$$

Check. $A + B + C = 180^\circ\ 0'\ 2''$.

How many logarithms had to be looked up in solving by this method?

How many would have to be looked up in finding the half angles by the sine formulas? How many by the cosine formulas?

EXERCISE

Determine A in the above example by the use of formula (34).

EXERCISES

Solve the following triangles:

1. Given $a = 10$, $b = 11$, $c = 13$.
   Ans. $A = 48° 22' 8''$, $B = 55° 18' 12''$, $C = 76° 19' 42''$.
2. Given $a = 342$, $b = 436$, $c = 388$.
   Ans. $A = 48° 36' 56''$, $B = 73° 2' 26''$, $C = 58° 20' 38''$.
3. Given $a = 3820$, $b = 4536$, $c = 2654$.
   Ans. $A = 57° 14' 44''$, $B = 87° 0' 4''$, $C = 35° 45' 8''$.
4. Given $a = 4004$, $b = 5056$, $c = 7012$.
   Ans. $A = 34° 7' 20''$, $B = 45° 5' 54''$, $C = 100° 46' 50''$.
5. Given $a = 126$, $b = 322.6$, $c = 253$.
   Ans. $A = 21° 11' 6''$, $B = 112° 17' 38''$, $C = 46° 31' 18''$.
6. Given $a = 78$, $b = 131$, $c = 146$.
   Ans. $A = 32° 5' 16''$, $B = 64° 1' 26''$, $C = 83° 53' 22''$.
7. Given $a = 0.506$, $b = 1.234$, $c = 0.936$.
   Ans. $A = 21° 56' 8''$, $B = 114° 21' 18''$, $C = 43° 42' 34''$.
8. Given $a = 200$, $b = 150$, $c = 174$.
   Ans. $A = 75° 50'$, $B = 46° 39' 6''$, $C = 57° 3'$.
9. Given $a = 16.7$, $b = 32.1$, $c = 23$.
   Ans. $A = 29° 51' 50''$, $B = 106° 50' 18''$, $C = 43° 17' 54''$.
10. Given $a = 19.61$, $b = 26.41$, $c = 13.54$.
    Ans. $A = 46° 3' 40''$, $B = 104° 7' 36''$, $C = 29° 48' 50''$.
11. Given $a = 20.9$, $b = 30.3$, $c = 41.8$.
12. Given $a = 521$, $b = 624$, $c = 728$.
13. Given $a = 0.921$, $b = 0.1264$, $c = 0.714$.
14. Given $a = 1045$, $b = 1523$, $c = 2286$.
15. Given $a = 426$, $b = 150$, $c = 174$.

55. To find the area of a triangle. We know, from geometry, that the area of a triangle equals one half the product of its base and altitude. That is, $K = \frac{1}{2} bp$, where $K =$ area, $b =$ base, $p =$ altitude.

1. To find the area in terms of *two sides and their included angle.*

Let the given parts be a, b, C. Draw the perpendicular $BD = p$ (Figs. 35 and 36). We have

$$K = \tfrac{1}{2} bp.$$

But $$p = a \sin C.$$

$$\therefore \mathbf{K = \tfrac{1}{2} ab \sin C.} \tag{38}$$

Or, *the area of a triangle equals one half the product of two sides times the sine of their included angle.*

2. If *one side and two angles* are given, find another side by the law of sines and apply formula (38).

3. To find the area in terms of *the three sides.*

By formula (38), $K = \tfrac{1}{2} ab \sin C.$

By formula (20), and the formulas for $\sin \tfrac{1}{2} C$ and $\cos \tfrac{1}{2} C$ in Secs. 52 and 51,

$$\sin C = 2 \sin \tfrac{1}{2} C \cos \tfrac{1}{2} C = 2\sqrt{\frac{(s-a)(s-b)}{ab}} \times \sqrt{\frac{s(s-c)}{ab}}$$
$$= \frac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}.$$

Substituting in formula (38),

$$\mathbf{K = \sqrt{s(s-a)(s-b)(s-c)}}. \tag{39}$$

To find the area of a right triangle, one leg may be taken as the base and the other as the altitude. If either leg is unknown, find it by using the given parts.

EXERCISES

Find the area of each of the following triangles:

1. $a = 480$, $b = 360$, $C = 62° 42' 16''$. *Ans.* 76778.
2. $b = 345$, $c = 423$, $A = 38° 17'$.
3. $A = 50° 50' 50''$, $B = 60° 22' 16''$, $a = 481.622$. *Ans.* 121197.
4. $a = 484$, $b = 346$, $c = 422$. *Ans.* 71256.4.
5. $a = 143$, $b = 257$, $c = 198.4$.

MISCELLANEOUS EXERCISES

1. From the ends of a base line, EW, 900 feet long, observations were taken on an object, B, and the following angles measured: $EWB = 85° 19'$, $WEB = 67° 51'$. Find EB and WB.
Ans. 1987.4 feet; 1846.3 feet.

2. Two cyclists start at 4 p.m. from the intersection of two straight roads which meet at an angle of 40°. One travels at the rate of 8 miles per hour and the other rides a 28-inch wheel, averaging 120 revolutions per minute. How far apart are they at 5 : 15 p.m.? *Ans.* 8.043 miles.

3. In a triangle ABC, $B = 75°$, $C = 40°$, $BC = 260$. BD is drawn to meet AC in D so that angle $ADB = 35°$. Find AD. *Ans.* 107.4.

4. An observer in a balloon finds that the angles of depression of two points on level ground and in the same vertical plane with the balloon are 33° 22′ and 22° 18′ respectively. If the distance between the objects is 1200 feet, find the height of the balloon. *Ans.* 1304.7 feet.

5. From a point, A, in a road running north and south, the bearing of a church steeple was found to be S. 36° 22′ E.; and from a point, B, 1986 feet south of A, it was found to bear S. 83° 15′ E. Find the shortest distance from the church to the road. *Ans.* 1602.1 feet.

6. A roof is inclined to the horizontal an angle of 45°, and the ridgepole runs east and west. At midday a lightning rod extending 8 feet above the ridgepole casts a shadow on the roof 15 feet long. Find the altitude of the sun. *Ans.* 60° 18′ 52″.

7. From the ends of a base line, MN, 450 feet long, observations were made on a point, P, with the following results: $\angle NMP = 49° 51'$, $\angle MNP = 110° 50'$. Find MP and NP. *Ans.* 1271.4; 1039.8.

8. A roof is inclined to the horizontal at an angle of 60°. A flagstaff standing on the ridgepole breaks at a point 12 feet from its base, and falls so that the end rests on the roof at a point 22 feet from the base of the staff and 16 feet from the ridgepole. Find the height of the staff.
Ans. 43 feet.

9. From two points, A' and B', in line with the foot of a tower (A the more remote), the angles of elevation of the top of the tower are A and B respectively. If $AB = a$, show that the height of the tower is

$$\frac{a \sin A \sin B}{\sin (B - A)}.$$

10. A and B are two points in the same horizontal plane with Q, the foot of a tower. At A and B the angles of elevation of P, the top of the tower, are measured; also the angles QAB, QBA. Denoting these four angles by A, B, A', B' respectively, and AB by c, and the height of the tower by h, show that $h = \dfrac{c \sin A'}{\sin (A' + B')} \tan A = \dfrac{c \sin B'}{\sin (A' + B')} \tan B$.

11. In a trapezoid $ABCD$, $AD = BC = c$, $AB = a$, $DC = b$, $DB = d$, $\angle ABD = m$, $\angle ADB = n$. Show that $d = \dfrac{b \sin (m + n)}{\sin (2m + n)}$.

12. If an angle of a triangle be doubled while the including sides remain constant, will the area be increased or diminished?

13. A base line, EW, 900 feet long was measured on the top of a dam and the following angles were read: $WEA = 110° 50'$, $BEW = 67° 51'$, $AWE = 49° 51'$, $EWB = 85° 19'$. Find AB. *Ans.* 1492.4 feet.

14. The angles of elevation of the top of a tower from two points, A and B, in line with its base are 45° and 30° respectively. If $AB = a$, show that the height of the tower is $\dfrac{a}{\sqrt{4 - 2\sqrt{3}}}$.

15. A boy flies a kite over a level plane when the sun is due south; the altitude of the sun is 60° and the wind blows in the direction N. 50° W. The line from the boy's position to the shadow of the kite is 430 feet long and bears N. 32° W. Find the height of the kite. *Ans.* 300.44 feet.

*CHAPTER VII

GEOMETRICAL APPLICATIONS

56. To find the radius R of the circumcircle. A circle passing through the vertices of a triangle is called the *circumscribed circle,* or *circumcircle* of the triangle. Its center is called the *circumcenter.* Let O (Fig. 37) be the circumcenter of the triangle ABC. Draw the perpendicular OD. Then,

$$\angle BOD = A. \quad \text{Why?}$$

$$\therefore \sin A = \frac{BD}{R} = \frac{\frac{1}{2}a}{R}.$$

$$\therefore R = \frac{a}{2\sin A}.$$

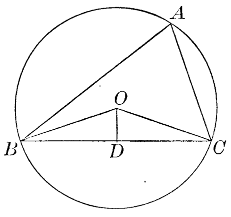

FIG. 37

Similarly, $R = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C}$.

Hence, $R = \dfrac{a}{2\sin A} = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C}$

57. To find the area of a triangle in terms of the radius of the circumcircle. By formula (38), we have

$$K = \tfrac{1}{2}ab\sin C.$$

But $$\sin C = \frac{c}{2R}.$$

$$\therefore K = \frac{abc}{4R}.$$

EXERCISE

Prove $K = 2R^2 \sin A \sin B \sin C$.

58. To find the radius of the incircle of a triangle. The inscribed circle of a triangle is called the *incircle*. Let O be the center of the incircle of the triangle ABC (Fig. 38) and r its radius. Then, by geometry, O is the intersection of the bisectors of the angles A, B, C. Draw OD, OE, OF to the points of contact. Then,

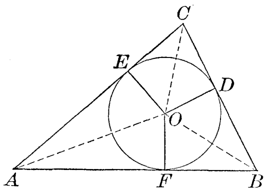

FIG. 38

$$BD + DC = a.$$
$$CE + EA = b.$$
$$AF + FB = c.$$

Adding, and remembering that, by geometry, $AF = AE$, $DC = CE$, $FB = DB$,

$$2\,AF + 2\,BD + 2\,CD = a + b + c = 2\,s.$$
$$\therefore AF + (BD + CD) = s,$$

or
$$AF + a = s.$$
$$\therefore AF = s - a.$$

Then,
$$\tan OAF = \frac{OF}{AF},$$

or
$$\tan \tfrac{1}{2} A = \frac{r}{s-a},$$

or
$$r = (s - a) \tan \tfrac{1}{2} A.$$

Similarly,
$$r = (s - b) \tan \tfrac{1}{2} B.$$
$$r = (s - c) \tan \tfrac{1}{2} C.$$

Hence, $r = (s - a) \tan \frac{1}{2} A = (s - b) \tan \frac{1}{2} B = (s - c) \tan \frac{1}{2} C.$

59. To find the radii of the escribed circles. A circle touching one side of a triangle and the other two produced is called an *escribed* circle of the triangle. Let O (Fig. 39) be the center of the escribed circle touching the side BC (a) of the triangle ABC. Denote its radius by r_a. The points of contact being D, E, F, we have, by geometry,

$$AE = AF, \quad BD = BF, \quad CD = CE.$$

$$\therefore 2s = AB + BD + DC + AC$$
$$= AB + BF + CE + AC$$
$$= AF + AE$$
$$= 2AF.$$
$$\therefore AF = s.$$

But $OAF = \frac{1}{2}A$. Why?

$$\therefore \tan\tfrac{1}{2}A = \frac{r_a}{s}.$$

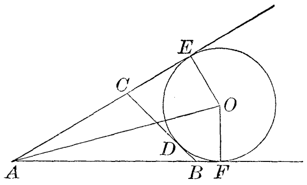

FIG. 39

$$\therefore r_a = s\tan\tfrac{1}{2}A.$$

Similarly,
$$r_b = s\tan\tfrac{1}{2}B.$$
$$r_c = s\tan\tfrac{1}{2}C.$$

60. To find the area of a triangle in terms of the radius of the incircle. We have, from Fig. 38,

$$\triangle BOC = \tfrac{1}{2}ra.$$
$$\triangle AOC = \tfrac{1}{2}rb.$$
$$\triangle AOB = \tfrac{1}{2}rc.$$

By addition,
$$\triangle ABC = \tfrac{1}{2}r(a + b + c),$$
or
$$K = rs.$$

61. To express K in terms of the radius of the incircle and the radii of the escribed circles. From Sec. 54,

$$\tan\tfrac{1}{2}A = \frac{r}{s-a}.$$

From Sec. 59,
$$\tan\tfrac{1}{2}A = \frac{r_a}{s}.$$
$$\therefore \frac{r}{s-a} = \frac{r_a}{s}.$$
$$\therefore rs = r_a(s-a).$$

But
$$K = rs.$$
$$\therefore K = r_a(s-a).$$

Similarly, $K = r_b(s-b)$.

$K = r_c(s-c)$.

By multiplication, $K^4 = rr_ar_br_cs(s-a)(s-b)(s-c)$.

But, by formula (39),

$s(s-a)(s-b)(s-c) = K^2$.

$\therefore K^2 = rr_ar_br_c$,

or $K = \sqrt{rr_ar_br_c}$.

62. We have, from equations in the preceding section,

$$\frac{1}{r_a} = \frac{s-a}{K}.$$

$$\frac{1}{r_b} = \frac{s-b}{K}.$$

$$\frac{1}{r_c} = \frac{s-c}{K}.$$

$$\therefore \text{by addition, } \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c} = \frac{3s-(a+b+c)}{K} = \frac{s}{K} = \frac{s}{rs} = \frac{1}{r}.$$

EXERCISE

Prove the relation $\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}$, where h_a, h_b, h_c are the altitudes of the triangle ABC and r the radius of the incircle.

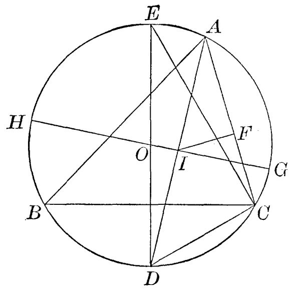

FIG. 40

63. To find the distance from circumcenter to the incenter in terms of R and r. Let O be the circumcenter and I the incenter of the triangle ABC (Fig. 40). Draw the diameter DE perpendicular to BC. I is in the bisector of the angle BAC. $\therefore AI$ produced passes through D. Through I draw the diameter HIG. Then, by geometry,

$$HI \times IG = AI \times ID,$$

or $$(R + OI)(R - OI) = AI \times ID.$$

$$\therefore R^2 - \overline{OI}^2 = AI \times ID.$$

To find the value of OI, therefore, it is necessary to find values for AI and ID. Draw IF perpendicular to AC. Then

$$AF = r,$$

and, from the right triangle AFI,

$$AI = \frac{r}{\sin \frac{1}{2} A}.$$

Now, prove $DI = DC$. $\angle DIC$ is an exterior angle of the triangle AIC.

$$\therefore \angle DIC = \angle IAC + \angle ACI = \tfrac{1}{2} A + \tfrac{1}{2} C.$$

From the figure, $$\angle DCI = \angle DCB + \angle BCI$$
$$= \tfrac{1}{2} A + \tfrac{1}{2} C. \quad \text{Why?}$$

$$\therefore \angle DIC = \angle DCI.$$

$$\therefore ID = DC.$$

Now, from the right triangle DCE,

$$DC = DE \sin DEC.$$

But $$DE = 2R,$$

and $$\angle DEC = \tfrac{1}{2} A$$

$$\therefore DC = ID = 2R \sin \tfrac{1}{2} A.$$

Substituting these values for AI and ID above,

$$R^2 - \overline{OI}^2 = \frac{2}{\sin \frac{1}{2} A} \times 2R \sin \tfrac{1}{2} A.$$

$$\therefore OI = \sqrt{R^2 - 2Rr}.$$

64. To find the area of an inscribed quadrilateral in terms of its sides. In Fig. 41 let

$AB = a,\ BC = b,\ CD = c,\ DA = a,$ area of $ABCD = S.$

Now, area of $\triangle ABC = \frac{1}{2} ab \sin B,$

and area of $\triangle ADC = \frac{1}{2} cd \sin D = \frac{1}{2} cd \sin B$,

since $\angle D$ is the supplement of $\angle B$.

Why so?

$$\therefore S = \tfrac{1}{2}(ab + cd)\sin B.$$

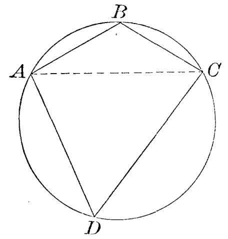

FIG. 41

It is necessary, therefore, to find sin B.

In triangle ABC we have, by the law of cosines (Sec. 49),

$$\overline{AC}^2 = a^2 + b^2 - 2\,ab \cos B,$$

and in triangle ADC,

$$\overline{AC}^2 = c^2 + d^2 - 2\,cd \cos D.$$

Equate the two values of $\overline{AC}^2$, and for $\cos D$ substitute $-\cos B$, and solve for $\cos B$. We find

$$\cos B = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}.$$

$$\therefore \sin^2 B = 1 - \cos^2 B = 1 - \left[\frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}\right]^2$$

$$= \frac{4(ab + cd)^2 - (a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2}$$

$$= \frac{[2(ab+cd)+(a^2+b^2-c^2-d^2)][2(ab+cd)-(a^2+b^2-c^2-d^2)]}{4(ab+cd)^2}$$

$$= \frac{[a^2+2ab+b^2-(c^2-2cd+d^2)][c^2+2cd+d^2-(a^2-2ab+b^2)]}{4(ab+cd)^2}$$

$$= \frac{[(a+b)^2-(c-d)^2][(c+d)^2-(a-b)^2]}{4(ab+cd)^2}$$

$$= \frac{(a+b+c-d)(a+b-c+d)(c+d+a-b)(c+d-a+b)}{4(ab+cd)^2}.$$

Let $2s = a + b + c + d$.

Then,
$$2s - 2a = b + c + d - a = 2(s - a).$$
$$2s - 2b = a - b + c + d = 2(s - b).$$
$$2s - 2c = a + b - c + d = 2(s - c).$$
$$2s - 2d = a + b + c - d = 2(s - d).$$

$$\therefore \sin^2 B = \frac{4(s-a)(s-b)(s-c)(s-d)}{(ab+cd)^2}.$$

$$\sin B = \frac{2}{ab+cd}\sqrt{(s-a)(s-b)(s-c)(s-d)}.$$

Substituting this value of $\sin B$ in the expression for S above, we have

$$S = \tfrac{1}{2}(ab+cd) \times \frac{2}{ab+cd}\sqrt{(s-a)(s-b)(s-c)(s-d)},$$

or $$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}.$$

65. Angles that can be constructed geometrically, and their Functions. Let us inquire what angles, expressible exactly in degrees, *i.e.*, containing an integral number of degrees, can be constructed with ruler and compass.

The construction can be made to depend in each case upon the inscription of a regular polygon in a circle. We can, by geometry, inscribe regular polygons of three, four, five, fifteen sides. The central angles subtended by the sides of these polygons are 120°, 90°, 72°, 24° respectively. The regular polygon of seventeen sides and certain others may also be inscribed with ruler and compass, but the corresponding central angles are not expressible exactly in degrees, and these cases will not be discussed.

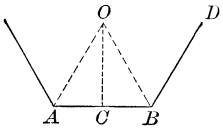

FIG. 42

Since an angle may be bisected, we may construct the following angles, without introducing fractions of a degree:

120°, 60°, 30°, 15°; 90°, 45°; 72°, 36°, 18°, 9°; 24°, 12°, 6°, 3°.

Each of these angles is a multiple of 3°. Corresponding to each of these central angles, we may construct a regular inscribed polygon, and the interior angles of the polygons must be added to the list of angles that can be constructed with ruler and compass. But if ABD (Fig. 42) be one of such interior angles, we have

$$OBA = \tfrac{1}{2}ABD = 90° - \tfrac{1}{2}O. \quad \therefore ABD = 180° - O.$$

And since 180° and O are multiples of 3°, ABD is a multiple of 3°; so also are the angles formed by bisecting these interior angles successively as many times as it can be done without introducing fractions of a degree. Hence, every angle expressible exactly in degrees, that can be constructed by ruler and compass, is a multiple of 3°. All such angles are included in the series 3°, 6°, 9°, 12°, 15°, ⋯.

The angle 3° is constructed, as we have seen, by beginning the central angle 24° and bisecting successively three times, giving the angles 12°, 6°, 3°.

To construct any multiple of 3° we have only to apply the chord of this angle the requisite number of times to the circumference, and join the first and last points of division to the center.

If we admit now angles not expressible exactly in degrees, we may, by successive bisections, construct any angle included in the form $\frac{N}{2^n}$, where N is 3°, or any multiple of 3°, and n any integer. And since angles may be added or subtracted, we may also construct any angle included in the more general form $\frac{N}{2^n} \pm \frac{M}{2^m}$, where N and M are multiples of 3°, and n and m any integers.

EXERCISES

1. Make a list of all the acute angles expressible in degrees that can be constructed geometrically.

2. Assume values for N and n in the expression $\frac{N}{2^n}$ and determine four angles, each less than 3°, that can be constructed geometrically. Thus, let

$$N = 6^\circ,\ n = 3. \quad \text{Then, } \frac{N}{2^n} = \frac{6^\circ}{8} = 45'.$$

We shall now show how to find the functions of 3°, and from them the functions of any multiple of 3°. Since

$3° = 18° - 15°$, we shall first show how to find the functions of 15° and 18°.

By formula (25),

$$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2},$$

and, by formula (26),

$$2\cos^2 15° = 1 + \cos 30° = 1 + \frac{\sqrt{3}}{2}.$$

$$\therefore \sin 15° = \frac{\sqrt{2-\sqrt{3}}}{2}.$$

$$\cos 15° = \frac{\sqrt{2+\sqrt{3}}}{2}.$$

From either of these the other functions may be found.

For the angle 18° let $AB = x$ be the side of the regular inscribed decagon, and let $AO = R$ (Fig. 42).

Then, by geometry,

$$R - x : x = x : R;$$

whence,
$$x = \frac{R(\sqrt{5}-1)}{2}.$$

$$\therefore \sin 18° = \frac{AC}{AO} = \frac{\sqrt{5}-1}{4}.$$

$$\therefore \cos 18° = \frac{\sqrt{5+\sqrt{5}}}{2}.$$

Now,

$$\sin 3° = \sin(18° - 15°) = \sin 18° \cos 15° - \cos 18° \sin 15°,$$

or
$$\sin 3° = \frac{\sqrt{5}-1}{2} \cdot \frac{\sqrt{2+\sqrt{3}}}{2} - \frac{\sqrt{5+\sqrt{5}}}{2} \cdot \frac{\sqrt{2-\sqrt{3}}}{2}.$$

The other functions of 3° may be found in a similar way, or by Sec. 32.

For sin 6°, put $x = 30°$ in formula (20).

For sin 9°, put $x = 6°$, $y = 3°$ in formula (9).

By successive applications of formulas (9) or (11) we can find the sine of the sum or difference of any two multiples of 3° whose sines and cosines have been previously found. Having found the sine and cosine of any angle in the series of multiples of 3°, we may find the sine and cosine of half the angle by formulas (25) and (26). We may, therefore, find the sine, the cosine, and the other functions of any angle included in the form $\frac{N}{2^n} \pm \frac{M}{2^m}$, given above.

MISCELLANEOUS EXERCISES

1. Find the values of the following in the form of surds as above: $\cos 3^\circ$, $\cos 33^\circ$, $\sin 57^\circ$, $\cos 6^\circ$, $\sin 93^\circ$, $\cos 12^\circ$, $\sin 36^\circ$, $\cos 7^\circ 30'$.

2. Prove the relation $P^2 = H^2 + D^2$, where P, H, D are the sides of the regular inscribed pentagon, hexagon, decagon respectively.

D has been determined above; from a figure find $P = 2r \sin 36^\circ$.

3. Find the length of the side of the regular inscribed polygon of 60 sides in terms of the radius R.

CHAPTER VIII

RADIAN MEASURE, OR CIRCULAR MEASURE; INVERSE FUNCTIONS; GENERAL EXPRESSIONS

66. Radian Measure, or Circular Measure. To measure any magnitude is to find its ratio to some magnitude of the same kind taken as a unit. So, to measure an angle is to find its ratio to some constant angle taken as a unit. What the unit angle shall be, is entirely arbitrary. A unit in common use is an angle of one degree; other units are angles of one minute, one second, a right angle. Still another unit, whose use is convenient in analytic work, is an angle called a *radian*. Let A (Fig. 43) be the center of a circle, and BAC a central angle whose sides intercept an arc BC equal in length to the radius r. The angle so determined is called a *radian*, or the *unit of circular measure*. The circular measure of an angle, therefore, is *its ratio to one radian*. To obtain a relation between degrees and radians, compare the angle BAC with the straight angle BAD. By geometry, central angles in the same circle have the same ratio as their arcs.

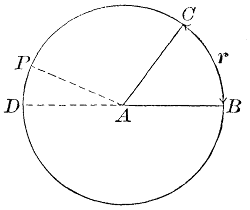

FIG. 43

$$\therefore \frac{\angle BAD}{\angle BAC} = \frac{\text{arc } BCD}{\text{arc } BC}.$$

But $\qquad \text{arc } BCD = \pi r$ [why?],

and, by hypothesis, $\qquad \text{arc } BC = r.$

Hence, $$\frac{\text{one straight angle}}{\text{one radian}} = \frac{\pi r}{r} = \pi.$$

$\therefore$ one straight angle equals π times one radian,

or **180 degrees = π radians.** (40)

This relation shows that *the radian is a constant angle* and enables us to convert degrees into radians, or radians into degrees. By dividing each member of formula (40) by π, we find one radian to be $\frac{180^\circ}{\pi} = 57^\circ.29577+$ or $57^\circ.3$ nearly.

EXAMPLES

1. How many radians in an angle of 105°? of 90°?

From formula (40), $$1^\circ = \frac{\pi}{180} \text{ radians.}$$

$$\therefore 105^\circ = \frac{105\,\pi}{180} \text{ radians} = \tfrac{7}{12}\,\pi \text{ radians.}$$

For π substitute 3.1416, and we find

$$105^\circ = 1.8326 \text{ radians.}$$

Similarly, $$90^\circ = \frac{90\,\pi}{180} \text{ radians} = \frac{\pi}{2} \text{ radians.}$$

The circular measure of a right angle is, therefore, $\frac{\pi}{2}$; *and for a straight angle it is* π.

2. Express $2\frac{1}{2}$ radians in degrees.

From formula (40), $$1 \text{ radian} = \frac{180}{\pi} \text{ degrees.}$$

$$\therefore 2\tfrac{1}{2} \text{ radians} = \frac{5}{2} \times \frac{180}{\pi} \text{ degrees.}$$

For π substitute 3.1416, and reduce.

The symbols °, ′, ″ for degrees, minutes, seconds are familiar. When the numerical value of an angle is given without the use of such symbols or without naming the unit, it is understood that the unit is one radian. Thus, an angle $\frac{2}{3}$ means $\frac{2}{3}$ of a radian; an angle $\frac{\pi}{2}$ means one half π radians.

As a symbol for the unit angle, one radian, it is proposed to use the Greek letter ρ, writing $2\,\rho$, $\frac{3}{4}\,\rho$, $\frac{3}{5}\,\pi\rho$, for two radians, three fourths of a radian, three fifths of π radians.

67. To express the length of an arc in terms of its central angle and the radius of the circle. Let s be the number of linear units in the arc BCP (Fig. 43); θ the number of radians in its central angle BAP. Then, by geometry,

$$\frac{\text{arc } BCP}{\text{arc } BC} = \frac{\angle BAP}{\angle BAC},$$

or

$$\frac{s}{r} = \frac{\theta}{1},$$

since $\angle BAC$ is one radian.

$$\therefore \mathbf{s} = \mathbf{r\theta}. \qquad (41)$$

EXERCISES

1. Express in radians 360°, 225°, 120°, 90°, 60°, 45°, 30°, 18°.

Give the results first in terms of π; then substitute the numerical value of π. Thus, $360° = 2\pi$ radians $= 6.2832\,\rho$.

2. Express in degrees one radian, $2.4\,\rho$, $\frac{4}{5}\pi\rho$, $\frac{5}{12}\pi\rho$, $1.6\,\rho$.

Reduce decimals of a degree to minutes and seconds.

3. Express in radians 42°.25, 64° 15′ 24″, 144° 18′ 36″, 5° 44′.

4. Find in degrees and also in radians the central angles subtended by the sides of regular polygons of 3, 7, 8, 9, 10, 15, n sides.

5. Find the lengths of the subtended arcs in Example 4, if the radius of the circle is 16 feet.

6. In a circle whose radius is 236 feet, what is the length of the arc corresponding to a central angle of 40°? of 128° 43′?

7. In a circle whose radius is 16 inches, the length of an arc is 3 feet. How many radians in its central angle? How many degrees?

8. Express in radians the angles of the following regular polygons: a square, a pentagon, a hexagon, an octagon, a decagon, a dodecagon.

9. Regarding the earth as a sphere, radius 3956 miles, what is the length of an arc of 1°? of 1″?

10. Prove that the area of a sector of a circle is $\frac{1}{2}r^2\theta$, where θ is the number of radians in the central angle and r the radius.

11. Find the number of seconds in one radian.

* **68. The 57.3 Rule.** Let ABC (Fig. 44) be a triangle in which the angle BAC is small and the angle B nearly 90°. Then the including sides will be nearly equal, and the short side BC, or a, will differ in length but little from the arc BD, whose center is at A and whose radius is c.

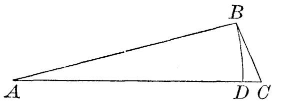

Fig. 44

Let A be the number of *degrees* and θ the number of *radians* in the angle BAC. Then, since (Sec. 66) one radian equals 57.3°, we have

$$\theta = \frac{A}{57.3}.$$

Also, by formula (41), $BD = c \cdot \theta = \dfrac{c \cdot A}{57.3}.$

Replacing BD by a, which is nearly equal to it, and clearing of fractions, we have approximately

$$a \times 57.3 = c \times A. \tag{42}$$

This relation is known as the *57.3 Rule.* It may be stated as follows:

The short side multiplied by 57.3 equals one of the long sides multiplied by the small angle in degrees.

Formula (42) may be used to find the short side BC when A is given, or A when BC is given. This method is sufficiently accurate for practical purposes when the small angle is less than 6°.

EXERCISES

Apply the 57.3 Rule to the solution of the following problems:

1. A straight roadway is 1320 feet long and has a rise of 21 feet. Find its inclination to the horizontal. *Ans.* 54′ 42″.

2. The diameter of the moon subtends an angle of 31′ at the earth, and the distance from the moon to the earth is about 240,000 miles. Find the diameter of the moon.
Ans. 2164 miles.

3. The diameter of the sun subtends an angle of 32′ at the earth, and the distance from the sun to the earth is about 93,000,000 miles. Find the diameter of the sun.
Ans. 865,620 miles.

4. The diameter of the earth, 7912 miles, subtends an angle of 17″.6 at the sun. Find the distance from the earth to the sun. *Ans.* 92,368,600 miles.

5. The radius of the earth's orbit is about 93,000,000 miles. If the angle formed by lines drawn from the nearest fixed star to the extremities of this radius form an angle of 0″.913, what is the distance from the star to the earth and how long does it take a ray of light to traverse this distance, the velocity of light being about 186,000 miles per second?

6. A surveyor, wishing to know the distance between two points, A and B, on opposite sides of a river, measured the angle $BAC = 5° 44'$; the distance BC was found by an assistant to be 326 feet. BC being nearly perpendicular to AB, find the length of AB. *Ans.* 3258 feet.

7. Telescopes at the ends of a line 200 feet long on the deck of a ship are turned upon a point on a distant fort. The lines of sight are found to make angles of 89° 17′ and 89° 13′ respectively with the given line. Find the distance from the ship to the fort. *Ans.* 7640 feet.

8. What is the angle subtended by the radius of the earth's orbit at the pole star, if it takes its light 42.4 years to reach the earth?

9. The diameter of a sphere is 1 yard. At what distance will it subtend an angle of 1″?

10. Find the length of an arc of 1′ on the earth's surface, regarding the earth as a sphere, radius 3956 miles.

INVERSE FUNCTIONS, OR ANTIFUNCTIONS

69. From the right triangle ACB (Fig. 45), we have

$$\sin A = \tfrac{3}{5}.$$

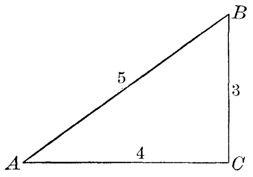

FIG. 45

We write $A = \sin^{-1}\frac{3}{5}$,

which is read: *A is an angle whose sine is $\frac{3}{5}$.*

The -1 is not used here as an exponent. From the figure, A is also an angle whose cosine is $\frac{4}{5}$. This is expressed, similarly, $A = \cos^{-1}\frac{4}{5}$; so for the other functions.

Likewise, $\sin^{-1}x$ is an abbreviation for the *angle whose sine is x.* $\sin^{-1}x$ is also called the *antisine of x,* or the inverse sine of x.

$A = \sin^{-1}x$ and $x = \sin A$ are equivalent statements. We have seen (Sec. 25) that one function does not determine an angle absolutely. For example, any angle and its supplement have the same sine; and if either of them be increased by 360°, the resulting angles will also have the same sine. The least positive value of an antifunction is called its *principal value.* Thus, the least positive angle having $\frac{1}{2}$ for its sine is 30°; *i.e.*, the principal value of $\sin^{-1}\frac{1}{2}$ is 30°; the principal value of $\sec^{-1}(-2)$ is 120°. The principal value of an antifunction is sometimes indicated by the use of a capital letter. Thus,

$$\mathrm{Sin}^{-1}\tfrac{1}{2} = 30°.$$

EXERCISES

1. Construct the following angles:

$$\text{Cos}^{-1}\tfrac{1}{2},\quad \text{Tan}^{-1}\tfrac{3}{4},\quad \text{Csc}^{-1}(-2),\quad \text{Tan}^{-1}\tfrac{8}{15}.$$

2. Find the remaining functions of each of the above angles.

3. What is the tangent of $\text{Sin}^{-1}\tfrac{12}{13}$?

4. Express $\tan^{-1} b$ as an anticosine.

Solution. Let $A = \tan^{-1} b.$

Then, $\tan A = b.$

$$\therefore\ \sec A = \sqrt{1 + b^2},$$

and $\cos A = \dfrac{1}{\sqrt{1 + b^2}}.$

$$\therefore\ A = \tan^{-1} b = \cos^{-1}\frac{1}{\sqrt{1 + b^2}}.$$

5. Show that $\text{Sin}^{-1} a + \text{Cos}^{-1} a = \dfrac{\pi}{2}\rho$, a being numerically not greater than 1.

6. Express in radians $\text{Cot}^{-1} 1$, $\text{Sec}^{-1}\dfrac{2}{\sqrt{3}}$, $\sin^{-1} 1$, $\cos^{-1} 0$.

7. Prove $\cos^{-1}\dfrac{r - y}{r} = \text{versin}^{-1}\dfrac{y}{r}.$

8. Prove $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\dfrac{a + b}{1 - ab}.$

Write $x = \tan^{-1} a, \qquad y = \tan^{-1} b.$

Then, $\tan x = a, \qquad \tan y = b.$

Find $\tan(x + y)$ in terms of a and b by formula (13).

Prove the following relations:

9. $\text{Tan}^{-1}(-1) + \text{Tan}^{-1}\tfrac{3}{4} = \text{Tan}^{-1}(-\tfrac{1}{7}).$

10. $\text{Tan}^{-1}\tfrac{1}{7} + 2\,\text{Tan}^{-1}\tfrac{1}{3} = \dfrac{\pi}{4}\rho.$

11. $\text{Sin}^{-1}\dfrac{1}{\sqrt{5}} + \text{Cot}^{-1} 3 = \dfrac{\pi}{4}\rho.$

12. $4\,\text{Tan}^{-1}\tfrac{1}{5} - \text{Tan}^{-1}\tfrac{1}{239} = \dfrac{\pi}{4}\rho.$

13. $\text{Tan}^{-1}\tfrac{1}{5} - \text{Tan}^{-1}\tfrac{1}{70} + \text{Tan}^{-1}\tfrac{11}{16} = \dfrac{\pi}{4}\rho.$

GENERAL VALUES OF ANTIFUNCTIONS

***70. To find an expression which shall include all angles having a given sine.** Let the value of the given sine be s. s must, of course, be less than 1, and we shall suppose it to be positive.

Let XOP (Fig. 46) be the least positive angle that has its sine equal to s. Let the value of XOP in radians be θ.

Its supplement, XOP', or $\pi - \theta$, will also have s for its sine; and if θ be increased by 2π radians, or m times 2π radians where m is any whole number, the terminal line will again come into the position OP and each of the angles formed will have its sine equal to s. All such angles are included in the expression

$$m \cdot 2\pi + \theta, \text{ or } 2m\pi + \theta. \quad \text{(a)}$$

Likewise, if XOP', or $\pi - \theta$, be increased by m times 2π, each of the resulting angles will have s for its sine. All such angles are included in the expression

$$m \cdot 2\pi + (\pi - \theta),$$

or

$$(2m + 1)\pi - \theta. \quad \text{(b)}$$

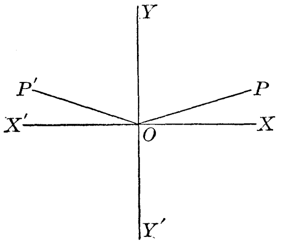

FIG. 46

And, conversely, if the sine of an angle be s, its terminal line must occupy one of the positions OP or OP'.

Comparing these two expressions, we see that when the coefficient of π is even, as in (a) ($2m$ being necessarily even), the sign of θ is $+$; when the coefficient of π is odd, as in (b) ($2m + 1$ being necessarily odd), the sign of θ is $-$. Therefore the angles θ, $2m\pi + \theta$, $(2m + 1)\pi - \theta$, and no others, all have the given sine s. They may all be included in the general expression $n\pi + (-1)^n\theta$, where n is 0 or any positive whole number. For, if $n = 0$, this expression reduces to θ; if n is an even number, say $2m$, it becomes $2m\pi + \theta$, since any even power of -1 is $+1$; and if n is an odd number, say $2m+1$, the expression becomes $(2m+1)\pi-\theta$, since any odd power of -1 is -1.

By giving to n different values, $0, 1, 2, 3, \cdots$, we can find any number of angles having the same sine as θ.

EXERCISES

1. Show that all angles having a given cosine are included in the general expression $2n\pi \pm \theta$.
2. Show that all angles having a given tangent are included in the general expression $n\pi + \theta$.
3. Name five angles that have the same sine as a given angle θ.
4. Name six values of $\cos^{-1}\frac{\sqrt{3}}{2}$.

71. Limiting Values of $\frac{\sin x}{x}$ and $\frac{\tan x}{x}$ as x approaches the Value Zero. Let AOX (Fig. 47) be any positive angle less than a right angle and let x be the number of radians it contains. To prove that as x approaches zero as a limit, $\frac{\sin x}{x}$ approaches 1 as its limit. With O as a center, and with any convenient radius, describe a circumference cutting OA at A and OX at F. Draw AC perpendicular to OX and produce it to meet the circumference at B. Draw the tangents AD, BD. Then, by geometry,

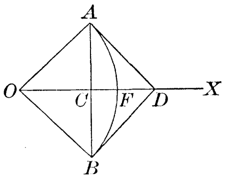

FIG. 47

$$\angle BOX = \angle AOX, \quad AC = CB,$$
$$AD = BD, \quad \text{arc } AF = \text{arc } BF.$$

It is assumed that the arc AFB is less than $AD + DB$. We have, then,

$$AF + FB > AC + CB, \text{ but } < AD + DB,$$

or

$$2AF > 2AC, \quad \text{but } < 2AD.$$

$$\therefore AF > AC, \quad \text{but} < AD.$$

$$\therefore \frac{AF}{OA} > \frac{AC}{OA}, \quad \text{but} < \frac{AD}{OA}.$$

But, by formula (41), $\frac{AF}{OA}$ equals the number of radians in the angle AOX, *i.e.*, $\frac{AF}{OA} = x$; also, $\frac{AC}{OA} = \sin x$ and $\frac{AD}{OA} = \tan x$.

We have, therefore,

$$x > \sin x, \text{ but} < \tan x.$$

Dividing by $\sin x$,

$$\frac{x}{\sin x} > 1, \qquad \text{but} < \frac{\tan x}{\sin x}, \text{ or } \frac{1}{\cos x}.$$

But the greater a number the less its reciprocal; $\therefore \frac{\sin x}{x}$ is less than 1, but greater than $\cos x$. Now, as x approaches the limit zero, $\cos x$ approaches the limit 1, and as $\frac{\sin x}{x}$ always lies between $\cos x$ and 1, the limit of $\frac{\sin x}{x}$ is also 1.

Again, $$\frac{\tan x}{x} = \frac{\sin x}{x} \times \frac{1}{\cos x},$$

and as x approaches the limit zero, $\frac{\sin x}{x}$ and $\frac{1}{\cos x}$ both approach the limit 1. Therefore, by the theory of limits, their product, $\frac{\sin x}{x} \times \frac{1}{\cos x}$, also approaches the limit 1.

***72. Special Solutions.** In the right triangle ACB (Fig. 48) let a and c be given. If they are nearly equal, A is nearly 90°, and its value cannot be determined with the usual degree of exactness from the equation

$$\sin A = \frac{a}{c}.$$

For example, let $a = 5235$, $c = 5237.1$ By the usual method,

$$\log \sin A = \log a - \log c.$$

$$\begin{aligned} \log 5235 &= 13.71892 - 10 \\ \log 5237.1 &= \underline{\ \ 3.71909\qquad\ } \\ \log \sin A &= \ \ 9.99983 - 10 \end{aligned}$$

And on referring to the table, we find it impossible to decide whether to take A to be 88° 21′ 30″, 88° 25′ 10″, or some intermediate value. In such cases, the following more accurate method should be employed.

First find $\frac{1}{2}A$ by formula (27). Thus, we have

$$\tan \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{1 - \dfrac{b}{c}}{1 + \dfrac{b}{c}}},$$

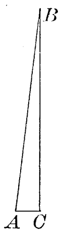

Fig. 48

or $$\mathbf{\tan \tfrac{1}{2} A = \sqrt{\frac{c - b}{c + b}}}. \qquad (43)$$

Since A is nearly 90°, $\frac{1}{2}A$ is nearly 45°, and the difficulty in determining A is obviated. Before applying the formula for $\tan \frac{1}{2} A$, however, it is necessary to find b. This is done from the relation $b^2 = (c + a)(c - a)$. We have

$$\begin{aligned} c + a &= 10472.1. \\ c - a &= 2.1. \end{aligned}$$

$$\begin{aligned} \log 10472.1 &= 4.02004 \\ \log &= \underline{0.32222} \\ 2\,\overline{)\underline{4.34226}} \\ \log b &= 2.17113 \end{aligned}$$

$$\begin{aligned} b &= 148.3. \\ c - b &= 5088.8. \\ c + b &= 5385.4. \end{aligned}$$

$$\begin{aligned} \log 5088.8 &= \ \ 3.70661 \\ \log 5385.4 &= \underline{\ \ 3.73122\qquad\ } \\ 2\,\overline{)\underline{19.97539 - 20}} \\ \log \tan \tfrac{1}{2} A &= \ \ 9.98770 - 10 \\ \tfrac{1}{2} A &= 44°\ 11'\ 19''. \\ A &= 88°\ 22'\ 38''. \end{aligned}$$

* **73**. The same difficulty is encountered in applying the law of sines in Case 3 of oblique triangles, when the unknown angle opposite one of the given sides is nearly 90°.

For example, let $b = 2$, $a = 1.06$, $A = 37°$. By the usual method,

$$\sin B = \frac{b \sin A}{a}.$$

$$\begin{aligned} \log 2 &= 0.30103 \\ \log \sin 37° &= \underline{9.72421 - 10} \\ & 10.02524 - 10 \\ \log 1.06 &= \underline{0.02531 } \\ \log \sin B &= 9.99993 - 10 \end{aligned}$$

And we cannot decide from the tables whether to take B as 88° 56′ 10″, 89° 0′ 30″, or some intermediate value.

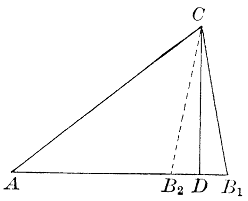

FIG. 49

To obtain the value of B more accurately, we may proceed as follows: Draw the perpendicular $CD = p$ (Fig. 49), and let $DB = t$. We first determine p from the right triangle ADC, then t from the relation $t^2 = a^2 - p^2$, and then apply the method of the last section to find B from the right triangle CDB.

Thus, in the above example, we have

$$\begin{aligned} p &= b \sin A = 2 \sin 37° = 1.05983. \\ t^2 &= (a + p)(a - p) = 2.11983 \times 0.00017. \\ \log 2.11983 &= 10.32630 - 10 \\ \log 0.00017 &= \underline{6.23045 - 10} \\ 2 \log t &= 16.55675 - 20 \\ \log t &= 8.27837 - 10. \\ t &= 0.01898. \end{aligned}$$

Then, by formula (43), $\tan \frac{1}{2} B = \sqrt{\dfrac{a - t}{a + t}} = \sqrt{\dfrac{1.04102}{1.07898}}.$

$$\begin{array}{rl}
\log 1.04102 = & 20.01746 - 20 \\
\log 1.07898 = & \underline{0.03301} \\
2\,\overline{)\,19.98445 - 20} & \\
\log \tan \tfrac{1}{2} B = & 9.99223 - 10
\end{array}$$

$$\tfrac{1}{2} B = 44^\circ\ 29'\ 26''.$$
$$B = 88^\circ\ 58'\ 52''.$$

The supplementary value 91° 1′ 8″ must also be accepted in this case (see Sec. 42), giving as the two solutions the triangles ACB_1, ACB_2. Let the student complete the solutions, finding the remaining parts for each triangle.

Ans. For triangle ACB_1, $C_1 = 54^\circ\ 1'\ 8''$, $c_1 = 1.6187$;
for triangle ACB_2, $C_2 = 51^\circ\ 58'\ 52''$, $c_2 = 1.5758$.

EXERCISES

1. Given $a = 999.7$, $c = 1000$, to find A.
Ans. $A = 88^\circ\ 35'\ 46''$.

2. Given $b = 1$, $c = 26$, to find A. *Ans.* $A = 87^\circ\ 47'\ 46''$.

3. Given $a = 148.228$, $b = 263$, $A = 34^\circ\ 17'$, to find B, C, c.
Ans. 1. $B_1 = 88^\circ\ 5'$, $C_1 = 57^\circ\ 38'$, $c_1 = 222.265$.
2. $B_2 = 91^\circ\ 55'$, $C_2 = 53^\circ\ 48'$, $c_2 = 212.35$.

* CHAPTER IX

SURVEYING

74. The fundamental problem in land surveying is the computation of the area of a tract of land from *data* which determine its boundaries.

Let $ABCDE$ (Fig. 50) be the plot of a farm. The bounding lines AB, BC, etc., are called *courses*. The usual *data* for finding the area of a farm are the lengths and bearings of the courses. The lengths, or *distances*, are measured by means of a chain, or tape. *Gunter's chain*, which is in general use, is 66 feet long and is divided into 100 *links*. In ordinary surveying, bearings are measured by means of a *compass*, an instrument by which angles can be read to quarter degrees. For more accurate work, the *engineer's transit*, reading to minutes, is used. The *field notes* for the tract plotted in Fig. 50 are as follows:

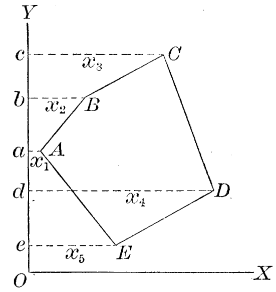

Fig. 50

Course	Bearing	Distance (chains)
AB	N. 40° E.	7.18
BC	N. 60° E.	9.20
CD	S. 20° E.	15.22
DE	S. 60° W.	11.42
EA	N. 38¼° W.	12.72

75. Units of Land Measure. The units of land measure commonly used are as follows:

1 rod, or pole = $16\frac{1}{2}$ feet.
1 chain = 4 poles = 66 feet.
1 acre = 10 square chains.

In some states Spanish-American units are in use. In this system the unit of length is the *vara* [vä′-rä]. In Texas a vara is taken to be $33\frac{1}{3}$ inches; in California, 33 inches. The vara used in Mexico is equivalent to 32.9927 inches. The area of a square whose side is 1000 varas is called a *labor* [lä-bōr′]; the area of a square whose side is 5000 varas is called a *league*.

EXERCISES

1. How many square poles in 1 acre? How many square feet? How many square yards?
2. How many acres in 1 labor? in 1 league?
3. How many varas in 1 mile?

76. Areas by Coördinates. In Sec. 78 we shall show how to find the area of a polygon when the bearings and distances of its boundary lines are given. The method will be more readily understood after the solution of the following.

Problem. *To find the area* S *of a polygon in terms of the coördinates of its vertices.*

Let the coördinates of the vertices A, B, C, D, E (Fig. 50) be $x_1y_1,\ x_2y_2,\ x_3y_3,\ x_4y_4,\ x_5y_5$. For convenience the axes are so chosen as to throw the polygon entirely in the first quadrant. Drawing the perpendiculars $Aa = x_1$, $Bb = x_2$, etc., we see that the polygon $ABCDE$ is equivalent to the polygon $cCDEe$ minus the sum of the trapezoids $aABb$, $bBCc$, and $eEAa$. The polygon $cCDEe$ is composed of the trapezoids $cCDd$ and $dDEe$.

$$\therefore S = cCDd + dDEe - (aABb + bBcC + eEAa). \qquad \text{(a)}$$

The area of each of these trapezoids can be expressed in terms of the coördinates of its vertices. For example, consider the trapezoid $cCDd$. Its bases cC and dD are equal respectively to x_3 and x_4; its altitude dc is equal to $y_3 - y_4$. Its area, therefore, is $\frac{1}{2}(x_3 + x_4)(y_3 - y_4)$; so for the other trapezoids.

We have, therefore, for the double area of the polygon,

$$\begin{aligned}2S = (x_3 + x_4)(y_3 - y_4)& \\ + (x_4 + x_5)(y_4 - y_5) - [(x_1 + x_2)&(y_2 - y_1) \\ + (x_2 + x_3)(y_3 - y_2) + (x_5 + x_1)&(y_1 - y_5)].\end{aligned}$$

Performing the indicated multiplications, reducing and rearranging, we have

$$\begin{aligned}2S = y_1(x_2 - x_5) + y_2(x_3 - x_1)& \\ + y_3(x_4 - x_2) + y_4(x_5 - x_3) + y_5&(x_1 - x_4).\end{aligned}$$

The proof applies to a polygon of any number of sides; hence, to find the double area of a polygon when the coördinates of its vertices are given, we have the following

Rule. *Multiply the ordinate of each vertex by the abscissa of the following vertex minus the abscissa of the preceding vertex and add the products.*

EXERCISES

1. Find the area of a polygon having its vertices at the following points, (12, 30), (30, 48), (60, 24), (18, 6), the linear unit being one chain. *Ans.* 104.4 *A.*

2. Draw the triangle which has its vertices at the points x_1y_1, x_2y_2, x_3y_3, and find its area by the *method* of this section without using the rule. Find it also by the rule.

77. Definitions. If the y-axis be a meridian and the x-axis an east-and-west line, the ordinate of a point is called its *latitude* and the abscissa is called its *longitude*. In Fig. 51 let DE be any course, and NS the meridian through its initial point; through its terminal point E, draw EP perpendicular to NS. Then DP is the difference in latitude of the points D and E, and is called the *latitude-difference* of the course DE. PE is the difference in longitude of the points D and E, and is called the *longitude-difference* of the course DE. Then, from the figure,

$$DP = DE \cos NDE,$$

and

$$PE = DE \sin NDE.$$

That is, *the latitude-difference of a course equals the length of the course multiplied by the cosine of the bearing; the longitude-difference equals the length of the course multiplied by the sine of the bearing.*

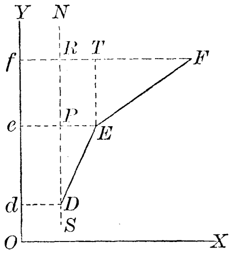

FIG. 51

When the latitude-difference is measured northward from the initial point of the course, it is called a *northing* and is taken to be positive; when it is measured southward, it is called a *southing* and is taken to be negative. Similarly, longitude-differences measured eastward are called *eastings* and are taken to be positive, and those measured westward are called *westings* and are taken to be negative.

The double longitude of a course is the sum of the longitudes of its ends. Thus, in Fig. 50, the double longitude of AB is $x_1 + x_2$.

78. Rule for finding the Area of a Tract when the Lengths and Bearings of the Courses are given. In the expression for S in equation (a), Sec. 76, we see that to every course in the tract $ABCDE$ there corresponds a trapezoid; and that to find S we add the trapezoids $cCDd$ and $dDEe$ corresponding to courses having southerly bearings, and from their sum subtract the sum of the trapezoids $aABb$, $bBCc$, and $eEAa$ corresponding to courses having northerly bearings. The areas of the first set are called *north* areas; of the second set, *south* areas. Again, considering the expression for the double area of any one of these trapezoids, as $bBCc$, *i.e.*, $(x_2 + x_3)(y_3 - y_2)$, we see that it is numerically equal to the product of the double longitude of the corresponding course by its latitude-difference. To find the double area of the tract, we have, therefore, the following

Rule. *Multiply the double longitude of each course by its latitude-difference, placing the product in a column of north areas when the bearing is northerly, and in a column of south areas when the bearing is southerly. Add up the two columns and take the difference of the results.*

79. Rule for finding the Double Longitudes. Let DE, EF (Fig. 51) be any two successive courses. Then, double longitude of

$$EF = eE + fF = \underline{eE + dD} + PE + TF.$$

That is, *the double longitude of any course equals the double longitude of the preceding course, plus the longitude-difference of that course, plus the longitude-difference of the given course.*

In applying this rule, remember to give the sign − to westings.

For convenience we shall take the reference meridian *one chain* west of the most westerly corner, and suppose the survey to begin at that corner, going around in regular order, keeping the tract on the right.

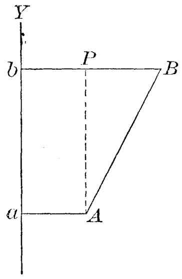

Fig. 52

Then, for the double longitude of the first course AB (Fig. 52), we have

$$aA + bB = aA + bP + PB$$
$$= 2 + PB,$$

since $aA = bP = 1$.

That is, *the double longitude of the first equals* 2, *plus its longitude-difference.*

The double longitudes of the remaining courses are then found by the rule given above.

80. Computation of Latitude-Differences and Longitude-Differences. Referring to the field notes in Sec. 74, we have, for the first course AB,

$$\text{lat.-diff.} = 7.18 \cos 40^\circ \quad = 7.18 \times 0.7660 = 5.50.$$
$$\text{long.-diff.} = 7.18 \times \sin 40^\circ = 7.18 \times 0.6428 = 4.61.$$

Making similar calculations for the remaining courses and tabulating the results, we have

COURSE	BEARING	DISTANCE	LAT.-DIFF.		LONG.-DIFF.	
			N.	S.	E.	W.
AB	N. 40° E.	7.18	5.50		4.61	
BC	N. 60° E.	9.20	4.60		7.97	
CD	S. 20° E.	15.22		14.30	5.21	
DE	S. 60° W.	11.42		5.71		9.89
EA	N. 38¼° W.	12.72	9.99			7.87
			20.09	20.01	17.79	17.76

It is obvious that if the measurements were exact, the sum of the northings would equal the sum of the southings, and that the sum of the eastings would equal the sum of the westings. But owing to the impossibility of making absolutely accurate measurements, this will not usually be the case. Thus, in the above example,

$$\begin{aligned} \text{sum of northings} &= 20.09 \\ \text{sum of southings} &= \underline{20.01} \\ \text{error} &= .08 \end{aligned}$$

$$\begin{aligned} \text{sum of eastings} &= 17.79 \\ \text{sum of westings} &= \underline{17.76} \\ \text{error} &= .03 \end{aligned}$$

The usual practice in compass surveys is to *balance* the work, as it is called, by distributing the errors among the several courses in proportion to their lengths. Thus, for the case in hand, the perimeter of the tract is 55.74 chains.

$$\therefore \text{ error in latitude for 1 chain } = \frac{.08}{55.74} = .0014.$$

$$\text{error in longitude for 1 chain} = \frac{.03}{55.74} = .0005.$$

Multiplying these unit errors by the lengths of the several courses, we have, correct to two places of decimals,

Course	Correction in Lat.-Diff.	Correction in Long.-Diff.
AB	.01	.00
BC	.01	.00
CD	.02	.01
DE	.02	.01
EA	.02	.01
	.08	.03

The northings are too great, and each must be diminished by the corresponding corrections; the southings are too small, and each must be increased; so for the eastings and westings.

After applying these corrections the work stands as follows:

Course	Bearing	Dist.	Lat.-Diff.		Long.-Diff.		Balanced			
			N.	S.	E.	W.	N.	S.	E.	W.
AB	N. 40° E.	7.18	5.50		4.61		5.49		4.61	
BC	N. 60° E.	9.20	4.60		7.97		4.59		7.97	
CD	S. 20° E.	15.22		14.30	5.21			14.32	5.20	
DE	S. 60° W.	11.42		5.71		9.89		5.73		9.90
EA	N. $38\frac{1}{4}$° W.	12.72	9.99			7.87	9.97			7.88
		55.74	20.09	20.01	17.79	17.76	20.05	20.05	17.78	17.78

81. Computation of Double Longitudes. Applying the method given in Sec. 79, the computation of the double longitudes of the several courses is as follows:

long.-diff. of 1st course	=	4.61
add		2
double longitude of 1st course	=	6.61
long.-diff. of 1st course	=	4.61
long.-diff. of 2d course	=	7.97
double longitude of 2d course	=	19.19
long.-diff. of 2d course	=	7.97
long.-diff. of 3d course	=	5.20
double longitude of 3d course	=	32.36
long.-diff. of 3d course	=	5.20
		37.56
long.-diff. of 4th course	= −	9.90
double longitude of 4th course	=	27.66
long.-diff. of 4th course	= −	9.90
		17.76
long.-diff. of 5th course	= −	7.88
double longitude of 5th course	=	9.88

Check. It is easy to show from a figure that the double longitude of the last course is numerically equal to its longitude-difference plus 2. This furnishes a check on the accuracy of the work. If, as is frequently done, the prime meridian is made to pass through the most westerly corner, double longitude of the last course will be equal numerically to its longitude-difference.

Inserting the above values for the double longitudes and computing the double areas by the rule given in Sec. 78, the work takes the final form given on the following page.

COURSE	BEARING	DIST.	LAT.-DIFF.		LONG.-DIFF.		BALANCED				DOUBLE LONGITUDE	DOUBLE AREAS	
			NORTHING	SOUTHING	EASTING	WESTING	N.	S.	E.	W.		NORTH	SOUTH
AB	N. 40° E.	7.18	5.50		4.61		5.49		4.61		6.61	36.2889	
BC	N. 60° E.	9.20	4.60		7.97		4.59		7.97		19.19	88.0821	
CD	S. 20° E.	15.22		14.30	5.21			14.32	5.20		32.36		463.3952
DE	S. 60° W.	11.42		5.71		9.89		5.73		9.90	27.66		158.4918
EA	N. 38¼° W.	12.72	9.99			7.87	9.97			7.88	9.88	98.5036	
		55.74	20.09	20.01	17.79	17.76	20.05	20.05	17.78	17.78		222.8746	621.8870
													222.8746
												2	)399.0124
												Area in square chains,	199.5062
												Area in acres,	19.95

MISCELLANEOUS EXERCISES

1. Find the area of the tract *ABCDE* from the following field notes :

Course	Bearing	Distance (poles)
AB	N. 41° E.	60
BC	S. 70° E.	40
CD	S. 29° E.	72
DE	West	75
EA	N. $49\frac{1}{4}$° W.	48

Ans. 32.89 *A.*

2.

Course	Bearing	Distance (chains)
AB	N. 21° E.	57.16
BC	N. 65° E.	7.68
CD	S. 33° E.	52.20
DE	S. $65\frac{1}{2}$° W.	50.96
EA	N. 48° W.	13.40

Ans. 187.13 *A.*

3.

Course	Bearing	Distance (varas)
AB	N. 47° E.	964
BC	East	822
CD	S. 32° E.	644
DE	South	523
EA	N. 77° 53′ W.	1961

Ans. 235.79 acres.

4.

COURSE	BEARING	DISTANCE (chains)
AB	S. 23° E.	48
BC	S. 80° E.	52
CD	N. 38° E.	46
DE	N. 42° W.	62
EA	S. $60\frac{1}{4}$° W.	64

Ans. 666.36 acres.

5. Assuming that all bearings and distances were accurately measured, supply the bearing and distance of *DE* in the following:

COURSE	BEARING	DISTANCE (chains)
AB	South	20
BC	S. 88° W.	18
CD	N. 42° W.	20
DE	. . . .	. .
EA	S. 18° E.	18

6. Find the bearing of a line from *B* to *E* in Exercise 1.

SPHERICAL TRIGONOMETRY

CHAPTER X

82. The Spherical Triangle. The portion of the surface of a sphere bounded by three arcs of great circles is called a *spherical triangle*. The bounding arcs are called the *sides* of the triangle, and the interior spherical angles formed by the sides are the *angles* of the triangle. Spherical trigonometry is concerned with the relations that subsist between the sides and the angles of a spherical triangle. We shall consider only triangles in which each side, and therefore each angle, is less than 180°.

83. Notation and Conditions. Let the vertices of the spherical triangle ABC (Fig. 53) be joined to the center of the sphere, O. Denote the angles of the triangles by A, B, C and the corresponding sides by a, b, c. Then we know, by geometry, that the angle A equals the plane angle of the dihedral angle whose edge is OA; so for B and C; and that the sides a, b, c measure the face angles BOC, AOC, BOA respectively.

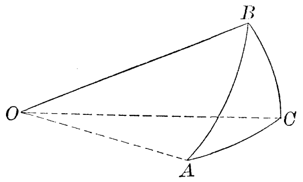

FIG. 53

It will be useful to recall the following propositions, which are proved in geometry.

In any spherical triangle

1. The greater side is opposite the greater angle, and conversely.
2. The sum of any two sides is greater than the third side.

3. The sum of the sides is less than 360°.

4. The sum of the angles is greater than 180° and less than 540°.

Also the following:

5. If $A'B'C'$ be the polar triangle of ABC, ABC is the polar triangle of $A'B'C'$.

6. In two polar triangles, ABC and $A'B'C'$,

$$A = 180° - a',\ B = 180° - b',\ C = 180° - c';$$
$$A' = 180° - a,\ B' = 180° - b,\ C' = 180° - c.$$

84. Like Parts and Unlike Parts. If two parts of a spherical triangle are both less than 90° or both greater than 90°, they are said to be *like* parts; if one is greater than 90° and the other less, they are said to be *unlike*.

The following propositions are established geometrically.

1. *If the two legs of a right spherical triangle are like parts, the hypotenuse is less than 90°; if the two legs are unlike parts, the hypotenuse is greater than 90°.*

2. *In a right spherical triangle a side and its opposite angle are like parts.*

Let ACB (Fig. 54) be a right spherical triangle, right angled at C, having the legs BC and AC each less than 90°.

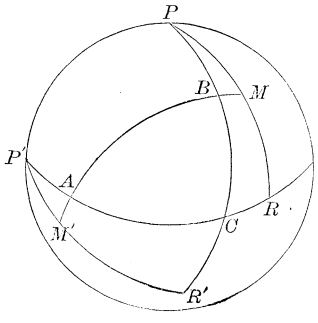

FIG. 54

Let P be the pole of the arc AC. Then CB produced passes through P. With A as a pole, and a quadrant as a radius, describe the arc PR, cutting AC produced in R. Then $PR = 90°$ and $CP = 90°$. Since CB is less than 90°, AB produced cuts PR in some point, as M, between P and R. Then $AM = 90°$. Hence, the hypotenuse AB is less than 90°.

Again, MR is the measure of the angle A, and since MR is less than 90°, A is less than 90°. In like manner, let P' be the pole of the arc BC. Then CA produced passes through P'. With B as a pole, and a quadrant as a radius, describe the arc $P'R'$, cutting BC produced in R'. Then $P'R' = 90°$. Since CA is less than 90°, BA produced cuts $P'R'$ at some point, as M', between P' and R', and $M'R'$, the measure of the angle B, is less than 90°. Hence, B is less than 90°. The propositions are true, therefore, for the case considered.

Next, let both a and b be greater than 90°, as in Fig. 55. Complete the line $CBC'A$. Then $\angle C = \angle C'$, and in the right triangle $AC'B$ each leg is less than 90°. Hence, the hypotenuse AB is less than 90°, and the angles BAC', ABC' are each less than 90°; therefore their supplements, A and B, are each greater than 90°.

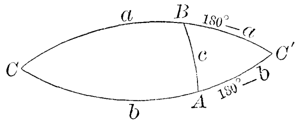

Fig. 55

Next, let one of the legs, as a (Fig. 56), be greater than 90°, and the other leg, b, be less than 90°. Complete the line $BAB'C$. Then in the right triangle ACB' each leg is less than 90°; therefore the hypotenuse AB' is less than 90°, and its supplement AB, the hypotenuse of ACB, is greater than 90°. Also, the angles B' and CAB' are each less than 90°. Hence, B, the equal of B', is less than 90°, and A, the supplement of CAB', is greater than 90°.

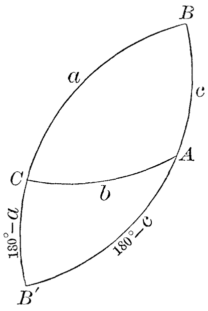

Fig. 56

Thus, the propositions are established for all cases.

85. The Right Spherical Triangle. Formulas. Let ACB (Fig. 57) be a right spherical triangle, C being the right angle, and let each of the legs a and b be less than 90°. Join each of the vertices to the center of the sphere, O. From any point, P, in OA draw PQ and PR perpendicular to OA, in the planes OAB, OAC respectively. Then QPR is the plane angle of the dihedral angle whose edge is OA.

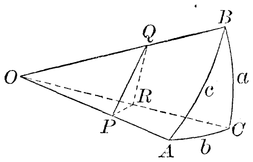

FIG. 57

$$\therefore \angle QPR = \angle A.$$

Join Q and R. Then, by geometry,* QR is perpendicular to the plane OAC, and therefore to RP and OR.

1. We have, then,

$$\sin QPR = \frac{QR}{PQ} = \frac{\frac{QR}{OQ}}{\frac{PQ}{OQ}} = \frac{\sin BOC}{\sin BOA}.$$

$$\therefore \sin A = \frac{\sin a}{\sin c}. \qquad [1]$$

Similarly, $$\sin B = \frac{\sin b}{\sin c}. \qquad [2]$$

---

* The proof is as follows:

1. OA is, by construction, perpendicular to PQ and PR. $\therefore OA$ is perpendicular to the plane QRP.

2. $\therefore$ the plane OAC, which contains OA, is perpendicular to the plane QRP.

3. Since C is a right angle, the plane OBC is also perpendicular to the plane OAC.

4. Since the intersecting planes OBC, QRP are perpendicular to the plane OAC, their intersection QR is perpendicular to OAC.

It should be added that the construction of the right triangle QRP is obviously impossible unless the dihedral angles whose edges are OA and OB and the face angle QOP are each less than 90°. But by the last article, since a and b are each less than 90°, A and B, which measure these dihedrals, are each less than 90°, and the hypotenuse c, which measures the face angle QOB, is also less than 90°.

2.
$$\cos QPR = \frac{PR}{PQ} = \frac{\dfrac{PR}{OP}}{\dfrac{PQ}{OP}} = \frac{\tan AOC}{\tan AOB}.$$

$$\therefore \cos A = \frac{\tan b}{\tan c}. \qquad [3]$$

Similarly,
$$\cos B = \frac{\tan a}{\tan c}. \qquad [4]$$

3.
$$\tan QPR = \frac{QR}{PR} = \frac{\dfrac{QR}{OR}}{\dfrac{PR}{OR}} = \frac{\tan BOC}{\sin AOC}.$$

$$\therefore \tan A = \frac{\tan a}{\sin b}. \qquad [5]$$

Similarly,
$$\tan B = \frac{\tan b}{\sin a}. \qquad [6]$$

4.
$$\cos QOP = \frac{OP}{OQ} = \frac{OP}{OR} \times \frac{OR}{OQ}$$
$$= \cos AOC \cos BOC.$$
$$\therefore \cos c = \cos a \cos b. \qquad [7]$$

5. From formula [3], we have

$$\cos A = \frac{\sin b}{\cos b} \times \frac{\cos c}{\sin c}.$$

But from formula [2],

$$\sin b = \sin c \sin B,$$

and from formula [7],

$$\cos c = \cos a \cos b.$$

$$\therefore \cos A = \frac{\sin c \sin B}{\cos b} \times \frac{\cos a \cos b}{\sin c},$$

or
$$\cos A = \cos a \sin B. \qquad [8]$$

Similarly,
$$\cos B = \cos b \sin A. \qquad [9]$$

From formulas [8] and [9],

$$\cos a = \frac{\cos A}{\sin B},$$

and

$$\cos b = \frac{\cos B}{\sin A}.$$

$$\therefore \cos a \cos b = \frac{\cos A}{\sin B} \times \frac{\cos B}{\sin A} = \cot A \cot B.$$

$\therefore$ by formula [7], $\cos c = \cot A \cot B.$ [10]

The formulas [1] to [10] suffice, as will be seen later, for the solution of right spherical triangles. For convenience they are here collected.

$$\sin A = \frac{\sin a}{\sin c}. \qquad [1] \qquad \sin B = \frac{\sin b}{\sin c}. \qquad [2]$$

$$\cos A = \frac{\tan b}{\tan c}. \qquad [3] \qquad \cos B = \frac{\tan a}{\tan c}. \qquad [4]$$

$$\tan A = \frac{\tan a}{\tan b}. \qquad [6] \qquad \tan B = \frac{\tan b}{\sin a}. \qquad [6]$$

$$\cos c = \cos a \cos b. \qquad [7] \qquad \cos A = \cos a \sin B. \qquad [8]$$

$$\cos B = \cos b \sin A. \qquad [9] \qquad \cos c = \cot A \cot B. \qquad [10]$$

86. In deriving the above formulas both of the legs a and b were taken to be less than 90°. The proof is extended so as to include all cases as follows:

1. Let both legs be greater than 90°, as in Fig. 55.

Produce the arcs CB, CA to meet in C', completing the line $CBC'A$. Then $C' = C = 90°$; and in the right triangle $AC'B$, $BC' = 180° - a$, $AC' = 180° - b$. Hence, both legs are less than 90°, and we may apply to this triangle the formulas of the last article.

By formula [1], $\sin BAC' = \dfrac{\sin BC'}{\sin BA},$

or

$$\sin(180° - A) = \frac{\sin(180° - a)}{\sin c}.$$

$\therefore$ by Sec. 42, $\sin A = \dfrac{\sin a}{\sin c}$,

which establishes formula [1] for this case.

Again, by formula [3],

$$\cos BAC' = \frac{\tan AC'}{\tan BA},$$

or $$\cos(180° - A) = \frac{\tan(180° - b)}{\tan c}.$$

$$\therefore -\cos A = \frac{-\tan b}{\tan c},$$

or $$\cos A = \frac{\tan b}{\tan c}.$$

So for the other formulas.

2. Let one of the legs, a, be greater than 90°, and the other, b, be less than 90°, as in Fig. 56.

Produce BC and BA to meet at B'. Then, in the right triangle ACB', $B' = B$, $CB' = 180° - a$, $AB' = 180° - c$.

Here, again, both legs are less than 90°, and we have, by formula [1],

$$\sin B'AC = \frac{\sin B'C}{\sin AB'},$$

or $$\sin(180° - A) = \frac{\sin(180° - a)}{\sin(180° - c)}.$$

$$\therefore \sin A = \frac{\sin a}{\sin c}.$$

Again, by formula [3],

$$\cos B'AC = \frac{\tan AC}{\tan AB'},$$

or $$\cos(180° - A) = \frac{\tan b}{\tan(180° - c)}.$$

$$\therefore \cos A = \frac{\tan b}{\tan c}.$$

So for the other formulas.

87. Napier's Rules. It is difficult to retain the formulas of the last article in the memory, and the necessity for doing so is obviated by the following device, known as *Napier's Rules.*

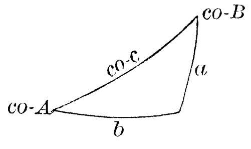

Fig. 58

Corresponding to the right spherical triangle ACB construct a triangle lettered as in Fig. 58, where a and b have the same values as in the given triangle ACB, and where co-A, co-B, co-c mean *complement of* A, *complement of* B, *complement of* c respectively. The five parts, co-A, b, a, co-B, co-c, occur in the figure in the same order in which the parts A, b, a, B, c occur in the triangle ABC, and are called the *circular parts* of the triangle ACB.

Note that the right angle C is not included among the circular parts.

If any one of these five circular parts be considered as a *middle* part, the two parts lying next to it are called *adjacent* parts, and the remaining two parts are called *opposite* parts.

Thus, if co-A be taken as a middle part, the adjacent parts are b and co-c; and the opposite parts are a and co-B.

Napier's Rules are as follows:

I. *The sine of a middle part equals the product of the tangent of the adjacent parts.*

II. *The sine of the middle part equals the product of the cosines of the opposite parts.*

These rules may be easily remembered by noting the occurrence of the letter ***a*** in tangents and adj***a***cent, and of the letter ***o*** in c***o***sines and ***o***pposite.

The rules are verified by considering each of the five circular parts in turn as below:

1. If we consider the circular part a, the adjacent parts are b and co-B.

According to Rule I,

$$\sin a = \tan b \tan (\text{co-}B),$$

or, since the tangent of the complement of B equals cot B,

$$\sin a = \tan b \cot B;$$

and this equation is true by formula [6].

The opposite parts are co-A, co-c.

According to Rule II,

$$\sin a = \cos(\text{co-}A)\cos(\text{co-}c),$$

or

$$\sin a = \sin A \sin c,$$

which is true by formula [1].

2. Consider next the circular part b. The adjacent parts are a and co-A.

According to Rule I,

$$\sin b = \tan a \tan(\text{co-}A),$$

or

$$\sin b = \tan a \cot A,$$

which is true by formula [c].

The opposite parts are co-c, co-B.

According to Rule II,

$$\sin b = \cos(\text{co-}c)\cos(\text{co-}B),$$

or

$$\sin b = \sin c \sin B,$$

which is true by formula [2].

3. Consider next the circular part co-A. The adjacent parts are b and co-c.

According to Rule I,

$$\sin(\text{co-}A) = \tan b \tan(\text{co-}c),$$

or

$$\cos A = \tan b \cot c,$$

which is true by formula [3].

The opposite parts are a and co-B.

According to Rule II,

$$\sin(\text{co-}A) = \cos a \cos(\text{co-}B),$$

or

$$\cos A = \cos a \sin B,$$

which is true by formula [8].

Let the student complete the verification by considering the circular parts co-c and co-B in turn as middle parts.

88. Solution of Right Spherical Triangles. If, besides the right angle, any two parts of a right spherical triangle be given, the remaining parts can be found by applying Napier's Rules. The Rules should be so applied that each equation formed shall contain only one unknown quantity. In any case the solution may be effected by the following steps:

1. Take one of the given parts as a middle part and apply to it that one of Napier's Rules which involves the other given part.
2. Take the other given part as a middle part and apply the *same* rule.

The equations thus formed determine two of the required parts. Then,

3. Take the third required part as a middle part and again apply the *same* rule.
4. As a check take the part last found as a middle part and apply the *other* rule.

The manner of applying these rules will be illustrated by examples.

Before beginning a solution the student should draw a figure showing the five circular parts, as in Sec. 87.

EXAMPLES

1. Given $A = 98° 16'$, $b = 57° 14'$, to find a, c, B.

Taking the circular part co-A as a middle part and applying Rule I, we have

$$\sin(\text{co-}A) = \tan b \tan(\text{co-}c),$$

or

$$\overset{-}{\cos} A = \overset{+}{\tan} b \overset{-}{\cot} c.$$

Since $A > 90°$, $\cos A$ is negative; since $b < 90°$, $\tan b$ is positive. $\therefore$ $\cot c$ is negative. Hence, c is greater than 90°, and we must take as the value of c the *supplement* of the angle found in the table. It will be found convenient to write the algebraic signs over the functions, as above. Applying logarithms to the above equation, we have

$$\log\cot c = \log\cos A - \log\tan b.$$

$$\begin{aligned} \log\cos 98^\circ\,16' &= 19.15770 - 10 \\ \log\tan 57^\circ\,14' &= \underline{10.19136 - 10} \\ \log\cot c &= 8.96634 - 10 \\ 180^\circ - c &= 84^\circ\,42'\,46''. \\ c &= 95^\circ\,17'\,14''. \end{aligned}$$

Next, apply the same rule to the part b, and we have

$$\overset{+}{\sin}\, b = \overset{-}{\cot}\, A\ \overset{-}{\tan}\, a.$$

$$\log\tan a = \log\sin b - \log\cot A.$$

$$\begin{aligned} \log\sin 57^\circ\,14' &= 19.92473 - 20 \\ \log\cot 98^\circ\,16' &= \underline{9.16224 - 10} \\ \log\tan a &= 10.76249 - 10 \\ 180^\circ - a &= 80^\circ\,11'\,49''. \\ a &= 99^\circ\,48'\,11''. \end{aligned}$$

Next, apply the same rule to the remaining part, co-B, and we have

$$\overset{+}{\cos}\, B = \overset{-}{\tan}\, a\ \overset{-}{\cot}\, c.$$

$$\log\cos B = \log\tan a + \log\cot c.$$

$$\begin{aligned} \log\tan a &= 10.76249 - 10 \\ \log\cot c &= \underline{8.96634 - 10} \\ \log\cos B &= 9.72883 - 10 \\ B &= 57^\circ\,37'. \end{aligned}$$

Check. Apply the *other* rule to the part last found, giving

$$\cos B = \cos b \sin A.$$

$$\log\cos B = \log\cos b + \log\sin A.$$

$$\begin{aligned} \log\cos 57^\circ\,14' &= 9.73337 - 10 \\ \log\sin 98^\circ\,16' &= \underline{9.99546 - 10} \\ \log\cos B &= 9.72883 - 10 \end{aligned}$$

which agrees with the value found above for $\log\cos B$.

The check formula may be modified when necessary, so that it shall contain only functions whose logarithms have already been found.

2. Given $a = 97^\circ\,22'$, $b = 46^\circ\,18'$, to find A, B, c.

Applying Rule I to the part a,

$$\overset{+}{\sin}\, a = \overset{+}{\tan}\, b\ \overset{+}{\cot}\, B.$$

$$\log\cot B = \log\sin a - \log\tan b.$$

$$\begin{aligned} \log\sin 97^\circ\,22' &= 19.99640 - 10 \\ \log\tan 46^\circ\,18' &= \underline{10.01971 - 10} \\ \log\cot B &= 9.97669 - 10 \\ B &= 46^\circ\,32'\,12''. \end{aligned}$$

Applying the same rule to the part b,

$$\overset{+}{\sin}\, b = \overset{-}{\tan}\, a \,\overset{-}{\cot}\, A.$$

$$\begin{aligned}
\log\cot A &= \log\sin b - \log\tan a.\\
\log\sin 46^\circ\, 18' &= 19.85912 - 10\\
\log\tan 97^\circ\, 22' &= \underline{10.88845 - 10}\\
\log\cot A &= 8.97067 - 10\\
180^\circ - A &= 84^\circ\, 39'\, 36''.\\
A &= 95^\circ\, 20'\, 24''.
\end{aligned}$$

Applying the same rule to the remaining circular part, co-c,

$$\overset{-}{\cos}\, c = \overset{-}{\cot}\, A \,\overset{+}{\cot}\, B.$$

$$\begin{aligned}
\log\cos c &= \log\cot A + \log\cot B.\\
\log\cot A &= 8.97067 - 10\\
\log\cot B &= \underline{9.97669 - 10}\\
\log\cos c &= 8.94736 - 10\\
180^\circ - c &= 84^\circ\, 55'\, 4''.\\
c &= 95^\circ\, 4'\, 56'.
\end{aligned}$$

As a check, apply Rule II to the part just found :

$$\begin{aligned}
\cos c &= \cos a \cos b.\\
\log\cos c &= \log\cos a + \log\cos b.\\
\log\cos a &= 9.10795 - 10\\
\log\cos b &= \underline{9.83940 - 10}\\
\log\cos c &= 9.94735 - 10
\end{aligned}$$

agreeing with the above value of $\log\cos c$ within one point.

3. *The ambiguous case.* Given $a = 64^\circ\, 29'$, $A = 78^\circ\, 10'$, to find b, c, B.

By Rule II,

$$\overset{+}{\sin}\, a = \overset{+}{\sin}\, A \,\overset{+}{\sin}\, c,$$

or

$$\sin c = \frac{\sin a}{\sin A}.$$

As c is to be found from its *sine*, there are, by Sec. 42, two solutions; namely, the angle found in the table, and its supplement.

Again, applying the same rule to co-A,

$$\cos A = \cos a \sin B,$$

or

$$\sin B = \frac{\cos A}{\cos a}.$$

And to find b, we have, by the same rule,

$$\sin b = \sin c \sin B\,;$$

so that B and b also are to be found from their sines, and the same ambiguity occurs in regard to them.

It is easily seen geometrically, also, that there are two solutions. For, let ACB (Fig. 59) be a right spherical triangle having the given parts A and a. Then, on completing the lune $ABA'C$, we form a second triangle, $A'CB$, which also fulfills the given conditions; for $A' = A$, $BC = a$, and $\angle BCA'$ is a right angle. It is evident also from the figure that the required parts BA', CA', $\angle CBA'$ are the supplements of the corresponding required parts in the triangle ACB.

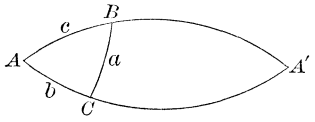

FIG. 59

The case in which a side and the opposite angle are the given parts is accordingly called the *ambiguous case.*

To complete the solution, apply logarithms to the equations stated above. Thus,

$$\log \sin c = \log \sin a - \log \sin A.$$

$$\begin{aligned} \log \sin 64^\circ 29' &= 19.95543 - 20 \\ \log \sin 78^\circ 10' &= \underline{9.99067 - 10} \\ \log \sin c &= 9.96476 - 10 \end{aligned}$$

$$\begin{aligned} c_1 &= 67^\circ 13' 48''. \\ c_2 &= 112^\circ 46' 12''. \end{aligned}$$

Also,

$$\log \sin B = \log \cos A - \log \cos a.$$

$$\begin{aligned} \log \cos 78^\circ 10' &= 19.31189 - 20 \\ \log \cos 64^\circ 29' &= \underline{9.63425 - 10} \\ \log \sin B &= 9.67764 - 10 \end{aligned}$$

$$\begin{aligned} B_1 &= 28^\circ 25' 37''. \\ B_2 &= 151^\circ 34' 23''. \end{aligned}$$

Also,

$$\log \sin b = \log \sin B + \log \sin c.$$

$$\begin{aligned} \log \sin B &= 9.67764 - 10 \\ \log \sin c &= \underline{9.96476 - 10} \\ \log \sin b &= 9.64240 - 10 \end{aligned}$$

$$\begin{aligned} b_1 &= 26^\circ 2' 9''. \\ b_2 &= 153^\circ 57' 51''. \end{aligned}$$

The two solutions, therefore, are

1. $b_1 = 26^\circ 2' 9''$, $B_1 = 28^\circ 25' 37''$, $c_1 = 67^\circ 13' 48''$.
2. $b_2 = 153^\circ 57' 51''$, $B_2 = 151^\circ 34' 23''$, $c_2 = 112^\circ 46' 12''$.

Check. $\sin b = \tan a \cot A$.

4. Given $a = 105° 18'$, $c = 98° 9'$, to find b, A, B.

By Rule II,
$$\sin a = \cos(\text{co-}c)\cdot\cos(\text{co-}A),$$
or
$$\overset{+}{\sin}\, a = \overset{+}{\sin}\, c\ \overset{+}{\sin}\, A.$$
$$\log\sin A = \log\sin a - \log\sin c.$$
$$\begin{aligned}\log\sin 105° 18' &= 19.98433 - 20\\ \log\sin 98° 9' &= \underline{\ 9.99559 - 10}\\ \log\sin A &= \ \ 9.98874 - 10\end{aligned}$$

Since $a > 90°$, $A > 90°$, and we must use as the value of A the supplement of the angle found in the table.
$$\begin{aligned}\therefore 180° - A &= 77° 0' 40''.\\ A &= 102° 59' 20''.\end{aligned}$$

Applying the same rule to the circular part co-c,
$$\sin(\text{co-}c) = \cos a \cos b,$$
or
$$\overset{-}{\cos}\, c = \overset{-}{\cos}\, a\ \overset{+}{\cos}\, b.$$
$$\log\cos b = \log\cos c - \log\cos a.$$
$$\begin{aligned}\log\cos 98° 9' &= 19.15157 - 20\\ \log\cos 105° 18' &= \underline{\ 9.42140 - 10}\\ \log\cos b &= \ \ 9.73017 - 10\\ b &= 57° 30' 15''.\end{aligned}$$

Applying the same rule to the remaining part,
$$\sin(\text{co-}B) = \cos b \cos(\text{co-}A),$$
or
$$\overset{+}{\cos}\, B = \overset{+}{\cos}\, b\ \overset{+}{\sin}\, A.$$
$$\log\cos B = \log\cos b + \log\sin A.$$
$$\begin{aligned}\log\cos b &= 9.73017 - 10\\ \log\sin A &= \underline{9.98874 - 10}\\ \log\cos B &= 9.71891 - 10\\ B &= 58° 21'.\end{aligned}$$

Write down a check formula and apply it.

Instead of the use of algebraic signs to determine whether a part of a right spherical triangle is greater or less than 90°, as in this section, we may apply the principles of *like parts* and *unlike parts* (Sec. 84). Thus in Example 1, since $A > 90°$ we have $a > 90$; also, a and b being unlike parts, c must be greater than $90°$; again, since $b < 90°$ we must have also $B < 90°$.

EXERCISES

Solve the following right spherical triangles, checking the work where the answers are not given:

1. Given $a = 48^\circ\ 15'$, $c = 74^\circ\ 12'$, to find A, b, B.
Ans. $A = 50^\circ\ 50'\ 12''$, $b = 65^\circ\ 15'\ 53''$, $B = 71^\circ\ 30'\ 57''$.

2. Given $a = 70^\circ\ 28'$, $c = 98^\circ\ 18'$, to find A, b, B.
Ans. $A = 72^\circ\ 15'\ 15''$, $b = 115^\circ\ 34'\ 42''$, $B = 114^\circ\ 16'\ 50''$.

3. Given $A = 36^\circ\ 20'$, $c = 81^\circ\ 24'$, to find b, B, a.
Ans. $b = 79^\circ\ 22'\ 3''$, $B = 42^\circ\ 16'\ 48''$, $a = 35^\circ\ 51'\ 39''$.

4. Given $A = 53^\circ\ 23'$, $c = 108^\circ$, to find b, B, a.
Ans. $b = 118^\circ\ 34'\ 46''$, $B = 112^\circ\ 33'\ 21''$, $a = 49^\circ\ 40'\ 55''$.

5. Given $a = 32^\circ\ 20'\ 40''$, $B = 56^\circ\ 20'\ 20''$, to find b, c, A.
Ans. $b = 38^\circ\ 46'\ 42''$, $c = 48^\circ\ 48'\ 19''$, $A = 45^\circ\ 19'$.

6. Given $a = 122^\circ\ 15'$, $B = 14^\circ\ 20'$, to find b, c, A.
Ans. $b = 12^\circ\ 11'\ 38''$, $c = 121^\circ\ 26'\ 19''$, $A = 97^\circ\ 35'\ 28''$.

7. Given $a = 56^\circ\ 13'$, $b = 34^\circ\ 22'$, to find B, A, c.
Ans. $B = 39^\circ\ 26'\ 51''$, $A = 69^\circ\ 18'\ 39''$, $c = 62^\circ\ 40'\ 43''$.

8. Given $a = 68^\circ\ 27'$, $b = 108^\circ\ 40'$, to find B, A, c.
Ans. $B = 107^\circ\ 26'\ 37''$, $A = 69^\circ\ 29'\ 13''$, $c = 96^\circ\ 45'\ 5''$.

9. Given $A = 76^\circ\ 15'$, $B = 49^\circ\ 3'$, to find a, b, c.
Ans. $a = 71^\circ\ 39'\ 27''$, $b = 47^\circ\ 33'\ 56''$, $c = 77^\circ\ 44'\ 26''$.

10. Given $A = 106^\circ$, $B = 64^\circ$, to find a, b, c.
Ans. $a = 107^\circ\ 51'\ 32''$, $b = 62^\circ\ 52'\ 5''$, $c = 98^\circ\ 2'\ 22''$.

11. Given $A = 32^\circ\ 10'$, $a = 24^\circ\ 8'$, to find B, c, b.
Ans. 1. $B_1 = 68^\circ\ 3'\ 36''$, $c_1 = 50^\circ\ 10'\ 27''$, $b_1 = 42^\circ\ 25'\ 45''$.
2. $B_2 = 111^\circ\ 56'\ 24''$, $c_2 = 129^\circ\ 49'\ 33''$, $b_2 = 137^\circ\ 34'\ 15''$.

12. Given $A = 105^\circ$, $a = 134^\circ$, to find B, c, b.
Ans. 1. $B_1 = 21^\circ\ 52'\ 31''$, $c_1 = 48^\circ\ 8'\ 5''$, $b_1 = 16^\circ\ 6'\ 36''$.
2. $B_2 = 158^\circ\ 7'\ 29''$, $c_2 = 131^\circ\ 51'\ 55''$, $b_2 = 163^\circ\ 53'\ 24''$.

13. Given $B = 60°$, $b = 50°$, to find A, c, a.

Ans. 1. $A_1 = 51° \, 3' \, 54''$, $c_1 = 62° \, 11' \, 43''$, $a_1 = 43° \, 28' \, 32''$.
2. $A_2 = 128° \, 56' \, 6''$, $c_2 = 117° \, 48' \, 17''$, $a_2 = 136° \, 31' \, 28''$.

14. Given $B = 100°$, $b = 106°$, to find A, c, a.

Ans. 1. $A_1 = 39° \, 21' \, 53''$, $c_1 = 77° \, 26' \, 40''$, $a_1 = 37° \, 56' \, 42''$.
2. $A_2 = 140° \, 38' \, 7''$, $c_2 = 102° \, 33' \, 20''$, $a_2 = 142° \, 3' \, 18''$.

15. Given $A = 110° \, 26'$, $B = 34° \, 19'$, to find a, b, c.
16. Given $A = 68° \, 12'$, $c = 73° \, 31'$, to find b, B, a.
17. Given $B = 106°$, $b = 112° \, 12'$, to find A, a, c.
18. Given $a = 72° \, 18'$, $b = 103°$, to find B, A, c.
19. Given $a = 53° \, 24'$, $c = 76° \, 12'$, to find A, b, B.
20. Given $a = 38° \, 16'$, $B = 69° \, 12'$, to find b, c, A.

89. Quadrantal Triangles. A *quadrantal* triangle is one in which one side is a quadrant. The polar triangle of such a triangle is a right triangle. WHY? After solving the polar triangle, the parts of the given triangle are readily found by Sec. 83, proposition 6.

EXERCISES

1. Given $a = 66° \, 32'$, $b = 59° \, 43'$, $c = 90°$,
to find $C = 104° \, 20' \, 38''$, $A = 62° \, 32' \, 20''$, $B = ?$

2. Given $c = 90°$, $B = 74° \, 45'$, $a = 18° \, 12'$,
to find $C = 104° \, 31' \, 13''$, $b = 83° \, 17' \, 16''$, $A = 17° \, 34' \, 2''$.

90. Isosceles Spherical Triangles. If, in a spherical triangle ABC (Fig. 60), we have $AB = AC$, then, by geometry,

$$\angle B = \angle C,$$

and AD, the arc of a great circle drawn from A to the mid-point of BC, divides the triangle ABC into two equal right triangles, each of which can be solved by Napier's Rules.

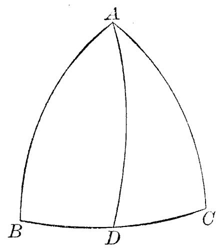

FIG. 60

EXERCISES

1. Given $b = c = 81° 24'$, $A = 72° 40'$,
to find $a = 71° 43' 18''$, $B = C = 42° 16' 48''$.

2. Given $b = c = 74° 13'$, $A = 98°$, find the remaining parts and check the results.

PROPERTIES OF SPHERICAL TRIANGLES

Oblique spherical triangles are solved by means of certain relations which subsist between the sides and the angles. These will now be established.

91. The Law of Sines. *In any spherical triangle the sines of the sides are to each other as the sines of the opposite angles.*

In the oblique spherical triangle ABC draw the arc BD perpendicular to AC. In Fig. 61, D falls between A and C; in Fig. 62, D falls on CA produced. By Napier's Rule II, we have, from the right spherical triangle ADB in Fig. 61,

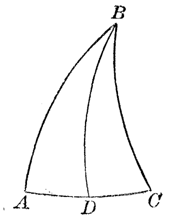

FIG. 61

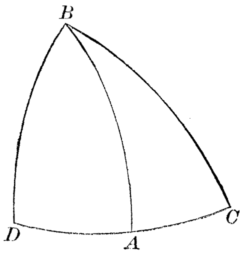

FIG. 62

$$\sin BD = \sin c \sin A;$$

in Fig. 62, $$\sin BD = \sin c \sin (180° - A) = \sin c \sin A.$$

Similarly, from the right triangle BDC in either figure,

$$\sin BD = \sin C \sin a.$$

$$\therefore \sin C \sin a = \sin c \sin A.$$

$$\therefore \frac{\sin a}{\sin c} = \frac{\sin A}{\sin C}.$$

Similarly, $$\frac{\sin a}{\sin b} = \frac{\sin A}{\sin B},$$

and $$\frac{\sin b}{\sin c} = \frac{\sin B}{\sin C}.$$

From these equations, we have

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \qquad [11]$$

92. Law of Cosines. *In a spherical triangle the cosine of any side equals the product of the cosines of the other two sides plus twice the product of their sines and the cosine of their included angle,* or, in symbols, taking the side a of the triangle ABC,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A. \qquad [12]$$

Draw the arc BD perpendicular to AC.

1. Let D fall between A and C, as in Fig. 61. From the right triangle BDC, we have, by Rule II,

$$\cos BC = \cos BD \cos DC,$$

or
$$\begin{aligned}\cos a &= \cos BD \cos (b - AD)\\ &= \cos BD (\cos b \cos AD + \sin b \sin AD)\\ &= \cos BD \cos AD (\cos b + \sin b \tan AD).\end{aligned}$$

Now, from the right triangle ADB, by Rule II,

$$\cos BD \cos AD = \cos c;$$

and, by Rule I, $\cos A = \cot c \tan AD$,

from which $\tan AD = \tan c \cos A$.

Substituting these values of $\cos BD \cos AD$ and $\tan AD$ in the expression for $\cos a$, we have

$$\cos a = \cos c (\cos b + \sin b \tan c \cos A).$$

$$\therefore \cos a = \cos b \cos c + \sin b \sin c \cos A.$$

2. Let D fall on CA produced, as in Fig. 62. Then, proceeding as before, we have, from the right triangle BDC,

$$\begin{aligned}\cos a &= \cos BD \cos CD\\ &= \cos BD \cos (b + AD)\\ &= \cos BD\,(\cos b \cos AD - \sin b \sin AD)\\ &= \cos BD \cos AD\,(\cos b - \sin b \tan AD).\end{aligned}$$

But $\cos BD \cos AD = \cos c$,

and $\cos BAD = \cot c \tan AD$.

But $\cos BAD = \cos (180° - A) = -\cos A$.

$\therefore -\cos A = \cot c \tan AD$,

from which $\tan AD = -\tan c \cos A$.

Substituting as before,

$$\cos a = \cos c\,(\cos b + \sin b \tan c \cos A),$$

or $$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Similarly, $$\cos b = \cos a \cos c + \sin a \sin c \cos B.$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

93. To express sin ½ A in terms of functions of a, b, c. By formula (25), we have

$$2 \sin^2 \tfrac{1}{2} A = 1 - \cos A.$$

From formula [12],

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Substituting this value,

$$\begin{aligned}2 \sin^2 \tfrac{1}{2} A &= 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}\\ &= \frac{\sin b \sin c + \cos b \cos c - \cos a}{\sin b \sin c}\\ &= \frac{\cos (b - c) - \cos a}{\sin b \sin c}\\ &= \frac{2 \sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin b \sin c},\end{aligned}$$

by formula (18).

Let $a + b + c = 2s$.

Then, $a + b - c = 2s - 2c = 2(s - c)$.

$a - b + c = 2s - 2b = 2(s - b)$.

Substituting, we have

$$2\sin^2 \tfrac{1}{2} A = \frac{2\sin(s-c)\sin(s-b)}{\sin b \sin c}.$$

$$\therefore \sin \tfrac{1}{2} A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}. \qquad [13]$$

Similarly, $\sin \tfrac{1}{2} B = \sqrt{\dfrac{\sin(s-a)\sin(s-c)}{\sin a \sin c}}$.

$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin a \sin b}}.$$

94. To express cos $\tfrac{1}{2}$ A in terms of functions of a, b, c. By formula (26),

$$\begin{aligned} 2\cos^2 \tfrac{1}{2} A &= 1 + \cos A \\ &= 1 + \frac{\cos a - \cos b \cos c}{\sin b \sin c} \\ &= \frac{\cos a - \cos b \cos c + \sin b \sin c}{\sin b \sin c} \\ &= \frac{\cos a - \cos(b+c)}{\sin b \sin c} \\ &= \frac{2\sin \tfrac{1}{2}(a+b+c)\sin(b+c-a)}{\sin b \sin c}, \end{aligned}$$

by formula (18).

$$\therefore \cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}. \qquad [14]$$

Similarly, $\cos \tfrac{1}{2} B = \sqrt{\dfrac{\sin s \sin(s-b)}{\sin a \sin c}}$.

$$\cos \tfrac{1}{2} C = \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin b}}.$$

95. To express tan $\frac{1}{2}$ A in terms of functions of a, b, c. From formulas [13] and [14], by division and reduction,

$$\tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}. \qquad [15]$$

$$\text{Similarly, } \tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin s\sin(s-b)}}.$$

$$\tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}.$$

As in the corresponding case in plane trigonometry, these formulas may be modified as follows:

In the formula for tan $\frac{1}{2}A$, multiply numerator and denominator of the fraction under the radical by $\sin(s-a)$, and we have

$$\begin{aligned}\tan\tfrac{1}{2}A &= \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s\sin^2(s-a)}}\\ &= \frac{1}{\sin(s-a)}\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.\end{aligned}$$

$$\text{Now, let} \qquad r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

$$\text{Then,} \qquad \tan\tfrac{1}{2}A = \frac{r}{\sin(s-a)}. \qquad [16]$$

$$\text{Similarly, } \tan\tfrac{1}{2}B = \frac{r}{\sin(s-b)}.$$

$$\tan\tfrac{1}{2}C = \frac{r}{\sin(s-c)}.$$

96. To express the sines, cosines, and tangents of the half sides in terms of functions of the angles.

1. *To express sin* $\frac{1}{2}$ a *in terms of functions of* A, B, C.

$$\text{We have} \qquad a = 180° - A'.$$

$$\therefore \tfrac{1}{2}a = 90° - \tfrac{1}{2}A'.$$

$$\therefore \sin\tfrac{1}{2}a = \cos\tfrac{1}{2}A'.$$

But in the polar triangle $A'B'C'$, we have, by formula [14],

$$\cos \tfrac{1}{2} A' = \sqrt{\frac{\sin s' \sin (s' - a')}{\sin b' \sin c'}},$$

where $a' + b' + c' = 2\,s'$.

We have now to express the quantity under the radical sign in terms of functions of A, B, C.

We have

$$\begin{aligned} a' &= 180° - A \\ b' &= 180° - B \\ c' &= 180° - C \\ \hline \therefore\ a' + b' + c' &= 540° - (A + B + C). \end{aligned}$$

Let $A + B + C = 2\,S$.

Then,

$$2\,s' = 540° - 2\,S.$$

$$\therefore\ s' = 270° - S.$$

$$s' - a' = 90° - (S - A).$$

$$\therefore\ \sin \tfrac{1}{2} a = \sqrt{\frac{\sin (270° - S) \sin [90° - (S - A)]}{\sin (180° - B) \sin (180° - C)}},$$

or, by Secs. 42 and 29,

$$\sin \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos (S - A)}{\sin B \sin C}}. \qquad [17]$$

Similarly,

$$\sin \tfrac{1}{2} b = \sqrt{\frac{-\cos S \cos (S - B)}{\sin A \sin C}}.$$

$$\sin \tfrac{1}{2} c = \sqrt{\frac{-\cos S \cos (S - C)}{\sin A \sin B}}.$$

2. *To express* $\cos \tfrac{1}{2}$ a *in terms of functions of* A, B, C.

Proceeding as above,

$$\tfrac{1}{2} a = 90° - \tfrac{1}{2} A'.$$

$$\cos \tfrac{1}{2} a = \sin \tfrac{1}{2} A' = \sqrt{\frac{\sin (s' - b') \sin (s' - c')}{\sin b' \sin c'}}$$

$$= \sqrt{\frac{\sin [90° - (S - B)] \sin [90° - (S - c)]}{\sin (180° - B) \sin (180° - C)}},$$

or
$$\cos \tfrac{1}{2} a = \sqrt{\frac{\cos (S - B) \cos (S - C)}{\sin B \sin C}}. \qquad [18]$$

Similarly,
$$\cos \tfrac{1}{2} b = \sqrt{\frac{\cos (S - A) \cos (S - C)}{\sin A \sin C}}.$$

$$\cos \tfrac{1}{2} c = \sqrt{\frac{\cos (S - A) \cos (S - B)}{\sin A \sin B}}.$$

3. *To express tan* $\tfrac{1}{2}$ a *in terms of functions of* A, B, C.

From formulas [17] and [18], we have, by division and reduction,

$$\tan \tfrac{1}{2} a = \sqrt{\frac{- \cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}}. \qquad [19]$$

Similarly,
$$\tan \tfrac{1}{2} b = \sqrt{\frac{- \cos S \cos (S - B)}{\cos (S - A) \cos (S - C)}}.$$

$$\tan \tfrac{1}{2} c = \sqrt{\frac{- \cos S \cos (S - B)}{\cos (S - A) \cos (S - B)}}.$$

97. Gauss's Equations. The following equations are known as *Gauss's Equations:*

$$[1\text{ G.}] \qquad \sin \tfrac{1}{2} (A + B) = \frac{\cos \tfrac{1}{2} (a - b)}{\cos \tfrac{1}{2} c} \cos \tfrac{1}{2} C.$$

$$[2\text{ G.}] \qquad \sin \tfrac{1}{2} (A - B) = \frac{\sin \tfrac{1}{2} (a - b)}{\sin \tfrac{1}{2} c} \cos \tfrac{1}{2} C.$$

$$[3\text{ G.}] \qquad \cos \tfrac{1}{2} (A + B) = \frac{\cos \tfrac{1}{2} (a + b)}{\cos \tfrac{1}{2} c} \sin \tfrac{1}{2} C.$$

$$[4\text{ G.}] \qquad \cos \tfrac{1}{2} (A - B) = \frac{\sin \tfrac{1}{2} (a + b)}{\sin \tfrac{1}{2} c} \sin \tfrac{1}{2} C.$$

To derive the first of these equations we proceed as follows:

We have, by formula (9),

$$\sin \tfrac{1}{2} (A + B) = \sin \tfrac{1}{2} A \cos \tfrac{1}{2} B + \cos \tfrac{1}{2} A \sin \tfrac{1}{2} B.$$

For $\sin \frac{1}{2} A$, $\cos \frac{1}{2} B$, substitute their values as given in Secs. 93 and 94, and we have

$$\sin \tfrac{1}{2} A \cos \tfrac{1}{2} B = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c} \times \frac{\sin s \sin(s-b)}{\sin a \sin c}}$$

$$= \frac{\sin(s-b)}{\sin c} \times \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin c}}$$

$$= \frac{\sin(s-b)}{\sin c} \times \cos \tfrac{1}{2} C. \qquad \text{By Sec. 94.}$$

Similarly,

$$\cos \tfrac{1}{2} A \sin \tfrac{1}{2} B = \frac{\sin(s-a)}{\sin c} \times \cos \tfrac{1}{2} C.$$

$$\therefore \sin \tfrac{1}{2}(A+B) = \frac{\sin(s-b) + \sin(s-a)}{\sin c} \cos \tfrac{1}{2} C.$$

But, by formula (15),

$$\sin(s-b) + \sin(s-a) = 2 \sin \tfrac{1}{2}(2s-a-b) \cos \tfrac{1}{2}(a-b)$$

$$= 2 \sin \tfrac{1}{2} c \cos \tfrac{1}{2}(a-b),$$

since $2s - a - b = c$.

Also, by formula (20),

$$\sin c = 2 \sin \tfrac{1}{2} c \cos \tfrac{1}{2} c.$$

$$\therefore \sin \tfrac{1}{2}(A+B) = \frac{2 \sin \frac{1}{2} c \cos \frac{1}{2}(a-b)}{2 \sin \frac{1}{2} c \cos \frac{1}{2} c} \cos \tfrac{1}{2} C,$$

or

$$\sin \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2} c} \cos \tfrac{1}{2} C. \qquad [1 \text{ G.}]$$

Proceeding in the same way, we find

$$\sin \tfrac{1}{2}(A-B) = \frac{\sin(s-b) - \sin(s-a)}{\sin c} \cos \tfrac{1}{2} C$$

$$= \frac{2 \cos \frac{1}{2}(2s-a-b) \sin \frac{1}{2}(a-b)}{2 \sin \frac{1}{2} c \cos \frac{1}{2} c} \cos \tfrac{1}{2} C,$$

or

$$\sin \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2} c} \cos \tfrac{1}{2} C. \qquad [2 \text{ G.}]$$

To derive formula [3 G.], we have

$$\cos \tfrac{1}{2}(A+B) = \cos \tfrac{1}{2} A \cos \tfrac{1}{2} B - \sin \tfrac{1}{2} A \sin \tfrac{1}{2} B.$$

As before,

$$\cos\tfrac{1}{2}A\cos\tfrac{1}{2}B=\sqrt{\frac{\sin s\sin(s-a)}{\sin b\sin c}\times\frac{\sin s\sin(s-b)}{\sin a\sin c}}$$

$$=\frac{\sin s}{\sin c}\times\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}}$$

$$=\frac{\sin s}{\sin c}\times\sin\tfrac{1}{2}C.$$

Similarly,

$$\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B=\frac{\sin(s-c)}{\sin c}\sin\tfrac{1}{2}C.$$

$$\therefore\cos\tfrac{1}{2}(A+B)=\frac{\sin s-\sin(s-c)}{\sin c}\sin\tfrac{1}{2}C$$

$$=\frac{2\cos\frac{1}{2}(2s-c)\sin\frac{1}{2}c}{\sin c}\sin\tfrac{1}{2}C$$

$$=\frac{2\cos\frac{1}{2}(a+b)\sin\frac{1}{2}c}{2\sin\frac{1}{2}c\cos\frac{1}{2}c}\sin\tfrac{1}{2}C,$$

or

$$\cos\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}c}\sin\tfrac{1}{2}C. \qquad [3\text{ G.}]$$

The derivation of formula [4 G.] is left as an exercise.

98. Napier's Analogies. The following equations are known as *Napier's Analogies.*

$$[1\text{ N.}]\qquad\tan\tfrac{1}{2}(A+B)=\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.$$

$$[2\text{ N.}]\qquad\tan\tfrac{1}{2}(A-B)=\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\tfrac{1}{2}C.$$

$$[3\text{ N.}]\qquad\tan\tfrac{1}{2}(a+b)=\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c.$$

$$[4\text{ N.}]\qquad\tan\tfrac{1}{2}(a-b)=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c.$$

Formula [1 N.] is derived from formulas [1 G.] and [3 G.] by division and reduction; [2 N.] similarly from [2 G.] and [4 G.]; [3 N.] from [4 G.] and [3 G.]; [4 N.] from [2 G.] and [1 G.].

Let the student perform the operations indicated.

SOLUTION OF OBLIQUE SPHERICAL TRIANGLES

99. If three parts of a spherical triangle be given, the remaining parts can be found by the use of formulas derived in this chapter. There are six cases, as follows :

1. *Given the three sides.*
2. *Given the three angles.*
3. *Given two sides and their included angle.*
4. *Given two angles and their included side.*
5. *Given two sides and the angle opposite one of them.*
6. *Given two angles and the side opposite one of them.*

The formulas for the tangents of the half angles (Sec. 94), the law of sines, and Gauss's Equations *or* Napier's Analogies will be found sufficient for all the cases. The results should be checked by formulas involving the required parts, not used in the solution. The method of solving will be illustrated by examples.

100. Case 1. *Given the three sides.*

Let $a = 72° 16'$, $b = 80° 44'$, $c = 41° 18'$, to find A, B, C.

By formula [15],
$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}}.$$

$$\begin{aligned} s &= \tfrac{1}{2}(a + b + c) = 97° 9'. \\ s - a &= 24° 53'. \\ s - b &= 16° 25'. \\ s - c &= 55° 51'. \end{aligned}$$

$$\begin{array}{rr|rr}
\log \sin 16° 25' = & 9.45120 - 10 & \log \sin 97° 9' = & 9.99661 - 10 \\
\log \sin 55° 51' = & \underline{9.91781 - 10} & \log \sin 24° 53' = & \underline{9.62405 - 10} \\
 & 19.36901 - 20 & & 9.62066 - 10 \\
 & \underline{9.62066 - 10} & & \\
 & 2\overline{)19.74835 - 20} & & \\
\log \tan \tfrac{1}{2} A = & 9.87418 - 10 & &
\end{array}$$

$$\begin{aligned} \tfrac{1}{2} A &= 36° 48' 47''. \\ A &= 73° 37' 34''. \end{aligned}$$

Similarly, $$\tan\tfrac{1}{2}B = \sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin s\sin(s-b)}}.$$

$$\begin{array}{rl|rl}
\log\sin 24^\circ 53' = & 9.62405-10 & \log\sin 97^\circ 9' = & 9.99661-10 \\
\log\sin 55^\circ 51' = & \underline{9.91781-10} & \log\sin 16^\circ 25' = & \underline{9.45120-10} \\
 & 19.54186-20 & & 9.44781-19 \\
 & \underline{9.44781-10} & & \\
2\,) & 20.09405-20 & & \\
\log\tan\tfrac{1}{2}B = & 10.04703-10 & & \\
\tfrac{1}{2}B = & 48^\circ 5' 48''. & & \\
B = & 96^\circ 11' 36''. & &
\end{array}$$

Similarly, $$\tan\tfrac{1}{2}C = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}.$$

$$\begin{array}{rl|rl}
\log\sin 24^\circ 53' = & 9.62405-10 & \log\sin 97^\circ 9' = & 9.99661-10 \\
\log\sin 16^\circ 25' = & \underline{9.45120-10} & \log\sin 55^\circ 51' = & \underline{9.91781-10} \\
 & 19.07525-20 & & 9.91442-10 \\
 & \underline{9.91442-10} & & \\
2\,) & 19.16083-20 & & \\
\log\tan\tfrac{1}{2}C = & 9.58042-10 & & \\
\tfrac{1}{2}C = & 20^\circ 50' 5''. & & \\
C = & 41^\circ 40' 10''. & &
\end{array}$$

Check. From the law of sines,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c},$$

from which

$$\log\sin A - \log\sin a = \log\sin B - \log\sin b = \log\sin C - \log\sin c.$$

$$\begin{array}{rl|rl|rl}
\log\sin A = & 9.98202-10 & \log\sin B = & 9.99746-10 & \log\sin C = & 9.82271-10 \\
\log\sin a = & \underline{9.97886-10} & \log\sin b = & \underline{9.99429-10} & \log\sin c = & \underline{9.81955-10} \\
 & 0.00316 & & 0.00317 & & 0.00316
\end{array}$$

A, B, C might also be calculated by the formulas of Sec. 95.

101. Case 2. *Given the three angles.*

Given $A = 96^\circ 45'$, $B = 108^\circ 30'$, $C = 116^\circ 15'$, to find a, b, c.

From the values of A, B, C find a', b', c', the sides of the polar triangle $A'B'C'$, by Sec. 83, proposition 6. Solve the triangle $A'B'C'$ by Case 1, finding A', B', C'. From the values of A', B', C' find a, b, c.

Ans. $a = 88^\circ 27' 49''$, $b = 107^\circ 19' 52''$, $c = 115^\circ 28' 13''$.

Check. Apply the law of sines, as in Case 1. This case could also be solved by the formulas of Sec. 96.

102. Case 3. *Given two sides and their included angle.*

Let $a = 95° 42'$, $b = 72° 22'$, $C = 62° 34'$, to find A, B, c.

By substituting the values of a, b, C in formulas [1 N.] and [2 N.], we can find A and B. Thus,

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos 11° 40'}{\cos 84° 2'} \cdot \cot 31° 17'.$$

$$\begin{aligned}
\log \cos 11° 40' &= 9.99093 - 10\\
\log \cot 31° 17' &= \underline{10.21637 - 10}\\
&\ 20.20730 - 20\\
\log \cos 84° 2' &= \underline{9.01682 - 10}\\
\log \tan \tfrac{1}{2}(A+B) &= 11.19048 - 10\\
\tfrac{1}{2}(A+B) &= 86° 18' 35'',
\end{aligned}$$

and
$$\tan \tfrac{1}{2}(A-B) = \frac{\sin 11° 40'}{\sin 84° 2'} \cdot \cot 31° 17'.$$

$$\begin{aligned}
\log \sin 11° 40' &= 9.30582 - 10\\
\log \cot 31° 17' &= \underline{10.21637 - 10}\\
&\ 19.52219 - 20\\
\log \sin 84° 2' &= \underline{9.99764 - 10}\\
\log \tan \tfrac{1}{2}(A-B) &= 9.52455 - 10\\
\tfrac{1}{2}(A-B) &= 18° 30' 4''.\\
\text{But}\qquad \tfrac{1}{2}(A+B) &= 86° 18' 35''.\\
\therefore A &= 104° 48' 39''.\\
B &= 67° 48' 31''.
\end{aligned}$$

To find c, we have
$$\frac{\sin c}{\sin C} = \frac{\sin b}{\sin B}.$$

$$\begin{aligned}
\log \sin 72° 22' &= 9.97910 - 10\\
\log \sin 62° 34' &= \underline{9.94819 - 10}\\
&\ 19.92729 - 20\\
\log \sin 67° 48' 31'' &= \underline{9.96655 - 10}\\
\log \sin c &= 9.96074 - 10\\
c &= 66° 0' 10''.
\end{aligned}$$

The supplement of this cannot be used as a value of c, for $C < B$. $\therefore c < b$.

c might also have been found from formulas [3 N.], [4 N.], or any one of Gauss's equations.

Check. By [1 G.], $\sin \frac{1}{2}(A+B) = \dfrac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2} c} \cos \frac{1}{2} C.$

103. Case 4. *Given two angles and their included side.*

Given $A = 116°$, $B = 84°$, $c = 72°$, to find a, b, C.

Solve the polar triangle by Case 3. (See Sec. 83, proposition 6.)

Ans. $a = 115° \, 27' \, 47''$, $b = 92° \, 27' \, 47''$, $C = 71° \, 12' \, 45''$.

104. Case 5. *Given two sides and the angle opposite one of them.*

Let $a = 41° \, 6'$, $b = 48° \, 22'$, $A = 54° \, 17'$, to find B, C, c.

To find B, we have, from the law of sines,

$$\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a}.$$

$$\begin{aligned}
\log \sin 48° \, 22' &= 9.87356 - 10 \\
\log \sin 54° \, 17' &= \underline{9.90951 - 10} \\
& 19.78307 - 20 \\
\log \sin 41° \, 6' &= \underline{9.81781 - 10} \\
\log \sin B &= 9.96526 - 10
\end{aligned}$$

$$\therefore B_1 = 67° \, 23' \, 12'', \quad B_2 = 112° \, 36' \, 48''.$$

Since $b > a$, we must have $B > A$; but both the values of B found above satisfy this condition, and we have two solutions. Case 5 is accordingly an *ambiguous case.* (See Fig. 63.)

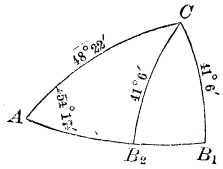

FIG. 63

To find C, we have, from formula [1 N.],

$$\cot \tfrac{1}{2} C = \frac{\cos \frac{1}{2}(b + a)}{\cos \frac{1}{2}(b - a)} \tan \tfrac{1}{2}(B + A).$$

1. Using the first value of B, we have

$$\tfrac{1}{2}(B_1 + A) = 60° \, 50' \, 6''.$$

Also,

$$\begin{aligned}
\tfrac{1}{2}(b + a) &= 44° \, 44'. \\
\tfrac{1}{2}(b - a) &= 3° \, 38'.
\end{aligned}$$

$$\begin{aligned}
\log \cos 44° \, 44' &= 9.85150 - 10 \\
\log \tan 60° \, 50' \, 6'' &= \underline{10.25330 - 10} \\
& 20.10480 - 20 \\
\log \cos 3° \, 38' &= \underline{9.99913 - 10} \\
\log \cot \tfrac{1}{2} C_1 &= 10.10567 - 10 \\
\tfrac{1}{2} C_1 &= 38° \, 5' \, 51''. \\
C_1 &= 76° \, 11' \, 42''.
\end{aligned}$$

To find the value of c_1, $\dfrac{\sin c_1}{\sin C} = \dfrac{\sin a}{\sin A}.$

$$\begin{array}{r@{\;=\;}l}
\log\sin 41^\circ\,6' & 9.81781-10 \\
\log\sin 76^\circ\,11'\,42'' & \underline{9.98727-10} \\
 & 19.80508-20 \\
\log\sin 54^\circ\,17' & \underline{9.90951-10} \\
\log\sin c_1 & 9.89557-10 \\
c_1 & 54^\circ\,2'\,40''.
\end{array}$$

2. Using the second value of B, we have

$$\tfrac{1}{2}(B_2+A)=83^\circ\,26'\,54''.$$

Solving as above,

$$\begin{array}{r@{\;=\;}l}
\log\cos 44^\circ\,44' & 9.85150-10 \\
\log\tan 83^\circ\,26'\,54'' & \underline{10.93987-10} \\
 & 20.79137-20 \\
\log\cos 3^\circ\,38' & \underline{9.99913-10} \\
\log\cot\tfrac{1}{2}C_2 & 10.79224-10 \\
\tfrac{1}{2}C_2 & 9^\circ\,9'\,56''. \\
C_2 & 18^\circ\,19'\,52''.
\end{array}$$

And to find the value of c_2,

$$\frac{\sin c_2}{\sin C_2}=\frac{\sin a}{\sin A}.$$

$$\begin{array}{r@{\;=\;}l}
\log\sin 41^\circ\,6' & 9.81781-10 \\
\log\sin 18^\circ\,19'\,52'' & \underline{9.49763-10} \\
 & 19.31544-20 \\
\log\sin 54^\circ\,17' & \underline{9.90951-10} \\
\log\sin c_2 & 9.40593-10 \\
c_2 & 14^\circ\,45'\,11''.
\end{array}$$

Hence, we have for the two solutions:

1. For the triangle AB_1C (Fig. 62),

$$B_1=67^\circ\,23'\,12'',\quad C_1=76^\circ\,11'\,42'',\quad c_1=54^\circ\,2'\,40''.$$

2. For the triangle AB_2C,

$$B_2=112^\circ\,36'\,48'',\quad C_2=18^\circ\,19'\,52'',\quad c_2=14^\circ\,45'\,11''.$$

Check. By formula [1 G.], $\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}c=\cos(a-b)\cos\tfrac{1}{2}C$.

If, in any example, after substituting the values of a, b, A in the equation $\dfrac{\sin B}{\sin b}=\dfrac{\sin A}{\sin a}$, $\sin B$ turns out to be greater than 1, or $\log\sin B$ positive, the solution is impossible. If we find $\sin B=1$, or $\log\sin B=0$, then $B=90^\circ$, and the solution can be completed by the method explained for right triangles.

If we find $\sin B$ less than 1, or $\log\sin B$ negative, there will be two corresponding values of B, — namely, the angle found in the table and its supplement, — and both must be accepted, giving *two solutions*, unless one of them should be incompatible with the condition that the greater angle lies opposite the greater side. Such a value of B would have to be rejected, giving *one solution*.

105. **Case** 6. *Given two angles and the side opposite one of them.* This is also an ambiguous case.

Given $A = 121°$, $B = 108°$, $a = 130°$, to find b, c, C.

By Case 5, solve the polar triangle $A'B'C'$, finding the parts B', C', c', and determine b, c, C by proposition 6, Sec. 83.

Ans. 1. $b_1 = 121° 47' 37''$, $c_1 = 59° 56' 16''$, $C_1 = 75° 34' 10''$.
2. $b_2 = 58° 12' 23''$, $c_2 = 160° 29' 42''$, $C_2 = 158° 3' 40''$.

EXERCISES

Solve the following triangles, checking the work where the answers are not given:

1. Given $a = 110° 4'$, $b = 74° 32'$, $c = 56° 30'$, to find A, B, C.
Ans. $A = 127° 35' 42''$, $B = 54° 23' 30''$, $C = 44° 42' 14''$.

2. Given $a = 58° 18'$, $b = 62° 40'$, $c = 78° 24'$, to find A, B, C. *Ans.* $A = 60° 8' 54''$, $B = 64° 54' 8''$, $C = 93° 2' 56''$.

3. Given $A = 53° 50'$, $B = 96° 18'$, $C = 64° 20'$, to find a, b, c. *Ans.* $a = 52° 43' 26''$, $b = 78° 26' 10''$, $c = 62° 40' 22''$.

4. Given $A = 80° 40'$, $B = 116° 20'$, $C = 92° 20'$, to find a, b, c. *Ans.* $a = 78° 23' 14''$, $b = 117° 10' 4''$, $c = 97° 19' 4''$.

5. Given $a = 96°$, $b = 64°$, $C = 108°$, to find A, B, c.
Ans. $A = 87° 32' 13''$, $B = 64° 32' 33''$, $c = 108° 47' 15''$.

6. Given $a = 76° 30'$, $b = 48° 10'$, $C = 36°$, to find A, B, c.
Ans. $A = 121° 32' 8''$, $B = 40° 46' 32''$, $c = 42° 6' 43''$.

7. Given $A = 137° 44'$, $B = 153° 48'$, $c = 107° 32'$, to find a, b, C.
Ans. $a = 102° 43' 32''$, $b = 140° 11' 40''$, $C = 138° 53' 28''$.

8. Given $A = 81^\circ 32'$, $B = 117^\circ 38'$, $c = 103^\circ 48'$, to find a, b, C. *Ans.* $a = 75^\circ 58' 44''$, $b = 119^\circ 39' 22''$, $C = 98^\circ 5'$.

9. Given $a = 100^\circ$, $b = 62^\circ$, $A = 95^\circ$, to find B, C, c.
Ans. $B = 63^\circ 16' 17''$, $C = 98^\circ 28' 4''$, $c = 102^\circ 5' 42''$.

10. Given $a = 59^\circ$, $b = 72^\circ$, $A = 50^\circ$, to find B, C, c.
Ans. 1. $B_1 = 58^\circ 12' 23''$, $C_1 = 120^\circ 3' 44''$, $c_1 = 104^\circ 25' 50''$.
2. $B_2 = 121^\circ 47' 37''$, $C_2 = 19^\circ 30' 18''$, $c_2 = 21^\circ 56' 20''$.

11. Given $A = 82^\circ$, $B = 116^\circ$, $a = 86^\circ$, to find C, b, c.
Ans. $C = 79^\circ 49' 14''$, $b = 115^\circ 7' 10''$, $c = 82^\circ 31' 40''$.

12. Given $A = 137^\circ 20'$, $B = 64^\circ$, $a = 149^\circ$, to find b, c, C.
Ans. 1. $b_1 = 136^\circ 55' 9''$, $c_1 = 19^\circ 45' 24''$, $C_1 = 26^\circ 24' 40''$.
2. $b_2 = 43^\circ 4' 51''$, $c_2 = 130^\circ 43' 42''$, $C_2 = 94^\circ 18' 32''$.

13. Given $a = 51^\circ 40'$, $b = 72^\circ 12'$, $A = 47^\circ 20'$, to find B, C, c.

14. Given $a = 76^\circ 20'$, $b = 110^\circ 24'$, $c = 58^\circ 12'$, to find A, B, C.

15. Given $A = 53^\circ$, $B = 46^\circ$, $C = 127^\circ$, to find a, b, c.

16. Given $A = 94^\circ$, $B = 62^\circ$, $C = 54^\circ$, to find a, b, c.

17. Given $a = 62^\circ 12'$, $b = 95^\circ$, $C = 107^\circ 10'$, to find A, B, c.

18. Given $a = 102^\circ$, $b = 139^\circ$, $C = 132^\circ$, to find A, B, c.

19. Given $A = 118^\circ$, $B = 82^\circ$, $c = 103^\circ 12'$, to find a, b, C.

20. Given $A = 156^\circ$, $B = 138^\circ$, $c = 106^\circ$, to find a, b, C.

106. Latitude and Longitude. The latitude and longitude of a point on the surface of the earth (regarded as a sphere) are its arc coördinates with reference to the equator and a prime meridian, usually taken to be the meridian passing through Greenwich. Let P and P' (Fig. 64) be the north and south poles respectively, PEP' the prime meridian, $ECDQ$ the equator,

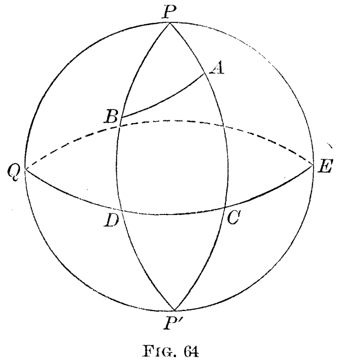

Fig. 64

and *A* and *B* any two points whose latitudes and longitudes are given. The longitude-difference of the two points is the arc *CD*, which measures the angle *APB*. The arcs *CA* and *DB* are the latitudes of *A* and *B* respectively, and determine the arcs *AP*, *BP*.

In the spherical triangle *APB*, therefore, we have given two sides and their included angle. The arc *AB* can be found in degrees by Case 3, Sec. 102.

To find *AB* in miles, the radius of the earth may be taken as 3956 miles, from which the length of an arc of one degree is found to be 69.05 miles approximately.

MISCELLANEOUS EXERCISES

1. Find the distance from San Francisco (latitude 37° 47′ 55″ N., longitude 122° 24′ 32″ W.) to Honolulu (latitude 21° 18′ N., longitude 157° 52′ W.). *Ans.* 2395 miles.

2. Find the distance from Paris (latitude 48° 50′ N., longitude 2° 20′ E.) to New York (latitude 40° 43′ N., longitude 74° 0′ W.). *Ans.* 3624 miles.

3. Find the distance from New Orleans (latitude 29° 58′ N., longitude 90° 3′ W.) to Rome (latitude 41° 54′ N., longitude 12° 29′ E.). *Ans.* 5444 miles.

4. Find the distance from San Francisco to Manila (latitude 14° 36′ N., longitude 120° 58′ E.). *Ans.* 5472 miles.

5. How many miles does a vessel go in sailing due west from a point in latitude 37° N., longitude 122° W., to a point in latitude 37° N., longitude 150° E.? How much would the distance be shortened by sailing along the arc of a great circle? *Ans.* 4651 miles; 202 miles.

APPENDIX

LOGARITHMIC AND TRIGONOMETRIC TABLES; EXPLANATION OF THEIR USE

TABLE I

Table I gives the logarithms of numbers from 1 to 10,000. On page 1, both the characteristic and the mantissa for numbers from 1 to 100 are given. In the rest of the table only the mantissa is given; the characteristic is to be determined by Sec. 12.

1. *To find the logarithm of a given integral number.*

(a) Let the number be less than 100.

Look for the number in column "N," page 1; opposite to it in the column "log" will be found its logarithm. Thus,

$$\log 38 = 1.57978.$$

(b) Let the number consist of three figures.

Look for the number in column "N," and find the mantissa on the same horizontal line under column "0." By Sec. 12, the characteristic is 2. Thus, we find (page 6)

$$\log 328 = 2.51587.$$

(c) Let the number consist of four figures.

Find the first three figures in column "N"; on the same horizontal line, in the column headed by the last figure, find the mantissa. By Sec. 12, the characteristic is 3. Thus, we find (page 15)

$$\log 7683 = 3.88553.$$

(d) Let the number consist of more than four figures. The logarithm will be found by the process of *interpolation,* which will be illustrated by an example.

Required the logarithm of 58736.

Place a decimal point after the fourth figure, giving 5873.6. By Sec. 12, this does not affect the mantissa. The number 5873.6 lies between 5873 and 5874.

By (c), the mantissa for 5874 is	.76893
and for 5873 it is	.76886
Difference for 1 unit (called tabular difference),	.00007

That is, when the number 5873 is increased by 1 unit, the corresponding mantissa is increased by 7 hundred-thousandths, or 7 *points*, as we may conveniently call this increase. It is assumed that if a number is increased by a small amount, the increase in the logarithm is proportional to the increase in the number. This assumption is not mathematically correct, but the error resulting from it is so small that it may be neglected in practical computations.

Now, 5873.6 exceeds 5873 by $\frac{6}{10}$ of a unit.

Then, by the above assumption, since the addition of a whole unit to 5873 increases the logarithm by 7 points, the addition of $\frac{6}{10}$ of a unit increases the logarithm by $\frac{6}{10} \times 7 = 4.2$, or 4 points, nearly.

Then, for 5873 the mantissa is	.76886
Add the correction,	4
and we have as the mantissa for 5873.6,	.76890

This is, as stated, also the mantissa for 58736. The characteristic is 4. Hence,

$$\log 58736 = 4.76890.$$

As another example, let it be required to find the logarithm of 367326.

As above, the mantissa is the same as for 3673.26.

The mantissa for 3674 is	.56514	
and for 3673 it is	.56490	
Difference for 1 unit,	84	points.

Hence, the correction is $.26 \times 84 = 21.84$, or 22 points, nearly.

Adding this correction, we have for the required mantissa, .56512. The characteristic is 5. Hence,

$$\log 367326 = 5.56512.$$

NOTE. When the fractional part of a "point" is less than .5, it is to be rejected, as in the first example in (d); while if it is .5 or more, it is to be counted as a whole point, as in the second example.

The same thing is done in computing the mantissas given in the tables. These mantissas, therefore, are only approximations, correct within 1 hundred thousandth. When the last figure of the mantissa is written $\underline{5}$, it means that the correct value is slightly less than 5.

(e) So far we have considered only integral numbers. Let the number whose logarithm is sought be a decimal fraction, or a whole number plus a decimal. The characteristic is to be determined by inspection in accordance with Sec. 12. The position of the decimal point does not affect the mantissa. Hence, we may in any case omit the decimal and find the mantissa, as in the case already explained. Thus, for 0.28 the mantissa is the same as for 28 and is found by (a); for 0.6834 it is the same as for 6834, and is found by (c); for 3.8757 or 0.38757 or 0.0038757 it is the same as for 38757, and is found by (d).

For example, let it be required to find the logarithm of 0.067374.

The mantissa is the same as for 67374, and is found to be .82849. By Sec. 12 the characteristic is $8 - 10$. Hence,

$$\log 0.067374 = 8.82849 - 10.$$

EXERCISES

Find the logarithms of the following numbers :

1. 632. 2. 6320. 3. 6327.

4. 6327.4. *Ans.* 3.80123.

5. 83545. *Ans.* 4.92192.

6. 297364. *Ans.* 5.47329.

7. 12.36.

8. 8.1267. *Ans.* 0.90992.

9. 0.31.

10. 0.0052683. *Ans.* 7.72167 − 10.

11. 0.43687. *Ans.* 9.64035 − 10.

12. 0.000786.

2. *To find the number corresponding to a given logarithm.*

The mantissa determines the figures of the required number; the characteristic determines the position of the decimal point.

(a) Let $\log x = 0.75785$.

The given mantissa is found in the table (page 11) in column "6"; and in the same horizontal line, in column "N," are found the figures 572. Hence, the figures of the required number, without regard to the position of the decimal point, are 5276. Placing the decimal point in accordance with Sec. 12, we have

$$x = 5.726.$$

Similarly, if $\log x = 2.70561$, $x = 507.7$;
and if $\log x = 8.61574 - 10$, $x = 0.04128$.

(b) Let $\log x = 7.45823 - 10$.

The next greater mantissa is .45834
The next less mantissa is $\underline{.45818}$ which corresponds to 2872.
Difference for 1 unit, 16 points.

The given mantissa exceeds .45818 by 5 points. Hence, 2872 must be increased by $\frac{5}{16}$ of a unit, giving $2872\frac{5}{16}$, or 2872.3, nearly. Placing the decimal point in accordance with Sec. 12, we have

$$x = 0.0028723.$$

EXERCISES

Find the numbers corresponding to the following logarithms. When interpolation is necessary, find the answer as far as *five* significant figures.

1. 1.26858.	*Ans.* 18.56.
2. 9.26858 – 10.	*Ans.* 0.1856.
3. 1.26850.	*Ans.* 18.5567.
4. 2.88572.	*Ans.* 768.63.
5. 0.66396.	*Ans.* 4612.8.
6. 7.61817 – 10.	*Ans.* 0.0041512.
7. 6.63914 – 10.	*Ans.* 0.00043565.
8. 3.46867.	*Ans.* 2942.2.
9. 8.31624.	*Ans.* 0.020713.

10. 0.23561. *Ans.* 1.7203.
11. 9.59794 – 10. *Ans.* 0.39623.
12. 8.55673 – 10. *Ans.* 0.036036.

TABLE II

LOGARITHMIC FUNCTIONS OF ANGLES

Table II gives the logarithms of the sines, cosines, tangents, and cotangents of angles from 0° to 90° at intervals of 1′, except as indicated. If the angle is less than 90°, the number of degrees it contains is to be found at the top of the page, the number of minutes in the column for minutes on the left-hand side of the page, and the name of the function at the top; if the angle is greater than 45°, the number of degrees is to be found at the bottom, the number of minutes on the right-hand column, and the name of the function at the bottom. The characteristic is the number at the top or bottom of the column diminished by 10. The numbers at the top and bottom are usually the same and apply to all the mantissas in the column; the only exceptions are found on pages 29 and 48. Here, the characteristics at the top of the column apply to all the mantissas above the bars drawn across the column, and the characteristics at the bottom to the mantissas below the bars.

1. *To find the logarithm of the sine, cosine, tangent, or cotangent of a given angle.*

The use of the table will be understood from the following examples:

(a) To find logarithm of the sine of 24° 27′.

This is written log sin 24° 17′, which is sometimes read *logarithmic sine of* 24° 17′.

On page 38, 24° is found at the top; in the column "log sin," in the same horizontal line with 27′, we find the mantissa .61689. The characteristic is 9 – 10. Hence,

$$\log \sin 24^\circ 27' = 9.61689 - 10.$$

(b) To find log sin 24° 27′ 13″.

$$\begin{array}{lrl} \text{As above,} & \log\sin 24^\circ\,27' = & 9.61689-10\\ & \log\sin 24^\circ\,28' = & \underline{9.61717-10}\\ \text{Difference for } 1',\text{ or } 60'', & & \qquad 28 \qquad \text{points.}\end{array}$$

The principle of proportional parts is assumed to hold here also; that is, it is assumed that if an angle be increased by a small amount, the change in logarithm of each function of the angle will be proportional to the increase in the angle.

Then if an increase of 1′, or 60″, produces a change of 28 points, 13″ will produce a change of $\frac{13}{60}\times 28 = 6$ points, nearly. Since the sine increases as the angle increases, this correction of 6 points must be added to log sin 24° 27′, giving

$$\log\sin 24^\circ\,27'\,13'' = 9.61695 - 10.$$

For sines and tangents, add the correction, for these functions increase as the angle increases; for cosines and cotangents, subtract the correction, for these functions decrease as the angle increases.

(c) To find log sin 61° 19′ 10″.

Here the angle is greater than 45°, and we find 61° on page 40 at the bottom; and looking in the right-hand column for minutes, we find

$$\begin{array}{lrl} & \log\sin 61^\circ\,19' = & 9.94314-10\\ & \log\sin 61^\circ\,20' = & \underline{9.94321-10}\\ \text{Difference for } 60'', & & \qquad 7 \qquad \text{points.}\end{array}$$

The difference for 10″ is accordingly $\frac{10}{60}\times 7 = 1$ point, nearly.

$$\therefore \log\sin 61^\circ\,19'\,10'' = 9.94315 - 10.$$

(d) To find log tan 73° 24′ 8″.

$$\begin{array}{lrl} & \log\tan 73^\circ\,24' = & 10.52562-10\\ & \log\tan 73^\circ\,25' = & \underline{10.52608-10}\\ \text{Difference for } 60'', & & \qquad 46 \qquad \text{points.}\end{array}$$

Difference for 8″ is, therefore, $\frac{8}{60}\times 46 = 6$ points, nearly. Adding this correction, we have

$$\log\tan 73^\circ\,24'\,8'' = 10.52568 - 10.$$

(e) To find log cos 40° 32′ 14″.

$$\begin{array}{lrl} & \log\cos 40^\circ\,32' = & 9.88083-10\\ & \log\cos 40^\circ\,33' = & \underline{9.88072-10}\\ \text{Difference for } 60'', & & \qquad 11 \qquad \text{points.}\end{array}$$

The difference for 14″ is, therefore, $\frac{14}{60} \times 11 = 3$ points, nearly. Subtracting this correction from log cos 40° 32′, we have

$$\log \cos 40° \, 32' \, 14'' = 9.88080 - 10.$$

(f) To find log cot 16° 40′ 12″.

$$\begin{array}{lrl} & \log \cot 16° \, 40' = & 10.52378 - 10 \\ & \log \cot 16° \, 41' = & \underline{10.52332 - 10} \\ \text{Difference for } 60'', & & \quad 46 \qquad \text{points.} \end{array}$$

The difference for 12″ is, therefore, $\frac{12}{60} \times 46 = 9$ points, nearly. Subtracting this correction from log cot 16° 40′, we have

$$\log \cot 16° \, 40' \, 12'' = 10.52369 - 10.$$

EXERCISES

Find the values of the following:

1. log sin 32° 37′ 21″. *Ans.* 9.73167 − 10.
2. log tan 36° 23′ 14″. *Ans.* 9.86742 − 10.
3. log cos 23° 12′ 28″. *Ans.* 9.96336 − 10.
4. log sin 63° 14′. *Ans.* 9.95078 − 10.
5. log sin 71° 23′ 41″. *Ans.* 9.97669 − 10.
6. log cot 13° 18′ 12″. *Ans.* 10.62626 − 10.
7. log cot 48° 42′ 14″. *Ans.* 10.94371 − 10.
8. log cos 73° 29′ 33″. *Ans.* 9.45353 − 10.
9. log sin 104° 26′. *Ans.* 9.97607 − 10.
10. log tan 112° 32′ 10″. *Ans.* 10.38200 − 10.
11. log sin 81° 52′ 43″. *Ans.* 9.99562 − 10.
12. log cos 47° 12′ 48″. *Ans.* 9.83205 − 10.

2. *To find the value of an angle when one of its logarithmic functions is given.*

(a) Given log sin $x = 9.77473 - 10$.

The given mantissa is found in the table (page 44) under 36°, in column "log sin" opposite 32′.

$$\therefore x = 36° \, 32'.$$

(b) Given log sin $x = 9.77478 - 10$, to find the value of x.

The given mantissa is not found in the table, but it lies between .77473 and .77490. Hence, the required angle lies between 36° 32′ and 36° 33′, the angles corresponding to these two mantissas.

The next greater mantissa is	.77490	
The next less mantissa is	.77473	which corresponds to 36° 32′.
Difference for 1′,	27	points.

The given mantissa exceeds the next less mantissa by 5 points; and as the addition of 27 points to the next less mantissa increases the corresponding angle by 1′, the addition of 5 points increases it by $\frac{5}{27} \times 60 = 11''$, nearly.

$$\therefore x = 36^\circ\ 32'\ 11''.$$

(c) Given log tan $x = 10.26171 - 10$, to find the value of x.

On page 40, in column marked "log tan" at the bottom of the page, we find

the next less mantissa to be	.26163	which corresponds to 61° 18′,
and the next greater mantissa,	.26193	
Difference for 1′,	30	points.

The given mantissa exceeds the next less mantissa by 8 points. Hence, the correction is $\frac{8}{30} \times 60 = 16''$. Adding this, we have

$$x = 61^\circ\ 18'\ 16''.$$

(d) Given log $x = 9.94809 - 10$, to find the value of x.

The next greater mantissa is	.94813	
The next less mantissa is	.94806	which corresponds to 27° 28′.
Difference for 1′,	7	points.

The given mantissa exceeds the next less mantissa by 3 points. Hence, the correction is $\frac{3}{7} \times 60 = 26''$, nearly. Subtracting this from 27° 28′, we have

$$x = 27^\circ\ 27'\ 34''.$$

(e) Given log cot $x = 10.13280 - 10$, to find the value of x.

The next greater mantissa is	.13291	
The next less mantissa is	.13264	which corresponds to 36° 23′.
Difference for 1′,	27	points.

The given mantissa exceeds the next less mantissa by 16 points. Hence, the correction is $\frac{16}{27} \times 60 = 36''$, nearly. Subtracting this from 36° 23′, we have

$$x = 36^\circ\ 22'\ 34''.$$

EXERCISES

Determine the value of x in each of the following equations:

1. $\log \sin x = 9.75515 - 10$. *Ans.* $34° 41' 3''$.
2. $\log \tan x = 9.71257 - 10$. *Ans.* $27° 17' 21''$.
3. $\log \tan x = 10.79774 - 10$. *Ans.* $80° 56' 53''$.
4. $\log \sin x = 9.97527 - 10$. *Ans.* $70° 50' 48''$.
5. $\log \sin x = 9.92939 - 10$. *Ans.* $58° 12' 23''$.
6. $\log \cos x = 9.82678 - 10$. *Ans.* $47° 50' 56''$.
7. $\log \cos x = 9.82431 - 10$. *Ans.* $48° 8' 32''$.
8. $\log \cos x = 9.58503 - 10$. *Ans.* $67° 22' 48''$.
9. $\log \cot x = 10.33857 - 10$. *Ans.* $24° 38' 9''$.
10. $\log \cot x = 10.76478 - 10$. *Ans.* $9° 45' 9''$.
11. $\log \cot x = 9.98060 - 10$. *Ans.* $46° 16' 46''$.
12. $\log \tan x = 10.24009 - 10$. *Ans.* $63° 5' 14''$.

TABLE III

For certain functions of angles near 0° or 90°, the assumption involved in the principle of proportional parts leads to errors too great to be neglected; and for more accurate results Table III should be used. By this table we can determine more accurately than by Table II,

1. log sin, log tan, log cot, for angles between 0° and 2°;
2. log cos, log tan, log cot, for angles between 88° and 90°;
3. the angle from the logarithmic functions in the corresponding cases.

The numbers S and T are calculated to satisfy the relations

$$\log \sin x = \log x + S,$$
$$\log \tan x = \log x + T,$$

x being expressed in *seconds;* it being understood also that -10 is to be written after each of the numbers in the columns "S" and "T."

EXAMPLES

1. To find log sin $1^\circ\,13'\,26''$.

$$1^\circ\,13'\,26'' = 4406''.$$

By referring to the column headed "x''," the number 4406 is found to lie between 4190 and 4840, and the corresponding number in the column "S" is 4.68554. We have, therefore,

$$\begin{aligned} \log 4406 &= 3.64404 \\ S &= \underline{4.68554 - 10} \\ \therefore \log\sin 1^\circ\,13'\,26'' &= 8.32958 - 10 \end{aligned}$$

2. To find log tan $1^\circ\,33'\,14''$.

$$\begin{aligned} 1^\circ\,33'\,14'' &= 5594'' \\ \log 5594 &= 3.74772 \\ T &= \underline{4.68568 - 10} \\ \therefore \log\tan 1^\circ\,33'\,14'' &= 8.43340 - 10 \end{aligned}$$

3. To find log cot $88^\circ\,26'\,46''$.

The cotangent of an angle equals the tangent of its complement. Hence,

$\log\cot 88^\circ\,26'\,46'' = \log\tan 1^\circ\,33'\,14'' = 8.43340 - 10$, by Example 2.

4. To find log cos $88^\circ\,31'\,28''$.

$$\log\cos 88^\circ\,31'\,28'' = \log\sin 1^\circ\,28'\,32'',$$

which may be found as in Example 1.

5. To find log tan $88^\circ\,26'\,46''$.

$$\tan 88^\circ\,26'\,46'' = \cot 1^\circ\,33'\,14'' = \frac{1}{\tan 1^\circ\,33'\,14''}.$$

$$\begin{aligned} \therefore \log\tan 88^\circ\,26'\,46'' &= \log 1 - \log\tan 1^\circ\,33'\,14'' \\ &= 10 - 10 - \log\tan 1^\circ\,33'\,14''. \end{aligned}$$

$$\begin{aligned} & 10 \quad\quad -10 \\ \text{By Example 2, } \log\tan 1^\circ\,33'\,14'' = {} & \underline{8.43340 - 10} \\ \text{By subtraction, } \log\tan 88^\circ\,26'\,46'' = {} & 1.56660 \end{aligned}$$

6. To find log cot $88^\circ\,26'\,46''$.

$$\log\cot 88^\circ\,26'\,46'' = \log\tan 1^\circ\,33'\,14'' = 8.43340 - 10.$$

Determine x from each of the following equations:

7. $\log \sin x = 8.38426 - 10.$

From the first of the above formulas, we have

$$\log x = \log \sin x - S.$$

$$\begin{aligned} \log \sin x &= 8.38426 - 10 \\ S &= \underline{4.68553 - 10} \\ \log x &= 3.69873 \\ x &= 4997''.2 = 1^\circ\ 23'\ 17''.2. \end{aligned}$$

8. $\operatorname{Log} \tan x = 1.63283.$

Let y be the complement of x. From the first of the above formulas, $\log y = \log \tan y - T.$

$$\log \tan y = \log \cot x = \log \left(\frac{1}{\tan x}\right) = 10 - 10 - \log \tan x.$$

$$\begin{aligned} & \quad 10 \qquad\quad -10 \\ \log \tan x &= \underline{\ 1.63283 \qquad\quad\ } \\ \log \tan y &= \ 8.36717 - 10 \\ T &= \underline{\ 4.68565 - 10} \\ \log y &= \ 3.68152 \\ y &= 4803''.1 = 1^\circ\ 20'\ 3''.1. \\ 90^\circ - y = x &= 88^\circ\ 39'\ 56''.9. \end{aligned}$$

EXERCISES

Find the values of the following:

1. $\log \sin 1^\circ\ 32'\ 18''.$ *Ans.* $8.42887 - 10$
2. $\log \tan 1^\circ\ 2'\ 13''.$ *Ans.* $8.25768 - 10$
3. $\log \cot 89^\circ\ 3'\ 12''.$ *Ans.* $8.21814 - 10$
4. $\log \cos 88^\circ\ 42'\ 15''.$ *Ans.* $8.35439 - 10$
5. $\log \tan 88^\circ\ 37'\ 25''.$ *Ans.* 1.61930
6. $\log \cot 88^\circ\ 32'\ 8''.$ *Ans.* $8.43169 - 10$

Determine x from each of the following equations:

7. $\log \sin x = 8.46872 - 10.$ *Ans.* $1^\circ\ 41'\ 10''.3.$
8. $\log \tan x = 8.36423 - 10.$ *Ans.* $1^\circ\ 19'\ 30''.7.$
9. $\log \cot x = 1.71326 - 10.$ *Ans.* $1^\circ\ 6'\ 31''.3.$
10. $\log \cos x = 8.42126 - 10.$ First find y, the complement of x. *Ans.* $88^\circ\ 29'\ 18''.2.$

TABLE IV

This table gives, to four places of decimals, the sines, cosines, tangents, and cotangents of angles from 0° to 90° at intervals of 1′. The principle of proportional parts is assumed, as in the case of the logarithmic functions.

EXAMPLES

1. To find $\sin 62^\circ\, 13'\, 28''$.

$$\begin{aligned} \sin 62^\circ\, 13' &= 0.8847 \\ \sin 62^\circ\, 14' &= \underline{0.8849} \end{aligned}$$

Difference for 1′, 2 points.

$\therefore$ difference for $28'' = \frac{28}{60} \times 2 = 1$ point, nearly.

Adding this correction to $\sin 62^\circ\, 13'$, we have

$$\sin 62^\circ\, 13'\, 28'' = 0.8848.$$

2. Given $\cot A = 1.8473$, to find A.

1.8473 lies between 1.8469 which corresponds to 28° 26′,
and $\underline{1.8482}$ which corresponds to 28° 25′.
Difference for 1′, 13 points.

1.8473 exceeds 1.8469 by 4 points. Therefore, the angle 28° 26′ must be diminished by $\frac{4}{13} \times 60'' = 18''$, nearly.

$$\therefore A = 28^\circ\, 25'\, 42''.$$

EXERCISES

Find the values of the following :

1. $\sin 36^\circ\, 12'\, 38''$.
2. $\cos 42^\circ\, 18'\, 15''$.
3. $\tan 62^\circ\, 18''\, 24''$.
4. $\cot 33^\circ\, 14'\, 50''$.
5. $\cos 48^\circ\, 37'\, 18''$.

Determine A from each of the following equations :

6. $\sin A = 0.4882$.
7. $\tan A = 2.2963$.
8. $\cos A = 0.8423$.
9. $\cot A = 1.1343$.

10. From the results of Exercises 1 to 5 find, by use of Table I, the corresponding logarithmic functions of the given angles; find the same by use of Table II, and compare results. Thus, in Example 1, we find

$$\sin 36^\circ\, 12'\, 38'' = 0.5907.$$

By Table I, $\log 0.5707 = 9.77137 - 10.$

$$\therefore \log \sin 36^\circ\, 12'\, 38'' = 9.77137 - 10.$$

By Table II, we find

$$\log \sin 36^\circ\, 12'\, 38'' = 9.77141 - 10\ ;$$

the discrepancy of 4 points being due to the fact that in Table IV the results are given only to four places, while in Table II they are given to five places.

11. Given $\log \sin A = 9.76574 - 10$, to find $\sin A$ by Table I. *Ans.* 0.5831.

12. Given $\log \cos A = 9.83264 - 10$, to find $\cos A$. *Ans.* 0.6802.

13. Given $\log \tan A = 0.97804$, to find $\tan A$. *Ans.* 9.507.

14. Given $\log \sin A = 9.59573 - 10$, to find A without using Table II. *Ans.* 23° 13′.

The following tables are reproduced, by permission, from electrotypes from Wentworth and Hill's Five Place Logarithmic and Trigonometric Tables.

TABLES

TABLE I.

THE

COMMON OR BRIGGS LOGARITHMS

OF THE

NATURAL NUMBERS

From 1 to 10000.

N	log	N	log	N	log	N	log	N	log
1	0. 00 000	**21**	1. 32 222	**41**	1. 61 278	**61**	1. 78 533	**81**	1. 90 849
2	0. 30 103	22	1. 34 242	42	1. 62 325	62	1. 79 239	82	1. 91 381
3	0. 47 712	23	1. 36 173	43	1. 63 347	63	1. 79 934	83	1. 91 908
4	0. 60 206	24	1. 38 021	44	1. 64 345	64	1. 80 618	84	1. 92 428
5	0. 69 897	25	1. 39 794	45	1. 65 321	65	1. 81 291	85	1. 92 942
6	0. 77 815	**26**	1. 41 497	**46**	1. 66 276	**66**	1. 81 954	**86**	1. 93 450
7	0. 84 510	27	1. 43 136	47	1. 67 210	67	1. 82 607	87	1. 93 952
8	0. 90 309	28	1. 44 716	48	1. 68 124	68	1. 83 251	88	1. 94 448
9	0. 95 424	29	1. 46 240	49	1. 69 020	69	1. 83 885	89	1. 94 939
10	1. 00 000	30	1. 47 712	50	1. 69 897	70	1. 84 510	90	1. 95 424
11	1. 04 139	**31**	1. 49 136	**51**	1. 70 757	**71**	1. 85 126	**91**	1. 95 904
12	1. 07 918	32	1. 50 515	52	1. 71 600	72	1. 85 733	92	1. 96 379
13	1. 11 394	33	1. 51 851	53	1. 72 428	73	1. 86 332	93	1. 96 848
14	1. 14 613	34	1. 53 148	54	1. 73 239	74	1. 86 923	94	1. 97 313
15	1. 17 609	35	1. 54 407	55	1. 74 036	75	1. 87 506	95	1. 97 772
16	1. 20 412	**36**	1. 55 630	**56**	1. 74 819	**76**	1. 88 081	**96**	1. 98 227
17	1. 23 045	37	1. 56 820	57	1. 75 587	77	1. 88 649	97	1. 98 677
18	1. 25 527	38	1. 57 978	58	1. 76 343	78	1. 89 209	98	1. 99 123
19	1. 27 875	39	1. 59 106	59	1. 77 085	79	1. 89 763	99	1. 99 564
20	1. 30 103	40	1. 60 206	60	1. 77 815	80	1. 90 309	100	2. 00 000
N	log	N	log	N	log	N	log	N	log

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
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256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
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264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
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281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
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283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
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291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
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295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
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300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
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N	0	1	2	3	4	5	6	7	8	9
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
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311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
322	50 786	50 799	50 813	50 826	50 840	50 853	50 866	50 880	50 893	50 907
323	50 920	50 934	50 947	50 961	50 974	50 987	51 001	51 014	51 028	51 041
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325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
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329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
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331	51 983	51 996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	52 140	52 153	52 166	52 179	52 192	52 205	52 218	52 231
333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
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337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
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340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
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343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	53 908	53 920	53 933	53 945	53 958	53 970	53 983	53 995	54 008	54 020
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348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
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351	54 531	54 543	54 555	54 568	54 580	54 593	54 605	54 617	54 630	54 642
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353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
358	55 388	55 400	55 413	55 425	55 437	55 449	55 461	55 473	55 485	55 497
359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	55 751	55 763	55 775	55 787	55 799	55 811	55 823	55 835	55 847	55 859
362	55 871	55 883	55 895	55 907	55 919	55 931	55 943	55 955	55 967	55 979
363	55 991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	56 122	56 134	56 146	56 158	56 170	56 182	56 194	56 205	56 217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
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373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
377	57 634	57 646	57 657	57 669	57 680	57 692	57 703	57 715	57 726	57 738
378	57 749	57 761	57 772	57 784	57 795	57 807	57 818	57 830	57 841	57 852
379	57 864	57 875	57 887	57 898	57 910	57 921	57 933	57 944	57 955	57 967
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381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
384	58 433	58 444	58 456	58 467	58 478	58 490	58 501	58 512	58 524	58 535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
388	58 883	58 894	58 906	58 917	58 928	58 939	58 950	58 961	58 973	58 984
389	58 995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

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400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 73$\underline{5}$
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 88$\underline{5}$	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 04$\underline{5}$	61 055
408	61 066	61 077	61 087	61 098	61 109	61 119	61 130	61 140	61 151	61 162
409	61 172	61 183	61 194	61 204	61 21$\underline{5}$	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 39$\underline{5}$	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 80$\underline{5}$	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 04$\underline{5}$	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 31$\underline{5}$
420	62 32$\underline{5}$	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 65$\underline{5}$	62 66$\underline{5}$	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 15$\underline{5}$	63 16$\underline{5}$	63 17$\underline{5}$	63 18$\underline{5}$	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
433	63 649	63 659	63 669	63 679	63 689	63 699	63 709	63 719	63 729	63 739
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436	63 949	63 959	63 969	63 979	63 988	63 998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	64 108	64 118	64 128	64 137
438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
439	64 246	64 256	64 266	64 276	64 286	64 296	64 306	64 316	64 326	64 335
440	64 345	64 355	64 365	64 37$\underline{5}$	64 38$\underline{5}$	64 39$\underline{5}$	64 404	64 414	64 424	64 434
441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 88$\underline{5}$	64 89$\underline{5}$	64 904	64 914	64 924
446	64 933	64 943	64 953	64 963	64 972	64 982	64 992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 21$\underline{5}$
449	65 22$\underline{5}$	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
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451	65 418	65 427	65 437	65 447	65 456	65 466	65 475	65 485	65 495	65 504
452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
453	65 610	65 619	65 629	65 639	65 648	65 658	65 667	65 677	65 686	65 696
454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
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663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
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700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
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702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
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715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
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717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
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719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
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727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
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734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
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738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
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743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
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746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
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791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
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837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
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842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
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844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
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856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
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862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
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866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545̲	95 55̲0	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665̲	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785̲	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895̲	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995̲
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095̲	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185̲
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275̲	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365̲	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445̲	96 45̲0	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525̲	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605̲	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685̲	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755̲	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825̲	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895̲	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965̲	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035̲	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165̲	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225̲	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 35̲0	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465̲	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525̲	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585̲
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635̲	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695̲	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745̲	97 749	97 754	97 759	97 763	97 768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

N	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

TABLE II.

THE LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS:

From 0° to 0° 3′, or 89° 57′ to 90°, for every second;
From 0° to 2°, or 88° to 90°, for every ten seconds:
From 1° to 89°, for every minute.

NOTE. To all the logarithms −10 is to be appended.

log sin **0°** log tan = log sin
log cos = 10. 00 000

″	0′	1′	2′	″	″	0′	1′	2′	″
0	—	6. 46 373	6. 76 476	**60**	**30**	6. 16 270	6. 63 982	6. 86 167	**30**
1	4. 68 557	6. 47 090	6. 76 836	59	31	6. 17 694	6. 64 462	6. 86 455	29
2	4. 98 660	6. 47 797	6. 77 193	58	32	6. 19 072	6. 64 936	6. 86 742	28
3	5. 16 270	6. 48 492	6. 77 548	57	33	6. 20 409	6. 65 406	6. 87 027	27
4	5. 28 763	6. 49 175	6. 77 900	56	34	6. 21 705	6. 65 870	6. 87 310	26
5	5. 38 454	6. 49 849	6. 78 248	**55**	**35**	6. 22 964	6. 66 330	6. 87 591	**25**
6	5. 46 373	6. 50 512	6. 78 595̲	54	36	6. 24 188	6. 66 785̲	6. 87 870	24
7	5. 53 067	6. 51 165̲	6. 78 938	53	37	6. 25 378	6. 67 235̲	6. 88 147	23
8	5. 58 866	6. 51 808	6. 79 278	52	38	6. 26 536	6. 67 680	6. 88 423	22
9	5. 63 982	6. 52 442	6. 79 616	51	39	6. 27 664	6. 68 121	6. 88 697	21
10	5. 68 557	6. 53 067	6. 79 952	**50**	**40**	6. 28 763	6. 68 557	6. 88 969	**20**
11	5. 72 697	6. 53 683	6. 80 285̲	49	41	6. 29 836	6. 68 990	6. 89 240	19
12	5. 76 476	6. 54 291	6. 80 615̲	48	42	6. 30 882	6. 69 418	6. 89 509	18
13	5. 79 952	6. 54 890	6. 80 943	47	43	6. 31 904	6. 69 841	6. 89 776	17
14	5. 83 170	6. 55 481	6. 81 268	46	44	6. 32 903	6. 70 261	6. 90 042	16
15	5. 86 167	6. 56 064	6. 81 591	**45**	**45**	6. 33 879	6. 70 676	6. 90 306	**15**
16	5. 88 969	6. 56 639	6. 81 911	44	46	6. 34 833	6. 71 088	6. 90 568	14
17	5. 91 602	6. 57 207	6. 82 230	43	47	6. 35 767	6. 71 496	6. 90 829	13
18	5. 94 085̲	6. 57 767	6. 82 545	42	48	6. 36 682	6. 71 900	6. 91 088	12
19	5. 96 433	6. 58 320	6. 82 859	41	49	6. 37 577	6. 72 300	6. 91 346	11
20	5. 98 660	6. 58 866	6. 83 170	**40**	**50**	6. 38 454	6. 72 697	6. 91 602	**10**
21	6. 00 779	6. 59 406	6. 83 479	39	51	6. 39 315̲	6. 73 090	6. 91 857	9
22	6. 02 800	6. 59 939	6. 83 786	38	52	6. 40 158	6. 73 479	6. 92 110	8
23	6. 04 730	6. 60 465	6. 84 091	37	53	6. 40 985	6. 73 865	6. 92 362	7
24	6. 06 579	6. 60 985	6. 84 394	36	54	6. 41 797	6. 74 248	6. 92 612	6
25	6. 08 351	6. 61 499	6. 84 694	**35**	**55**	6. 42 594	6. 74 627	6. 92 861	**5**
26	6. 10 055̲	6. 62 007	6. 84 993	34	56	6. 43 376	6. 75 003	6. 93 109	4
27	6. 11 694	6. 62 509	6. 85 289	33	57	6. 44 145̲	6. 75 376	6. 93 355̲	3
28	6. 13 273	6. 63 006	6. 85 584	32	58	6. 44 900	6. 75 746	6. 93 599	2
29	6. 14 797	6. 63 496	6. 85 876	31	59	6. 45 643	6. 76 112	6. 93 843	1
30	6. 16 270	6. 63 982	6. 86 167	**30**	**60**	6. 46 373	6. 76 476	6. 94 085̲	**0**
″	59′	58′	57′	″	″	59′	58′	57′	″

log cot = log cos
log sin = 10. 00 000 **89°** **log cos**

0°

′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
0 0	—	—	10.00000	0 **60**	**10** 0	7.46 373	7.46 373	10.00000	0 **50**
10	5.68 557	5.68 557	10.00000	50	10	7.47 090	7.47 091	10.00000	50
20	5.98 660	5.98 660	10.00000	40	20	7.47 797	7.47 797	10.00000	40
30	6.16 270	6.16 270	10.00000	30	30	7.48 491	7.48 492	10.00000	30
40	6.28 763	6.28 763	10.00000	20	40	7.49 175	7.49 176	10.00000	20
50	6.38 454	6.38 454	10.00000	10	50	7.49 849	7.49 849	10.00000	10
1 0	6.46 373	6.46 373	10.00000	0 **59**	**11** 0	7.50 512	7.50 512	10.00000	0 **49**
10	6.53 067	6.53 067	10.00000	50	10	7.51 165	7.51 165	10.00000	50
20	6.58 866	6.58 866	10.00000	40	20	7.51 808	7.51 809	10.00000	40
30	6.63 982	6.63 982	10.00000	30	30	7.52 442	7.52 443	10.00000	30
40	6.68 557	6.68 557	10.00000	20	40	7.53 067	7.53 067	10.00000	20
50	6.72 697	6.72 697	10.00000	10	50	7.53 683	7.53 683	10.00000	10
2 0	6.76 476	6.76 476	10.00000	0 **58**	**12** 0	7.54 291	7.54 291	10.00000	0 **48**
10	6.79 952	6.79 952	10.00000	50	10	7.54 890	7.54 890	10.00000	50
20	6.83 170	6.83 170	10.00000	40	20	7.55 481	7.55 481	10.00000	40
30	6.86 167	6.86 167	10.00000	30	30	7.56 064	7.56 064	10.00000	30
40	6.88 969	6.88 969	10.00000	20	40	7.56 639	7.56 639	10.00000	20
50	6.91 602	6.91 602	10.00000	10	50	7.57 206	7.57 207	10.00000	10
3 0	6.94 085	6.94 085	10.00000	0 **57**	**13** 0	7.57 767	7.57 767	10.00000	0 **47**
10	6.96 433	6.96 433	10.00000	50	10	7.58 320	7.58 320	10.00000	50
20	6.98 660	6.98 661	10.00000	40	20	7.58 866	7.58 867	10.00000	40
30	7.00 779	7.00 779	10.00000	30	30	7.59 406	7.59 406	10.00000	30
40	7.02 800	7.02 800	10.00000	20	40	7.59 939	7.59 939	10.00000	20
50	7.04 730	7.04 730	10.00000	10	50	7.60 465	7.60 466	10.00000	10
4 0	7.06 579	7.06 579	10.00000	0 **56**	**14** 0	7.60 985	7.60 986	10.00000	0 **46**
10	7.08 351	7.08 352	10.00000	50	10	7.61 499	7.61 500	10.00000	50
20	7.10 055	7.10 055	10.00000	40	20	7.62 007	7.62 008	10.00000	40
30	7.11 694	7.11 694	10.00000	30	30	7.62 509	7.62 510	10.00000	30
40	7.13 273	7.13 273	10.00000	20	40	7.63 006	7.63 006	10.00000	20
50	7.14 797	7.14 797	10.00000	10	50	7.63 496	7.63 497	10.00000	10
5 0	7.16 270	7.16 270	10.00000	0 **55**	**15** 0	7.63 982	7.63 982	10.00000	0 **45**
10	7.17 694	7.17 694	10.00000	50	10	7.64 461	7.64 462	10.00000	50
20	7.19 072	7.19 073	10.00000	40	20	7.64 936	7.64 937	10.00000	40
30	7.20 409	7.20 409	10.00000	30	30	7.65 406	7.65 406	10.00000	30
40	7.21 705	7.21 705	10.00000	20	40	7.65 870	7.65 871	10.00000	20
50	7.22 964	7.22 964	10.00000	10	50	7.66 330	7.66 330	10.00000	10
6 0	7.24 188	7.24 188	10.00000	0 **54**	**16** 0	7.66 784	7.66 785	10.00000	0 **44**
10	7.25 378	7.25 378	10.00000	50	10	7.67 235	7.67 235	10.00000	50
20	7.26 536	7.26 536	10.00000	40	20	7.67 680	7.67 680	10.00000	40
30	7.27 664	7.27 664	10.00000	30	30	7.68 121	7.68 121	10.00000	30
40	7.28 763	7.28 764	10.00000	20	40	7.68 557	7.68 558	9.99999	20
50	7.29 836	7.29 836	10.00000	10	50	7.68 989	7.68 990	9.99999	10
7 0	7.30 882	7.30 882	10.00000	0 **53**	**17** 0	7.69 417	7.69 418	9.99 999	0 **43**
10	7.31 904	7.31 904	10.00000	50	10	7.69 841	7.69 842	9.99 999	50
20	7.32 903	7.32 903	10.00000	40	20	7.70 261	7.70 261	9.99 999	40
30	7.33 879	7.33 879	10.00000	30	30	7.70 676	7.70 677	9.99 999	30
40	7.34 833	7.34 833	10.00000	20	40	7.71 088	7.71 088	9.99 999	20
50	7.35 767	7.35 767	10.00000	10	50	7.71 496	7.71 496	9.99 999	10
8 0	7.36 682	7.36 682	10.00000	0 **52**	**18** 0	7.71 900	7.71 900	9.99 999	0 **42**
10	7.37 577	7.37 577	10.00000	50	10	7.72 300	7.72 301	9.99 999	50
20	7.38 454	7.38 455	10.00000	40	20	7.72 697	7.72 697	9.99 999	40
30	7.39 314	7.39 315	10.00000	30	30	7.73 090	7.73 090	9.99 999	30
40	7.40 158	7.40 158	10.00000	20	40	7.73 479	7.73 480	9.99 999	20
50	7.40 985	7.40 985	10.00000	10	50	7.73 865	7.73 866	9.99 999	10
9 0	7.41 797	7.41 797	10.00000	0 **51**	**19** 0	7.74 248	7.74 248	9.99 999	0 **41**
10	7.42 594	7.42 594	10.00000	50	10	7.74 627	7.74 628	9.99 999	50
20	7.43 376	7.43 376	10.00000	40	20	7.75 003	7.75 004	9.99 999	40
30	7.44 145	7.44 145	10.00000	30	30	7.75 376	7.75 377	9.99 999	30
40	7.44 900	7.44 900	10.00000	20	40	7.75 745	7.75 746	9.99 999	20
50	7.45 643	7.45 643	10.00000	10	50	7.76 112	7.76 113	9.99 999	10
10 0	7.46 373	7.46 373	10.00000	0 **50**	**20** 0	7.76 475	7.76 476	9.99 999	0 **40**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
20 0	7. 76 475	7. 76 476	9. 99 999	0 **40**	**30** 0	7. 94 084	7. 94 086	9. 99 998	0 **30**
10	7. 76 836	7. 76 837	9. 99 999	50	10	7. 94 325	7. 94 326	9. 99 998	50
20	7. 77 193	7. 77 194	9. 99 999	40	20	7. 94 564	7. 94 566	9. 99 998	40
30	7. 77 548	7. 77 549	9. 99 999	30	30	7. 94 802	7. 94 804	9. 99 998	30
40	7. 77 899	7. 77 900	9. 99 999	20	40	7. 95 039	7. 95 040	9. 99 998	20
50	7. 78 248	7. 78 249	9. 99 999	10	50	7. 95 274	7. 95 276	9. 99 998	10
21 0	7. 78 594	7. 78 595	9. 99 999	0 **39**	**31** 0	7. 95 508	7. 95 510	9. 99 998	0 **29**
10	7. 78 938	7. 78 938	9. 99 999	50	10	7. 95 741	7. 95 743	9. 99 998	50
20	7. 79 278	7. 79 279	9. 99 999	40	20	7. 95 973	7. 95 974	9. 99 998	40
30	7. 79 616	7. 79 617	9. 99 999	30	30	7. 96 203	7. 96 205	9. 99 998	30
40	7. 79 952	7. 79 952	9. 99 999	20	40	7. 96 432	7. 96 434	9. 99 998	20
50	7. 80 284	7. 80 285	9. 99 999	10	50	7. 96 660	7. 96 662	9. 99 998	10
22 0	7. 80 615	7. 80 615	9. 99 999	0 **38**	**32** 0	7. 96 887	7. 96 889	9. 99 998	0 **28**
10	7. 80 942	7. 80 943	9. 99 999	50	10	7. 97 113	7. 97 114	9. 99 998	50
20	7. 81 268	7. 81 269	9. 99 999	40	20	7. 97 337	7. 97 339	9. 99 998	40
30	7. 81 591	7. 81 591	9. 99 999	30	30	7. 97 560	7. 97 562	9. 99 998	30
40	7. 81 911	7. 81 912	9. 99 999	20	40	7. 97 782	7. 97 784	9. 99 998	20
50	7. 82 229	7. 82 230	9. 99 999	10	50	7. 98 003	7. 98 005	9. 99 998	10
23 0	7. 82 545	7. 82 546	9. 99 999	0 **37**	**33** 0	7. 98 223	7. 98 225	9. 99 998	0 **27**
10	7. 82 859	7. 82 860	9. 99 999	50	10	7. 98 442	7. 98 444	9. 99 998	50
20	7. 83 170	7. 83 171	9. 99 999	40	20	7. 98 660	7. 98 662	9. 99 998	40
30	7. 83 479	7. 83 480	9. 99 999	30	30	7. 98 876	7. 98 878	9. 99 998	30
40	7. 83 786	7. 83 787	9. 99 999	20	40	7. 99 092	7. 99 094	9. 99 998	20
50	7. 84 091	7. 84 092	9. 99 999	10	50	7. 99 306	7. 99 308	9. 99 998	10
24 0	7. 84 393	7. 84 394	9. 99 999	0 **36**	**34** 0	7. 99 520	7. 99 522	9. 99 998	0 **26**
10	7. 84 694	7. 84 695	9. 99 999	50	10	7. 99 732	7. 99 734	9. 99 998	50
20	7. 84 992	7. 84 994	9. 99 999	40	20	7. 99 943	7. 99 946	9. 99 998	40
30	7. 85 289	7. 85 290	9. 99 999	30	30	8. 00 154	8. 00 156	9. 99 998	30
40	7. 85 583	7. 85 584	9. 99 999	20	40	8. 00 363	8. 00 365	9. 99 998	20
50	7. 85 876	7. 85 877	9. 99 999	10	50	8. 00 571	8. 00 574	9. 99 998	10
25 0	7. 86 166	7. 86 167	9. 99 999	0 **35**	**35** 0	8. 00 779	8. 00 781	9. 99 998	0 **25**
10	7. 86 455	7. 86 456	9. 99 999	50	10	8. 00 985	8. 00 987	9. 99 998	50
20	7. 86 741	7. 86 743	9. 99 999	40	20	8. 01 190	8. 01 193	9. 99 998	40
30	7. 87 026	7. 87 027	9. 99 999	30	30	8. 01 395	8. 01 397	9. 99 998	30
40	7. 87 309	7. 87 310	9. 99 999	20	40	8. 01 598	8. 01 600	9. 99 998	20
50	7. 87 590	7. 87 591	9. 99 999	10	50	8. 01 801	8. 01 803	9. 99 998	10
26 0	7. 87 870	7. 87 871	9. 99 999	0 **34**	**36** 0	8. 02 002	8. 02 004	9. 99 998	0 **24**
10	7. 88 147	7. 88 148	9. 99 999	50	10	8. 02 203	8. 02 205	9. 99 998	50
20	7. 88 423	7. 88 424	9. 99 999	40	20	8. 02 402	8. 02 405	9. 99 998	40
30	7. 88 697	7. 88 698	9. 99 999	30	30	8. 02 601	8. 02 604	9. 99 998	30
40	7. 88 969	7. 88 970	9. 99 999	20	40	8. 02 799	8. 02 801	9. 99 998	20
50	7. 89 240	7. 89 241	9. 99 999	10	50	8. 02 996	8. 02 998	9. 99 998	10
27 0	7. 89 509	7. 89 510	9. 99 999	0 **33**	**37** 0	8. 03 192	8. 03 194	9. 99 997	0 **23**
10	7. 89 776	7. 89 777	9. 99 999	50	10	8. 03 387	8. 03 390	9. 99 997	50
20	7. 90 041	7. 90 043	9. 99 999	40	20	8. 03 581	8. 03 584	9. 99 997	40
30	7. 90 305	7. 90 307	9. 99 999	30	30	8. 03 775	8. 03 777	9. 99 997	30
40	7. 90 568	7. 90 569	9. 99 999	20	40	8. 03 967	8. 03 970	9. 99 997	20
50	7. 90 829	7. 90 830	9. 99 999	10	50	8. 04 159	8. 04 162	9. 99 997	10
28 0	7. 91 088	7. 91 089	9. 99 999	0 **32**	**38** 0	8. 04 350	8. 04 353	9. 99 997	0 **22**
10	7. 91 346	7. 91 347	9. 99 999	50	10	8. 04 540	8. 04 543	9. 99 997	50
20	7. 91 602	7. 91 603	9. 99 999	40	20	8. 04 729	8. 04 732	9. 99 997	40
30	7. 91 857	7. 91 858	9. 99 999	30	30	8. 04 918	8. 04 921	9. 99 997	30
40	7. 92 110	7. 92 111	9. 99 998	20	40	8. 05 105	8. 05 108	9. 99 997	20
50	7. 92 362	7. 92 363	9. 99 998	10	50	8. 05 292	8. 05 295	9. 99 997	10
29 0	7. 92 612	7. 92 613	9. 99 998	0 **31**	**39** 0	8. 05 478	8. 05 481	9. 99 997	0 **21**
10	7. 92 861	7. 92 862	9. 99 998	50	10	8. 05 663	8. 05 666	9. 99 997	50
20	7. 93 108	7. 93 110	9. 99 998	40	20	8. 05 848	8. 05 851	9. 99 997	40
30	7. 93 354	7. 93 356	9. 99 998	30	30	8. 06 031	8. 06 034	9. 99 997	30
40	7. 93 599	7. 93 601	9. 99 998	20	40	8. 06 214	8. 06 217	9. 99 997	20
50	7. 93 842	7. 93 844	9. 99 998	10	50	8. 06 396	8. 06 399	9. 99 997	10
30 0	7. 94 084	7. 94 086	9. 99 998	0 **30**	**40** 0	8. 06 578	8. 06 581	9. 99 997	0 **20**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′
40 0	8. 06 578	8. 06 581	9. 99 997	0 **20**
10	8. 06 758	8. 06 761	9. 99 997	50
20	8. 06 938	8. 06 941	9. 99 997	40
30	8. 07 117	8. 07 120	9. 99 997	30
40	8. 07 295	8. 07 299	9. 99 997	20
50	8. 07 473	8. 07 476	9. 99 997	10
41 0	8. 07 650	8. 07 653	9. 99 997	0 **19**
10	8. 07 826	8. 07 829	9. 99 997	50
20	8. 08 002	8. 08 005	9. 99 997	40
30	8. 08 176	8. 08 180	9. 99 997	30
40	8. 08 350	8. 08 354	9. 99 997	20
50	8. 08 524	8. 08 527	9. 99 997	10
42 0	8. 08 696	8. 08 700	9. 99 997	0 **18**
10	8. 08 868	8. 08 872	9. 99 997	50
20	8. 09 040	8. 09 043	9. 99 997	40
30	8. 09 210	8. 09 214	9. 99 997	30
40	8. 09 380	8. 09 384	9. 99 997	20
50	8. 09 550	8. 09 553	9. 99 997	10
43 0	8. 09 718	8. 09 722	9. 99 997	0 **17**
10	8. 09 886	8. 09 890	9. 99 997	50
20	8. 10 054	8. 10 057	9. 99 997	40
30	8. 10 220	8. 10 224	9. 99 997	30
40	8. 10 386	8. 10 390	9. 99 997	20
50	8. 10 552	8. 10 555	9. 99 996	10
44 0	8. 10 717	8. 10 720	9. 99 996	0 **16**
10	8. 10 881	8. 10 884	9. 99 996	50
20	8. 11 044	8. 11 048	9. 99 996	40
30	8. 11 207	8. 11 211	9. 99 996	30
40	8. 11 370	8. 11 373	9. 99 996	20
50	8. 11 531	8. 11 535	9. 99 996	10
45 0	8. 11 693	8. 11 696	9. 99 996	0 **15**
10	8. 11 853	8. 11 857	9. 99 996	50
20	8. 12 013	8. 12 017	9. 99 996	40
30	8. 12 172	8. 12 176	9. 99 996	30
40	8. 12 331	8. 12 335	9. 99 996	20
50	8. 12 489	8. 12 493	9. 99 996	10
46 0	8. 12 647	8. 12 651	9. 99 996	0 **14**
10	8. 12 804	8. 12 808	9. 99 996	50
20	8. 12 961	8. 12 965	9. 99 996	40
30	8. 13 117	8. 13 121	9. 99 996	30
40	8. 13 272	8. 13 276	9. 99 996	20
50	8. 13 427	8. 13 431	9. 99 996	10
47 0	8. 13 581	8. 13 585	9. 99 996	0 **13**
10	8. 13 735	8. 13 739	9. 99 996	50
20	8. 13 888	8. 13 892	9. 99 996	40
30	8. 14 041	8. 14 045	9. 99 996	30
40	8. 14 193	8. 14 197	9. 99 996	20
50	8. 14 344	8. 14 348	9. 99 996	10
48 0	8. 14 495	8. 14 500	9. 99 996	0 **12**
10	8. 14 646	8. 14 650	9. 99 996	50
20	8. 14 796	8. 14 800	9. 99 996	40
30	8. 14 945	8. 14 950	9. 99 996	30
40	8. 15 094	8. 15 099	9. 99 996	20
50	8. 15 243	8. 15 247	9. 99 996	10
49 0	8. 15 391	8. 15 395	9. 99 996	0 **11**
10	8. 15 538	8. 15 543	9. 99 996	50
20	8. 15 685	8. 15 690	9. 99 996	40
30	8. 15 832	8. 15 836	9. 99 996	30
40	8. 15 978	8. 15 982	9. 99 995	20
50	8. 16 123	8. 16 128	9. 99 995	10
50 0	8. 16 268	8. 16 273	9. 99 995	0 **10**
′ ″	log cos	log cot	log sin	″ ′

′ ″	log sin	log tan	log cos	″ ′
50 0	8. 16 268	8. 16 273	9. 99 995	0 **10**
10	8. 16 413	8. 16 417	9. 99 995	50
20	8. 16 557	8. 16 561	9. 99 995	40
30	8. 16 700	8. 16 705	9. 99 995	30
40	8. 16 843	8. 16 848	9. 99 995	20
50	8. 16 986	8. 16 991	9. 99 995	10
51 0	8. 17 128	8. 17 133	9. 99 995	0 **9**
10	8. 17 270	8. 17 275	9. 99 995	50
20	8. 17 411	8. 17 416	9. 99 995	40
30	8. 17 552	8. 17 557	9. 99 995	30
40	8. 17 692	8. 17 697	9. 99 995	20
50	8. 17 832	8. 17 837	9. 99 995	10
52 0	8. 17 971	8. 17 976	9. 99 995	0 **8**
10	8. 18 110	8. 18 115	9. 99 995	50
20	8. 18 249	8. 18 254	9. 99 995	40
30	8. 18 387	8. 18 392	9. 99 995	30
40	8. 18 524	8. 18 530	9. 99 995	20
50	8. 18 662	8. 18 667	9. 99 995	10
53 0	8. 18 798	8. 18 804	9. 99 995	0 **7**
10	8. 18 935	8. 18 940	9. 99 995	50
20	8. 19 071	8. 19 076	9. 99 995	40
30	8. 19 206	8. 19 212	9. 99 995	30
40	8. 19 341	8. 19 347	9. 99 995	20
50	8. 19 476	8. 19 481	9. 99 995	10
54 0	8. 19 610	8. 19 616	9. 99 995	0 **6**
10	8. 19 744	8. 19 749	9. 99 995	50
20	8. 19 877	8. 19 883	9. 99 995	40
30	8. 20 010	8. 20 016	9. 99 995	30
40	8. 20 143	8. 20 149	9. 99 995	20
50	8. 20 275	8. 20 281	9. 99 994	10
55 0	8. 20 407	8. 20 413	9. 99 994	0 **5**
10	8. 20 538	8. 20 544	9. 99 994	50
20	8. 20 669	8. 20 675	9. 99 994	40
30	8. 20 800	8. 20 806	9. 99 994	30
40	8. 20 930	8. 20 936	9. 99 994	20
50	8. 21 060	8. 21 066	9. 99 994	10
56 0	8. 21 189	8. 21 195	9. 99 994	0 **4**
10	8. 21 319	8. 21 324	9. 99 994	50
20	8. 21 447	8. 21 453	9. 99 994	40
30	8. 21 576	8. 21 581	9. 99 994	30
40	8. 21 703	8. 21 709	9. 99 994	20
50	8. 21 831	8. 21 837	9. 99 994	10
57 0	8. 21 958	8. 21 964	9. 99 994	0 **3**
10	8. 22 085	8. 22 091	9. 99 994	50
20	8. 22 211	8. 22 217	9. 99 994	40
30	8. 22 337	8. 22 343	9. 99 994	30
40	8. 22 463	8. 22 469	9. 99 994	20
50	8. 22 588	8. 22 595	9. 99 994	10
58 0	8. 22 713	8. 22 720	9. 99 994	0 **2**
10	8. 22 838	8. 22 844	9. 99 994	50
20	8. 22 962	8. 22 968	9. 99 994	40
30	8. 23 086	8. 23 092	9. 99 994	30
40	8. 23 210	8. 23 216	9. 99 994	20
50	8. 23 333	8. 23 339	9. 99 994	10
59 0	8. 23 456	8. 23 462	9. 99 994	0 **1**
10	8. 23 578	8. 23 585	9. 99 994	50
20	8. 23 700	8. 23 707	9. 99 994	40
30	8. 23 822	8. 23 829	9. 99 993	30
40	8. 23 944	8. 23 950	9. 99 993	20
50	8. 24 065	8. 24 071	9. 99 993	10
60 0	8. 24 186	8. 24 192	9. 99 993	0 **0**
′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
0 0	8. 24 186	8. 24 192	9. 99 993	0 **60**	**10** 0	8. 30 879	8. 30 888	9. 99 991	0 **50**
10	8. 24 306	8. 24 313	9. 99 993	50	10	8. 30 983	8. 30 992	9. 99 991	50
20	8. 24 426	8. 24 433	9. 99 993	40	20	8. 31 086	8. 31 095	9. 99 991	40
30	8. 24 546	8. 24 553	9. 99 993	30	30	8. 31 188	8. 31 198	9. 99 991	30
40	8. 24 665	8. 24 672	9. 99 993	20	40	8. 31 291	8. 31 300	9. 99 991	20
50	8. 24 785	8. 24 791	9. 99 993	10	50	8. 31 393	8. 31 403	9. 99 991	10
1 0	8. 24 903	8. 24 910	9. 99 993	0 **59**	**11** 0	8. 31 495	8. 31 505	9. 99 991	0 **49**
10	8. 25 022	8. 25 029	9. 99 993	50	10	8. 31 597	8. 31 606	9. 99 991	50
20	8. 25 140	8. 25 147	9. 99 993	40	20	8. 31 699	8. 31 708	9. 99 991	40
30	8. 25 258	8. 25 265	9. 99 993	30	30	8. 31 800	8. 31 809	9. 99 991	30
40	8. 25 375	8. 25 382	9. 99 993	20	40	8. 31 901	8. 31 911	9. 99 991	20
50	8. 25 493	8. 25 500	9. 99 993	10	50	8. 32 002	8. 32 012	9. 99 991	10
2 0	8. 25 609	8. 25 616	9. 99 993	0 **58**	**12** 0	8. 32 103	8. 32 112	9. 99 990	0 **48**
10	8. 25 726	8. 25 733	9. 99 993	50	10	8. 32 203	8. 32 213	9. 99 990	50
20	8. 25 842	8. 25 849	9. 99 993	40	20	8. 32 303	8. 32 313	9. 99 990	40
30	8. 25 958	8. 25 965	9. 99 993	30	30	8. 32 403	8. 32 413	9. 99 990	30
40	8. 26 074	8. 26 081	9. 99 993	20	40	8. 32 503	8. 32 513	9. 99 990	20
50	8. 26 189	8. 26 196	9. 99 993	10	50	8. 32 602	8. 32 612	9. 99 990	10
3 0	8. 26 304	8. 26 312	9. 99 993	0 **57**	**13** 0	8. 32 702	8. 32 711	9. 99 990	0 **47**
10	8. 26 419	8. 26 426	9. 99 993	50	10	8. 32 801	8. 32 811	9. 99 990	50
20	8. 26 533	8. 26 541	9. 99 993	40	20	8. 32 899	8. 32 909	9. 99 990	40
30	8. 26 648	8. 26 655	9. 99 993	30	30	8. 32 998	8. 33 008	9. 99 990	30
40	8. 26 761	8. 26 769	9. 99 993	20	40	8. 33 096	8. 33 106	9. 99 990	20
50	8. 26 875	8. 26 882	9. 99 993	10	50	8. 33 195	8. 33 205	9. 99 990	10
4 0	8. 26 988	8. 26 996	9. 99 992	0 **56**	**14** 0	8. 33 292	8. 33 302	9. 99 990	0 **46**
10	8. 27 101	8. 27 109	9. 99 992	50	10	8. 33 390	8. 33 400	9. 99 990	50
20	8. 27 214	8. 27 221	9. 99 992	40	20	8. 33 488	8. 33 498	9. 99 990	40
30	8. 27 326	8. 27 334	9. 99 992	30	30	8. 33 585	8. 33 595	9. 99 990	30
40	8. 27 438	8. 27 446	9. 99 992	20	40	8. 33 682	8. 33 692	9. 99 990	20
50	8. 27 550	8. 27 558	9. 99 992	10	50	8. 33 779	8. 33 789	9. 99 990	10
5 0	8. 27 661	8. 27 669	9. 99 992	0 **55**	**15** 0	8. 33 875	8. 33 886	9. 99 990	0 **45**
10	8. 27 773	8. 27 780	9. 99 992	50	10	8. 33 972	8. 33 982	9. 99 990	50
20	8. 27 883	8. 27 891	9. 99 992	40	20	8. 34 068	8. 34 078	9. 99 990	40
30	8. 27 994	8. 28 002	9. 99 992	30	30	8. 34 164	8. 34 174	9. 99 990	30
40	8. 28 104	8. 28 112	9. 99 992	20	40	8. 34 260	8. 34 270	9. 99 989	20
50	8. 28 215	8. 28 223	9. 99 992	10	50	8. 34 355	8. 34 366	9. 99 989	10
6 0	8. 28 324	8. 28 332	9. 99 992	0 **54**	**16** 0	8. 34 450	8. 34 461	9. 99 989	0 **44**
10	8. 28 434	8. 28 442	9. 99 992	50	10	8. 34 546	8. 34 556	9. 99 989	50
20	8. 28 543	8. 28 551	9. 99 992	40	20	8. 34 640	8. 34 651	9. 99 989	40
30	8. 28 652	8. 28 660	9. 99 992	30	30	8. 34 735	8. 34 746	9. 99 989	30
40	8. 28 761	8. 28 769	9. 99 992	20	40	8. 34 830	8. 34 840	9. 99 989	20
50	8. 28 869	8. 28 877	9. 99 992	10	50	8. 34 924	8. 34 935	9. 99 989	10
7 0	8. 28 977	8. 28 986	9. 99 992	0 **53**	**17** 0	8. 35 018	8. 35 029	9. 99 989	0 **43**
10	8. 29 085	8. 29 094	9. 99 992	50	10	8. 35 112	8. 35 123	9. 99 989	50
20	8. 29 193	8. 29 201	9. 99 992	40	20	8. 35 206	8. 35 217	9. 99 989	40
30	8. 29 300	8. 29 309	9. 99 992	30	30	8. 35 299	8. 35 310	9. 99 989	30
40	8. 29 407	8. 29 416	9. 99 992	20	40	8. 35 392	8. 35 403	9. 99 989	20
50	8. 29 514	8. 29 523	9. 99 992	10	50	8. 35 485	8. 35 497	9. 99 989	10
8 0	8. 29 621	8. 29 629	9. 99 992	0 **52**	**18** 0	8. 35 578	8. 35 590	9. 99 989	0 **42**
10	8. 29 727	8. 29 736	9. 99 991	50	10	8. 35 671	8. 35 682	9. 99 989	50
20	8. 29 833	8. 29 842	9. 99 991	40	20	8. 35 764	8. 35 775	9. 99 989	40
30	8. 29 939	8. 29 947	9. 99 991	30	30	8. 35 856	8. 35 867	9. 99 989	30
40	8. 30 044	8. 30 053	9. 99 991	20	40	8. 35 948	8. 35 959	9. 99 989	20
50	8. 30 150	8. 30 158	9. 99 991	10	50	8. 36 040	8. 36 051	9. 99 989	10
9 0	8. 30 255	8. 30 263	9. 99 991	0 **51**	**19** 0	8. 36 131	8. 36 143	9. 99 989	0 **41**
10	8. 30 359	8. 30 368	9. 99 991	50	10	8. 36 223	8. 36 235	9. 99 988	50
20	8. 30 464	8. 30 473	9. 99 991	40	20	8. 36 314	8. 36 326	9. 99 988	40
30	8. 30 568	8. 30 577	9. 99 991	30	30	8. 36 405	8. 36 417	9. 99 988	30
40	8. 30 672	8. 30 681	9. 99 991	20	40	8. 36 496	8. 36 508	9. 99 988	20
50	8. 30 776	8. 30 785	9. 99 991	10	50	8. 36 587	8. 36 599	9. 99 988	10
10 0	8. 30 879	8. 30 888	9. 99 991	0 **50**	**20** 0	8. 36 678	8. 36 689	9. 99 988	0 **40**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
20 0	8. 36 678	8. 36 689	9. 99 988	0 **40**	**30** 0	8. 41 792	8. 41 807	9. 99 985	0 **30**
10	8. 36 768	8. 36 780	9. 99 988	50	10	8. 41 872	8. 41 887	9. 99 985	50
20	8. 36 858	8. 36 870	9. 99 988	40	20	8. 41 952	8. 41 967	9. 99 985	40
30	8. 36 948	8. 36 960	9. 99 988	30	30	8. 42 032	8. 42 048	9. 99 985	30
40	8. 37 038	8. 37 050	9. 99 988	20	40	8. 42 112	8. 42 127	9. 99 985	20
50	8. 37 128	8. 37 140	9. 99 988	10	50	8. 42 192	8. 42 207	9. 99 985	10
21 0	8. 37 217	8. 37 229	9. 99 988	0 **39**	**31** 0	8. 42 272	8. 42 287	9. 99 985	0 **29**
10	8. 37 306	8. 37 318	9. 99 988	50	10	8. 42 351	8. 42 366	9. 99 985	50
20	8. 37 395	8. 37 408	9. 99 988	40	20	8. 42 430	8. 42 446	9. 99 985	40
30	8. 37 484	8. 37 497	9. 99 988	30	30	8. 42 510	8. 42 525	9. 99 985	30
40	8. 37 573	8. 37 585	9. 99 988	20	40	8. 42 589	8. 42 604	9. 99 985	20
50	8. 37 662	8. 37 674	9. 99 988	10	50	8. 42 667	8. 42 683	9. 99 985	10
22 0	8. 37 750	8. 37 762	9. 99 988	0 **38**	**32** 0	8. 42 746	8. 42 762	9. 99 984	0 **28**
10	8. 37 838	8. 37 850	9. 99 988	50	10	8. 42 825	8. 42 840	9. 99 984	50
20	8. 37 926	8. 37 938	9. 99 988	40	20	8. 42 903	8. 42 919	9. 99 984	40
30	8. 38 014	8. 38 026	9. 99 987	30	30	8. 42 982	8. 42 997	9. 99 984	30
40	8. 38 101	8. 38 114	9. 99 987	20	40	8. 43 060	8. 43 075	9. 99 984	20
50	8. 38 189	8. 38 202	9. 99 987	10	50	8. 43 138	8. 43 154	9. 99 984	10
23 0	8. 38 276	8. 38 289	9. 99 987	0 **37**	**33** 0	8. 43 216	8. 43 232	9. 99 984	0 **27**
10	8. 38 363	8. 38 376	9. 99 987	50	10	8. 43 293	8. 43 309	9. 99 984	50
20	8. 38 450	8. 38 463	9. 99 987	40	20	8. 43 371	8. 43 387	9. 99 984	40
30	8. 38 537	8. 38 550	9. 99 987	30	30	8. 43 448	8. 43 464	9. 99 984	30
40	8. 38 624	8. 38 636	9. 99 987	20	40	8. 43 526	8. 43 542	9. 99 984	20
50	8. 38 710	8. 38 723	9. 99 987	10	50	8. 43 603	8. 43 619	9. 99 984	10
24 0	8. 38 796	8. 38 809	9. 99 987	0 **36**	**34** 0	8. 43 680	8. 43 696	9. 99 984	0 **26**
10	8. 38 882	8. 38 895	9. 99 987	50	10	8. 43 757	8. 43 773	9. 99 984	50
20	8. 38 968	8. 38 981	9. 99 987	40	20	8. 43 834	8. 43 850	9. 99 984	40
30	8. 39 054	8. 39 067	9. 99 987	30	30	8. 43 910	8. 43 927	9. 99 984	30
40	8. 39 139	8. 39 153	9. 99 987	20	40	8. 43 987	8. 44 003	9. 99 984	20
50	8. 39 225	8. 39 238	9. 99 987	10	50	8. 44 063	8. 44 080	9. 99 983	10
25 0	8. 39 310	8. 39 323	9. 99 987	0 **35**	**35** 0	8. 44 139	8. 44 156	9. 99 983	0 **25**
10	8. 39 395	8. 39 408	9. 99 987	50	10	8. 44 216	8. 44 232	9. 99 983	50
20	8. 39 480	8. 39 493	9. 99 987	40	20	8. 44 292	8. 44 308	9. 99 983	40
30	8. 39 565	8. 39 578	9. 99 987	30	30	8. 44 367	8. 44 384	9. 99 983	30
40	8. 39 649	8. 39 663	9. 99 987	20	40	8. 44 443	8. 44 460	9. 99 983	20
50	8. 39 734	8. 39 747	9. 99 986	10	50	8. 44 519	8. 44 536	9. 99 983	10
26 0	8. 39 818	8. 39 832	9. 99 986	0 **34**	**36** 0	8. 44 594	8. 44 611	9. 99 983	0 **24**
10	8. 39 902	8. 39 916	9. 99 986	50	10	8. 44 669	8. 44 686	9. 99 983	50
20	8. 39 986	8. 40 000	9. 99 986	40	20	8. 44 745	8. 44 762	9. 99 983	40
30	8. 40 070	8. 40 083	9. 99 986	30	30	8. 44 820	8. 44 837	9. 99 983	30
40	8. 40 153	8. 40 167	9. 99 986	20	40	8. 44 895	8. 44 912	9. 99 983	20
50	8. 40 237	8. 40 251	9. 99 986	10	50	8. 44 969	8. 44 987	9. 99 983	10
27 0	8. 40 320	8. 40 334	9. 99 986	0 **33**	**37** 0	8. 45 044	8. 45 061	9. 99 983	0 **23**
10	8. 40 403	8. 40 417	9. 99 986	50	10	8. 45 119	8. 45 136	9. 99 983	50
20	8. 40 486	8. 40 500	9. 99 986	40	20	8. 45 193	8. 45 210	9. 99 983	40
30	8. 40 569	8. 40 583	9. 99 986	30	30	8. 45 267	8. 45 285	9. 99 983	30
40	8. 40 651	8. 40 665	9. 99 986	20	40	8. 45 341	8. 45 359	9. 99 982	20
50	8. 40 734	8. 40 748	9. 99 986	10	50	8. 45 415	8. 45 433	9. 99 982	10
28 0	8. 40 816	8. 40 830	9. 99 986	0 **32**	**38** 0	8. 45 489	8. 45 507	9. 99 982	0 **22**
10	8. 40 898	8. 40 913	9. 99 986	50	10	8. 45 563	8. 45 581	9. 99 982	50
20	8. 40 980	8. 40 995	9. 99 986	40	20	8. 45 637	8. 45 655	9. 99 982	40
30	8. 41 062	8. 41 077	9. 99 986	30	30	8. 45 710	8. 45 728	9. 99 982	30
40	8. 41 144	8. 41 158	9. 99 986	20	40	8. 45 784	8. 45 802	9. 99 982	20
50	8. 41 225	8. 41 240	9. 99 986	10	50	8. 45 857	8. 45 875	9. 99 982	10
29 0	8. 41 307	8. 41 321	9. 99 985	0 **31**	**39** 0	8. 45 930	8. 45 948	9. 99 982	0 **21**
10	8. 41 388	8. 41 403	9. 99 985	50	10	8. 46 003	8. 46 021	9. 99 982	50
20	8. 41 469	8. 41 484	9. 99 985	40	20	8. 46 076	8. 46 094	9. 99 982	40
30	8. 41 550	8. 41 565	9. 99 985	30	30	8. 46 149	8. 46 167	9. 99 982	30
40	8. 41 631	8. 41 646	9. 99 985	20	40	8. 46 222	8. 46 240	9. 99 982	20
50	8. 41 711	8. 41 726	9. 99 985	10	50	8. 46 294	8. 46 312	9. 99 982	10
30 0	8. 41 792	8. 41 807	9. 99 985	0 **30**	**40** 0	8. 46 366	8. 46 385	9. 99 982	0 **20**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′ ″	log sin	log tan	log cos	″ ′	′ ″	log sin	log tan	log cos	″ ′
40 0	8. 46 366	8. 46 385	9. 99 982	0 **20**	**50** 0	8. 50 504	8. 50 527	9. 99 978	0 **10**
10	8. 46 439	8. 46 457	9. 99 982	50	10	8. 50 570	8. 50 593	9. 99 978	50
20	8. 46 511	8. 46 529	9. 99 982	40	20	8. 50 636	8. 50 658	9. 99 978	40
30	8. 46 583	8. 46 602	9. 99 981	30	30	8. 50 701	8. 50 724	9. 99 978	30
40	8. 46 655	8. 46 674	9. 99 981	20	40	8. 50 767	8. 50 789	9. 99 977	20
50	8. 46 727	8. 46 745	9. 99 981	10	50	8. 50 832	8. 50 855	9. 99 977	10
41 0	8. 46 799	8. 46 817	9. 99 981	0 **19**	**51** 0	8. 50 897	8. 50 920	9. 99 977	0 **9**
10	8. 46 870	8. 46 889	9. 99 981	50	10	8. 50 963	8. 50 985	9. 99 977	50
20	8. 46 942	8. 46 960	9. 99 981	40	20	8. 51 028	8. 51 050	9. 99 977	40
30	8. 47 013	8. 47 032	9. 99 981	30	30	8. 51 092	8. 51 115	9. 99 977	30
40	8. 47 084	8. 47 103	9. 99 981	20	40	8. 51 157	8. 51 180	9. 99 977	20
50	8. 47 155	8. 47 174	9. 99 981	10	50	8. 51 222	8. 51 245	9. 99 977	10
42 0	8. 47 226	8. 47 245	9. 99 981	0 **18**	**52** 0	8. 51 287	8. 51 310	9. 99 977	0 **8**
10	8. 47 297	8. 47 316	9. 99 981	50	10	8. 51 351	8. 51 374	9. 99 977	50
20	8. 47 368	8. 47 387	9. 99 981	40	20	8. 51 416	8. 51 439	9. 99 977	40
30	8. 47 439	8. 47 458	9. 99 981	30	30	8. 51 480	8. 51 503	9. 99 977	30
40	8. 47 509	8. 47 528	9. 99 981	20	40	8. 51 544	8. 51 568	9. 99 977	20
50	8. 47 580	8. 47 599	9. 99 981	10	50	8. 51 609	8. 51 632	9. 99 977	10
43 0	8. 47 650	8. 47 669	9. 99 981	0 **17**	**53** 0	8. 51 673	8. 51 696	9. 99 977	0 **7**
10	8. 47 720	8. 47 740	9. 99 980	50	10	8. 51 737	8. 51 760	9. 99 976	50
20	8. 47 790	8. 47 810	9. 99 980	40	20	8. 51 801	8. 51 824	9. 99 976	40
30	8. 47 860	8. 47 880	9. 99 980	30	30	8. 51 864	8. 51 888	9. 99 976	30
40	8. 47 930	8. 47 950	9. 99 980	20	40	8. 51 928	8. 51 952	9. 99 976	20
50	8. 48 000	8. 48 020	9. 99 980	10	50	8. 51 992	8. 52 015	9. 99 976	10
44 0	8. 48 069	8. 48 090	9. 99 980	0 **16**	**54** 0	8. 52 055	8. 52 079	9. 99 976	0 **6**
10	8. 48 139	8. 48 159	9. 99 980	50	10	8. 52 119	8. 52 143	9. 99 976	50
20	8. 48 208	8. 48 228	9. 99 980	40	20	8. 52 182	8. 52 206	9. 99 976	40
30	8. 48 278	8. 48 298	9. 99 980	30	30	8. 52 245	8. 52 269	9. 99 976	30
40	8. 48 347	8. 48 367	9. 99 980	20	40	8. 52 308	8. 52 332	9. 99 976	20
50	8. 48 416	8. 48 436	9. 99 980	10	50	8. 52 371	8. 52 396	9. 99 976	10
45 0	8. 48 485	8. 48 505	9. 99 980	0 **15**	**55** 0	8. 52 434	8. 52 459	9. 99 976	0 **5**
10	8. 48 554	8. 48 574	9. 99 980	50	10	8. 52 497	8. 52 522	9. 99 976	50
20	8. 48 622	8. 48 643	9. 99 980	40	20	8. 52 560	8. 52 584	9. 99 976	40
30	8. 48 691	8. 48 711	9. 99 980	30	30	8. 52 623	8. 52 647	9. 99 975	30
40	8. 48 760	8. 48 780	9. 99 979	20	40	8. 52 685	8. 52 710	9. 99 975	20
50	8. 48 828	8. 48 849	9. 99 979	10	50	8. 52 748	8. 52 772	9. 99 975	10
46 0	8. 48 896	8. 48 917	9. 99 979	0 **14**	**56** 0	8. 52 810	8. 52 835	9. 99 975	0 **4**
10	8. 48 965	8. 48 985	9. 99 979	50	10	8. 52 872	8. 52 897	9. 99 975	50
20	8. 49 033	8. 49 053	9. 99 979	40	20	8. 52 935	8. 52 960	9. 99 975	40
30	8. 49 101	8. 49 121	9. 99 979	30	30	8. 52 997	8. 53 022	9. 99 975	30
40	8. 49 169	8. 49 189	9. 99 979	20	40	8. 53 059	8. 53 084	9. 99 975	20
50	8. 49 236	8. 49 257	9. 99 979	10	50	8. 53 121	8. 53 146	9. 99 975	10
47 0	8. 49 304	8. 49 325	9. 99 979	0 **13**	**57** 0	8. 53 183	8. 53 208	9. 99 975	0 **3**
10	8. 49 372	8. 49 393	9. 99 979	50	10	8. 53 245	8. 53 270	9. 99 975	50
20	8. 49 439	8. 49 460	9. 99 979	40	20	8. 53 306	8. 53 332	9. 99 975	40
30	8. 49 506	8. 49 528	9. 99 979	30	30	8. 53 368	8. 53 393	9. 99 975	30
40	8. 49 574	8. 49 595	9. 99 979	20	40	8. 53 429	8. 53 455	9. 99 975	20
50	8. 49 641	8. 49 662	9. 99 979	10	50	8. 53 491	8. 53 516	9. 99 974	10
48 0	8. 49 708	8. 49 729	9. 99 979	0 **12**	**58** 0	8. 53 552	8. 53 578	9. 99 974	0 **2**
10	8. 49 775	8. 49 796	9. 99 979	50	10	8. 53 614	8. 53 639	9. 99 974	50
20	8. 49 842	8. 49 863	9. 99 978	40	20	8. 53 675	8. 53 700	9. 99 974	40
30	8. 49 908	8. 49 930	9. 99 978	30	30	8. 53 736	8. 53 762	9. 99 974	30
40	8. 49 975	8. 49 997	9. 99 978	20	40	8. 53 797	8. 53 823	9. 99 974	20
50	8. 50 042	8. 50 063	9. 99 978	10	50	8. 53 858	8. 53 884	9. 99 974	10
49 0	8. 50 108	8. 50 130	9. 99 978	0 **11**	**59** 0	8. 53 919	8. 53 945	9. 99 974	0 **1**
10	8. 50 174	8. 50 196	9. 99 978	50	10	8. 53 979	8. 54 005	9. 99 974	50
20	8. 50 241	8. 50 263	9. 99 978	40	20	8. 54 040	8. 54 066	9. 99 974	40
30	8. 50 307	8. 50 329	9. 99 978	30	30	8. 54 101	8. 54 127	9. 99 974	30
40	8. 50 373	8. 50 395	9. 99 978	20	40	8. 54 161	8. 54 187	9. 99 974	20
50	8. 50 439	8. 50 461	9. 99 978	10	50	8. 54 222	8. 54 248	9. 99 974	10
50 0	8. 50 504	8. 50 527	9. 99 978	0 **10**	**60** 0	8. 54 282	8. 54 308	9. 99 974	0 **0**
′ ″	log cos	log cot	log sin	″ ′	′ ″	log cos	log cot	log sin	″ ′

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′	log sin	log tan	log cot	log cos	′
	8	**8**	**11**	**9**	
0	24 186	24 192	75 808	99 993	**60**
1	24 903	24 910	75 090	99 993	59
2	25 609	25 616	74 384	99 993	58
3	26 304	26 312	73 688	99 993	57
4	26 988	26 996	73 004	99 992	56
5	27 661	27 669	72 331	99 992	**55**
6	28 324	28 332	71 668	99 992	54
7	28 977	28 986	71 014	99 992	53
8	29 621	29 629	70 371	99 992	52
9	30 255	30 263	69 737	99 991	51
10	30 879	30 888	69 112	99 991	**50**
11	31 495	31 505	68 495	99 991	49
12	32 103	32 112	67 888	99 990	48
13	32 702	32 711	67 289	99 990	47
14	33 292	33 302	66 698	99 990	46
15	33 875	33 886	66 114	99 990	**45**
16	34 450	34 461	65 539	99 989	44
17	35 018	35 029	64 971	99 989	43
18	35 578	35 590	64 410	99 989	42
19	36 131	36 143	63 857	99 989	41
20	36 678	36 689	63 311	99 988	**40**
21	37 217	37 229	62 771	99 988	39
22	37 750	37 762	62 238	99 988	38
23	38 276	38 289	61 711	99 987	37
24	38 796	38 809	61 191	99 987	36
25	39 310	39 323	60 677	99 987	**35**
26	39 818	39 832	60 168	99 986	34
27	40 320	40 334	59 666	99 986	33
28	40 816	40 830	59 170	99 986	32
29	41 307	41 321	58 679	99 985	31
30	41 792	41 807	58 193	99 985	**30**
31	42 272	42 287	57 713	99 985	29
32	42 746	42 762	57 238	99 984	28
33	43 216	43 232	56 768	99 984	27
34	43 680	43 696	56 304	99 984	26
35	44 139	44 156	55 844	99 983	**25**
36	44 594	44 611	55 389	99 983	24
37	45 044	45 061	54 939	99 983	23
38	45 489	45 507	54 493	99 982	22
39	45 930	45 948	54 052	99 982	21
40	46 366	46 385	53 615	99 982	**20**
41	46 799	46 817	53 183	99 981	19
42	47 226	47 245	52 755	99 981	18
43	47 650	47 669	52 331	99 981	17
44	48 069	48 089	51 911	99 980	16
45	48 485	48 505	51 495	99 980	**15**
46	48 896	48 917	51 083	99 979	14
47	49 304	49 325	50 675	99 979	13
48	49 708	49 729	50 271	99 979	12
49	50 108	50 130	49 870	99 978	11
50	50 504	50 527	49 473	99 978	**10**
51	50 897	50 920	49 080	99 977	9
52	51 287	51 310	48 690	99 977	8
53	51 673	51 696	48 304	99 977	7
54	52 055	52 079	47 921	99 976	6
55	52 434	52 459	47 541	99 976	**5**
56	52 810	52 835	47 165	99 975	4
57	53 183	53 208	46 792	99 975	3
58	53 552	53 578	46 422	99 974	2
59	53 919	53 945	46 055	99 974	1
60	54 282	54 308	45 692	99 974	**0**
	8	**8**	**11**	**9**	
′	log cos	log cot	log tan	log sin	′

88°

2°

′	log sin	log tan	log cot	log cos	′
	8	**8**	**11**	**9**	
0	54 282	54 308	45 692	99 974	**60**
1	54 642	54 669	45 331	99 973	59
2	54 999	55 027	44 973	99 973	58
3	55 354	55 382	44 618	99 972	57
4	55 705	55 734	44 266	99 972	56
5	56 054	56 083	43 917	99 971	**55**
6	56 400	56 429	43 571	99 971	54
7	56 743	56 773	43 227	99 970	53
8	57 084	57 114	42 886	99 970	52
9	57 421	57 452	42 548	99 969	51
10	57 757	57 788	42 212	99 969	**50**
11	58 089	58 121	41 879	99 968	49
12	58 419	58 451	41 549	99 968	48
13	58 747	58 779	41 221	99 967	47
14	59 072	59 105	40 895	99 967	46
15	59 395	59 428	40 572	99 967	**45**
16	59 715	59 749	40 251	99 966	44
17	60 033	60 068	39 932	99 966	43
18	60 349	60 384	39 616	99 965	42
19	60 662	60 698	39 302	99 964	41
20	60 973	61 009	38 991	99 964	**40**
21	61 282	61 319	38 681	99 963	39
22	61 589	61 626	38 374	99 963	38
23	61 894	61 931	38 069	99 962	37
24	62 196	62 234	37 766	99 962	36
25	62 497	62 535	37 465	99 961	**35**
26	62 795	62 834	37 166	99 961	34
27	63 091	63 131	36 869	99 960	33
28	63 385	63 426	36 574	99 960	32
29	63 678	63 718	36 282	99 959	31
30	63 968	64 009	35 991	99 959	**30**
31	64 256	64 298	35 702	99 958	29
32	64 543	64 585	35 415	99 958	28
33	64 827	64 870	35 130	99 957	27
34	65 110	65 154	34 846	99 956	26
35	65 391	65 435	34 565	99 956	**25**
36	65 670	65 715	34 285	99 955	24
37	65 947	65 993	34 007	99 955	23
38	66 223	66 269	33 731	99 954	22
39	66 497	66 543	33 457	99 954	21
40	66 769	66 816	33 184	99 953	**20**
41	67 039	67 087	32 913	99 952	19
42	67 308	67 356	32 644	99 952	18
43	67 575	67 624	32 376	99 951	17
44	67 841	67 890	32 110	99 951	16
45	68 104	68 154	31 846	99 950	**15**
46	68 367	68 417	31 583	99 949	14
47	68 627	68 678	31 322	99 949	13
48	68 886	68 938	31 062	99 948	12
49	69 144	69 196	30 804	99 948	11
50	69 400	69 453	30 547	99 947	**10**
51	69 654	69 708	30 292	99 946	9
52	69 907	69 962	30 038	99 946	8
53	70 159	70 214	29 786	99 945	7
54	70 409	70 465	29 535	99 944	6
55	70 658	70 714	29 286	99 944	**5**
56	70 905	70 962	29 038	99 943	4
57	71 151	71 208	28 792	99 942	3
58	71 395	71 453	28 547	99 942	2
59	71 638	71 697	28 303	99 941	1
60	71 880	71 940	28 060	99 940	**0**
	8	**8**	**11**	**9**	
′	log cos	log cot	log tan	log sin	′

87°

3°

′	log sin 8	log tan 8	log cot 11	log cos 9	′
0	71 880	71 940	28 060	99 940	**60**
1	72 120	72 181	27 819	99 940	59
2	72 359	72 420	27 580	99 939	58
3	72 597	72 659	27 341	99 938	57
4	72 834	72 896	27 104	99 938	56
5	73 069	73 132	26 868	99 937	**55**
6	73 303	73 366	26 634	99 936	54
7	73 535	73 600	26 400	99 936	53
8	73 767	73 832	26 168	99 935	52
9	73 997	74 063	25 937	99 934	51
10	74 226	74 292	25 708	99 934	**50**
11	74 454	74 521	25 479	99 933	49
12	74 680	74 748	25 252	99 932	48
13	74 906	74 974	25 026	99 932	47
14	75 130	75 199	24 801	99 931	46
15	75 353	75 423	24 577	99 930	**45**
16	75 575	75 645	24 355	99 929	44
17	75 795	75 867	24 133	99 929	43
18	76 015	76 087	23 913	99 928	42
19	76 234	76 306	23 694	99 927	41
20	76 451	76 525	23 475	99 926	**40**
21	76 667	76 742	23 258	99 926	39
22	76 883	76 958	23 042	99 925	38
23	77 097	77 173	22 827	99 924	37
24	77 310	77 387	22 613	99 923	36
25	77 522	77 600	22 400	99 923	**35**
26	77 733	77 811	22 189	99 922	34
27	77 943	78 022	21 978	99 921	33
28	78 152	78 232	21 768	99 920	32
29	78 360	78 441	21 559	99 920	31
30	78 568	78 649	21 351	99 919	**30**
31	78 774	78 855	21 145	99 918	29
32	78 979	79 061	20 939	99 917	28
33	79 183	79 266	20 734	99 917	27
34	79 386	79 470	20 530	99 916	26
35	79 588	79 673	20 327	99 915	**25**
36	79 789	79 875	20 125	99 914	24
37	79 990	80 076	19 924	99 913	23
38	80 189	80 277	19 723	99 913	22
39	80 388	80 476	19 524	99 912	21
40	80 585	80 674	19 326	99 911	**20**
41	80 782	80 872	19 128	99 910	19
42	80 978	81 068	18 932	99 909	18
43	81 173	81 264	18 736	99 909	17
44	81 367	81 459	18 541	99 908	16
45	81 560	81 653	18 347	99 907	**15**
46	81 752	81 846	18 154	99 906	14
47	81 944	82 038	17 962	99 905	13
48	82 134	82 230	17 770	99 904	12
49	82 324	82 420	17 580	99 904	11
50	82 513	82 610	17 390	99 903	**10**
51	82 701	82 799	17 201	99 902	9
52	82 888	82 987	17 013	99 901	8
53	83 075	83 175	16 825	99 900	7
54	83 261	83 361	16 639	99 899	6
55	83 446	83 547	16 453	99 898	**5**
56	83 630	83 732	16 268	99 898	4
57	83 813	83 916	16 084	99 897	3
58	83 996	84 100	15 900	99 896	2
59	84 177	84 282	15 718	99 895	1
60	84 358	84 464	15 536	99 894	**0**
′	8 log cos	8 log cot	11 log tan	9 log sin	′

86°

4°

′	log sin 8	log tan 8	log cot 11	log cos 9	′
0	84 358	84 464	15 536	99 894	**60**
1	84 539	84 646	15 354	99 893	59
2	84 718	84 826	15 174	99 892	58
3	84 897	85 006	14 994	99 891	57
4	85 075	85 185	14 815	99 891	56
5	85 252	85 363	14 637	99 890	**55**
6	85 429	85 540	14 460	99 889	54
7	85 605	85 717	14 283	99 888	53
8	85 780	85 893	14 107	99 887	52
9	85 955	86 069	13 931	99 886	51
10	86 128	86 243	13 757	99 885	**50**
11	86 301	86 417	13 583	99 884	49
12	86 474	86 591	13 409	99 883	48
13	86 645	86 763	13 237	99 882	47
14	86 816	86 935	13 065	99 881	46
15	86 987	87 106	12 894	99 880	**45**
16	87 156	87 277	12 723	99 879	44
17	87 325	87 447	12 553	99 879	43
18	87 494	87 616	12 384	99 878	42
19	87 661	87 785	12 215	99 877	41
20	87 829	87 953	12 047	99 876	**40**
21	87 995	88 120	11 880	99 875	39
22	88 161	88 287	11 713	99 874	38
23	88 326	88 453	11 547	99 873	37
24	88 490	88 618	11 382	99 872	36
25	88 654	88 783	11 217	99 871	**35**
26	88 817	88 948	11 052	99 870	34
27	88 980	89 111	10 889	99 869	33
28	89 142	89 274	10 726	99 868	32
29	89 304	89 437	10 563	99 867	31
30	89 464	89 598	10 402	99 866	**30**
31	89 625	89 760	10 240	99 865	29
32	89 784	89 920	10 080	99 864	28
33	89 943	90 080	09 920	99 863	27
34	90 102	90 240	09 760	99 862	26
35	90 260	90 399	09 601	99 861	**25**
36	90 417	90 557	09 443	99 860	24
37	90 574	90 715	09 285	99 859	23
38	90 730	90 872	09 128	99 858	22
39	90 885	91 029	08 971	99 857	21
40	91 040	91 185	08 815	99 856	**20**
41	91 195	91 340	08 660	99 855	19
42	91 349	91 495	08 505	99 854	18
43	91 502	91 650	08 350	99 853	17
44	91 655	91 803	08 197	99 852	16
45	91 807	91 957	08 043	99 851	**15**
46	91 959	92 110	07 890	99 850	14
47	92 110	92 262	07 738	99 848	13
48	92 261	92 414	07 586	99 847	12
49	92 411	92 565	07 435	99 846	11
50	92 561	92 716	07 284	99 845	**10**
51	92 710	92 866	07 134	99 844	9
52	92 859	93 016	06 984	99 843	8
53	93 007	93 165	06 835	99 842	7
54	93 154	93 313	06 687	99 841	6
55	93 301	93 462	06 538	99 840	**5**
56	93 448	93 609	06 391	99 839	4
57	93 594	93 756	06 244	99 838	3
58	93 740	93 903	06 097	99 837	2
59	93 885	94 049	05 951	99 836	1
60	94 030	94 195	05 805	99 834	**0**
′	8 log cos	8 log cot	11 log tan	9 log sin	′

85°

5°

′	log sin	log tan	log cot	log cos	′
	8	8	11	9	
0	94 030	94 195	05 805	99 834	**60**
1	94 174	94 340	05 660	99 833	59
2	94 317	94 485	05 515	99 832	58
3	94 461	94 630	05 370	99 831	57
4	94 603	94 773	05 227	99 830	56
5	94 746	94 917	05 083	99 829	**55**
6	94 887	95 060	04 940	99 828	54
7	95 029	95 202	04 798	99 827	53
8	95 170	95 344	04 656	99 825	52
9	95 310	95 486	04 514	99 824	51
10	95 450	95 627	04 373	99 823	**50**
11	95 589	95 767	04 233	99 822	49
12	95 728	95 908	04 092	99 821	48
13	95 867	96 047	03 953	99 820	47
14	96 005	96 187	03 813	99 819	46
15	96 143	96 325	03 675	99 817	**45**
16	96 280	96 464	03 536	99 816	44
17	96 417	96 602	03 398	99 815	43
18	96 553	96 739	03 261	99 814	42
19	96 689	96 877	03 123	99 813	41
20	96 825	97 013	02 987	99 812	**40**
21	96 960	97 150	02 850	99 810	39
22	97 095	97 285	02 715	99 809	38
23	97 229	97 421	02 579	99 808	37
24	97 363	97 556	02 444	99 807	36
25	97 496	97 691	02 309	99 806	**35**
26	97 629	97 825	02 175	99 804	34
27	97 762	97 959	02 041	99 803	33
28	97 894	98 092	01 908	99 802	32
29	98 026	98 225	01 775	99 801	31
30	98 157	98 358	01 642	99 800	**30**
31	98 288	98 490	01 510	99 798	29
32	98 419	98 622	01 378	99 797	28
33	98 549	98 753	01 247	99 796	27
34	98 679	98 884	01 116	99 795	26
35	98 808	99 015	00 985	99 793	**25**
36	98 937	99 145	00 855	99 792	24
37	99 066	99 275	00 725	99 791	23
38	99 194	99 405	00 595	99 790	22
39	99 322	99 534	00 466	99 788	21
40	99 450	99 662	00 338	99 787	**20**
41	99 577	99 791	00 209	99 786	19
42	99 704	99 919	00 081	99 785	18
43	99 830	00 046	99 954	99 783	17
44	99 956	00 174	99 826	99 782	16
45	00 082	00 301	99 699	99 781	**15**
46	00 207	00 427	99 573	99 780	14
47	00 332	00 553	99 447	99 778	13
48	00 456	00 679	99 321	99 777	12
49	00 581	00 805	99 195	99 776	11
50	00 704	00 930	99 070	99 775	**10**
51	00 828	01 055	98 945	99 773	9
52	00 951	01 179	98 821	99 772	8
53	01 074	01 303	98 697	99 771	7
54	01 196	01 427	98 573	99 769	6
55	01 318	01 550	98 450	99 768	**5**
56	01 440	01 673	98 327	99 767	4
57	01 561	01 796	98 204	99 765	3
58	01 682	01 918	98 082	99 764	2
59	01 803	02 040	97 960	99 763	1
60	01 923	02 162	97 838	99 761	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

84°

6°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	01 923	02 162	97 838	99 761	**60**
1	02 043	02 283	97 717	99 760	59
2	02 163	02 404	97 596	99 759	58
3	02 283	02 525	97 475	99 757	57
4	02 402	02 645	97 355	99 756	56
5	02 520	02 766	97 234	99 755	**55**
6	02 639	02 885	97 115	99 753	54
7	02 757	03 005	96 995	99 752	53
8	02 874	03 124	96 876	99 751	52
9	02 992	03 242	96 758	99 749	51
10	03 109	03 361	96 639	99 748	**50**
11	03 226	03 479	96 521	99 747	49
12	03 342	03 597	96 403	99 745	48
13	03 458	03 714	96 286	99 744	47
14	03 574	03 832	96 168	99 742	46
15	03 690	03 948	96 052	99 741	**45**
16	03 805	04 065	95 935	99 740	44
17	03 920	04 181	95 819	99 738	43
18	04 034	04 297	95 703	99 737	42
19	04 149	04 413	95 587	99 736	41
20	04 262	04 528	95 472	99 734	**40**
21	04 376	04 643	95 357	99 733	39
22	04 490	04 758	95 242	99 731	38
23	04 603	04 873	95 127	99 730	37
24	04 715	04 987	95 013	99 728	36
25	04 828	05 101	94 899	99 727	**35**
26	04 940	05 214	94 786	99 726	34
27	05 052	05 328	94 672	99 724	33
28	05 164	05 441	94 559	99 723	32
29	05 275	05 553	94 447	99 721	31
30	05 386	05 666	94 334	99 720	**30**
31	05 497	05 778	94 222	99 718	29
32	05 607	05 890	94 110	99 717	28
33	05 717	06 002	93 998	99 716	27
34	05 827	06 113	93 887	99 714	26
35	05 937	06 224	93 776	99 713	**25**
36	06 046	06 335	93 665	99 711	24
37	06 155	06 445	93 555	99 710	23
38	06 264	06 556	93 444	99 708	22
39	06 372	06 666	93 334	99 707	21
40	06 481	06 775	93 225	99 705	**20**
41	06 589	06 885	93 115	99 704	19
42	06 696	06 994	93 006	99 702	18
43	06 804	07 103	92 897	99 701	17
44	06 911	07 211	92 789	99 699	16
45	07 018	07 320	92 680	99 698	**15**
46	07 124	07 428	92 572	99 696	14
47	07 231	07 536	92 464	99 695	13
48	07 337	07 643	92 357	99 693	12
49	07 442	07 751	92 249	99 692	11
50	07 548	07 858	92 142	99 690	**10**
51	07 653	07 964	92 036	99 689	9
52	07 758	08 071	91 929	99 687	8
53	07 863	08 177	91 823	99 686	7
54	07 968	08 283	91 717	99 684	6
55	08 072	08 389	91 611	99 683	**5**
56	08 176	08 495	91 505	99 681	4
57	08 280	08 600	91 400	99 680	3
58	08 383	08 705	91 295	99 678	2
59	08 486	08 810	91 190	99 677	1
60	08 589	08 914	91 086	99 675	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

83°

7° 8°

′	log sin	log tan	log cot	log cos	′	′	log sin	log tan	log cot	log cos	′
	9	9	10	9			9	9	10	9	
0	08 589	08 914	91 086	99 675	**60**	**0**	14 356	14 780	85 220	99 575	**60**
1	08 692	09 019	90 981	99 674	59	1	14 445	14 872	85 128	99 574	59
2	08 795	09 123	90 877	99 672	58	2	14 535	14 963	85 037	99 572	58
3	08 897	09 227	90 773	99 670	57	3	14 624	15 054	84 946	99 570	57
4	08 999	09 330	90 670	99 669	56	4	14 714	15 145	84 855	99 568	56
5	09 101	09 434	90 566	99 667	**55**	**5**	14 803	15 236	84 764	99 566	**55**
6	09 202	09 537	90 463	99 666	54	6	14 891	15 327	84 673	99 565	54
7	09 304	09 640	90 360	99 664	53	7	14 980	15 417	84 583	99 563	53
8	09 405	09 742	90 258	99 663	52	8	15 069	15 508	84 492	99 561	52
9	09 506	09 845	90 155	99 661	51	9	15 157	15 598	84 402	99 559	51
10	09 606	09 947	90 053	99 659	**50**	**10**	15 245	15 688	84 312	99 557	**50**
11	09 707	10 049	89 951	99 658	49	11	15 333	15 777	84 223	99 556	49
12	09 807	10 150	89 850	99 656	48	12	15 421	15 867	84 133	99 554	48
13	09 907	10 252	89 748	99 655	47	13	15 508	15 956	84 044	99 552	47
14	10 006	10 353	89 647	99 653	46	14	15 596	16 046	83 954	99 550	46
15	10 106	10 454	89 546	99 651	**45**	**15**	15 683	16 135	83 865	99 548	**45**
16	10 205	10 555	89 445	99 650	44	16	15 770	16 224	83 776	99 546	44
17	10 304	10 656	89 344	99 648	43	17	15 857	16 312	83 688	99 545	43
18	10 402	10 756	89 244	99 647	42	18	15 944	16 401	83 599	99 543	42
19	10 501	10 856	89 144	99 645	41	19	16 030	16 489	83 511	99 541	41
20	10 599	10 956	89 044	99 643	**40**	**20**	16 116	16 577	83 423	99 539	**40**
21	10 697	11 056	88 944	99 642	39	21	16 203	16 665	83 335	99 537	39
22	10 795	11 155	88 845	99 640	38	22	16 289	16 753	83 247	99 535	38
23	10 893	11 254	88 746	99 638	37	23	16 374	16 841	83 159	99 533	37
24	10 990	11 353	88 647	99 637	36	24	16 460	16 928	83 072	99 532	36
25	11 087	11 452	88 548	99 635	**35**	**25**	16 545	17 016	82 984	99 530	**35**
26	11 184	11 551	88 449	99 633	34	26	16 631	17 103	82 897	99 528	34
27	11 281	11 649	88 351	99 632	33	27	16 716	17 190	82 810	99 526	33
28	11 377	11 747	88 253	99 630	32	28	16 801	17 277	82 723	99 524	32
29	11 474	11 845	88 155	99 629	31	29	16 886	17 363	82 637	99 522	31
30	11 570	11 943	88 057	99 627	**30**	**30**	16 970	17 450	82 550	99 520	**30**
31	11 666	12 040	87 960	99 625	29	31	17 055	17 536	82 464	99 518	29
32	11 761	12 138	87 862	99 624	28	32	17 139	17 622	82 378	99 517	28
33	11 857	12 235	87 765	99 622	27	33	17 223	17 708	82 292	99 515	27
34	11 952	12 332	87 668	99 620	26	34	17 307	17 794	82 206	99 513	26
35	12 047	12 428	87 572	99 618	**25**	**35**	17 391	17 880	82 120	99 511	**25**
36	12 142	12 525	87 475	99 617	24	36	17 474	17 965	82 035	99 509	24
37	12 236	12 621	87 379	99 615	23	37	17 558	18 051	81 949	99 507	23
38	12 331	12 717	87 283	99 613	22	38	17 641	18 136	81 864	99 505	22
39	12 425	12 813	87 187	99 612	21	39	17 724	18 221	81 779	99 503	21
40	12 519	12 909	87 091	99 610	**20**	**40**	17 807	18 306	81 694	99 501	**20**
41	12 612	13 004	86 996	99 608	19	41	17 890	18 391	81 609	99 499	19
42	12 706	13 099	86 901	99 607	18	42	17 973	18 475	81 525	99 497	18
43	12 799	13 194	86 806	99 605	17	43	18 055	18 560	81 440	99 495	17
44	12 892	13 289	86 711	99 603	16	44	18 137	18 644	81 356	99 494	16
45	12 985	13 384	86 616	99 601	**15**	**45**	18 220	18 728	81 272	99 492	**15**
46	13 078	13 478	86 522	99 600	14	46	18 302	18 812	81 188	99 490	14
47	13 171	13 573	86 427	99 598	13	47	18 383	18 896	81 104	99 488	13
48	13 263	13 667	86 333	99 596	12	48	18 465	18 979	81 021	99 486	12
49	13 355	13 761	86 239	99 595	11	49	18 547	19 063	80 937	99 484	11
50	13 447	13 854	86 146	99 593	**10**	**50**	18 628	19 146	80 854	99 482	**10**
51	13 539	13 948	86 052	99 591	9	51	18 709	19 229	80 771	99 480	9
52	13 630	14 041	85 959	99 589	8	52	18 790	19 312	80 688	99 478	8
53	13 722	14 134	85 866	99 588	7	53	18 871	19 395	80 605	99 476	7
54	13 813	14 227	85 773	99 586	6	54	18 952	19 478	80 522	99 474	6
55	13 904	14 320	85 680	99 584	**5**	**55**	19 033	19 561	80 439	99 472	**5**
56	13 994	14 412	85 588	99 582	4	56	19 113	19 643	80 357	99 470	4
57	14 085	14 504	85 496	99 581	3	57	19 193	19 725	80 275	99 468	3
58	14 175	14 597	85 403	99 579	2	58	19 273	19 807	80 193	99 466	2
59	14 266	14 688	85 312	99 577	1	59	19 353	19 889	80 111	99 464	1
60	14 356	14 780	85 220	99 575	**0**	**60**	19 433	19 971	80 029	99 462	**0**
	9	9	10	9			9	9	10	9	
′	log cos	log cot	log tan	log sin	′	′	log cos	log cot	log tan	log sin	′

82° 81°

9°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	19 433	19 971	80 029	99 462	**60**
1	19 513	20 053	79 947	99 460	59
2	19 592	20 134	79 866	99 458	58
3	19 672	20 216	79 784	99 456	57
4	19 751	20 297	79 703	99 454	56
5	19 830	20 378	79 622	99 452	**55**
6	19 909	20 459	79 541	99 450	54
7	19 988	20 540	79 460	99 448	53
8	20 067	20 621	79 379	99 446	52
9	20 145	20 701	79 299	99 444	51
10	20 223	20 782	79 218	99 442	**50**
11	20 302	20 862	79 138	99 440	49
12	20 380	20 942	79 058	99 438	48
13	20 458	21 022	78 978	99 436	47
14	20 535	21 102	78 898	99 434	46
15	20 613	21 182	78 818	99 432	**45**
16	20 691	21 261	78 739	99 429	44
17	20 768	21 341	78 659	99 427	43
18	20 845	21 420	78 580	99 425	42
19	20 922	21 499	78 501	99 423	41
20	20 999	21 578	78 422	99 421	**40**
21	21 076	21 657	78 343	99 419	39
22	21 153	21 736	78 264	99 417	38
23	21 229	21 814	78 186	99 415	37
24	21 306	21 893	78 107	99 413	36
25	21 382	21 971	78 029	99 411	**35**
26	21 458	22 049	77 951	99 409	34
27	21 534	22 127	77 873	99 407	33
28	21 610	22 205	77 795	99 404	32
29	21 685	22 283	77 717	99 402	31
30	21 761	22 361	77 639	99 400	**30**
31	21 836	22 438	77 562	99 398	29
32	21 912	22 516	77 484	99 396	28
33	21 987	22 593	77 407	99 394	27
34	22 062	22 670	77 330	99 392	26
35	22 137	22 747	77 253	99 390	**25**
36	22 211	22 824	77 176	99 388	24
37	22 286	22 901	77 099	99 385	23
38	22 361	22 977	77 023	99 383	22
39	22 435	23 054	76 946	99 381	21
40	22 509	23 130	76 870	99 379	**20**
41	22 583	23 206	76 794	99 377	19
42	22 657	23 283	76 717	99 375	18
43	22 731	23 359	76 641	99 372	17
44	22 805	23 435	76 565	99 370	16
45	22 878	23 510	76 490	99 368	**15**
46	22 952	23 586	76 414	99 366	14
47	23 025	23 661	76 339	99 364	13
48	23 098	23 737	76 263	99 362	12
49	23 171	23 812	76 188	99 359	11
50	23 244	23 887	76 113	99 357	**10**
51	23 317	23 962	76 038	99 355	9
52	23 390	24 037	75 963	99 353	8
53	23 462	24 112	75 888	99 351	7
54	23 535	24 186	75 814	99 348	6
55	23 607	24 261	75 739	99 346	**5**
56	23 679	24 335	75 665	99 344	4
57	23 752	24 410	75 590	99 342	3
58	23 823	24 484	75 516	99 340	2
59	23 895	24 558	75 442	99 337	1
60	23 967	24 632	75 368	99 335	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

80°

10°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	23 967	24 632	75 368	99 335	**60**
1	24 039	24 706	75 294	99 333	59
2	24 110	24 779	75 221	99 331	58
3	24 181	24 853	75 147	99 328	57
4	24 253	24 926	75 074	99 326	56
5	24 324	25 000	75 000	99 324	**55**
6	24 395	25 073	74 927	99 322	54
7	24 466	25 146	74 854	99 319	53
8	24 536	25 219	74 781	99 317	52
9	24 607	25 292	74 708	99 315	51
10	24 677	25 365	74 635	99 313	**50**
11	24 748	25 437	74 563	99 310	49
12	24 818	25 510	74 490	99 308	48
13	24 888	25 582	74 418	99 306	47
14	24 958	25 655	74 345	99 304	46
15	25 028	25 727	74 273	99 301	**45**
16	25 098	25 799	74 201	99 299	44
17	25 168	25 871	74 129	99 297	43
18	25 237	25 943	74 057	99 294	42
19	25 307	26 015	73 985	99 292	41
20	25 376	26 086	73 914	99 290	**40**
21	25 445	26 158	73 842	99 288	39
22	25 514	26 229	73 771	99 285	38
23	25 583	26 301	73 699	99 283	37
24	25 652	26 372	73 628	99 281	36
25	25 721	26 443	73 557	99 278	**35**
26	25 790	26 514	73 486	99 276	34
27	25 858	26 585	73 415	99 274	33
28	25 927	26 655	73 345	99 271	32
29	25 995	26 726	73 274	99 269	31
30	26 063	26 797	73 203	99 267	**30**
31	26 131	26 867	73 133	99 264	29
32	26 199	26 937	73 063	99 262	28
33	26 267	27 008	72 992	99 260	27
34	26 335	27 078	72 922	99 257	26
35	26 403	27 148	72 852	99 255	**25**
36	26 470	27 218	72 782	99 252	24
37	26 538	27 288	72 712	99 250	23
38	26 605	27 357	72 643	99 248	22
39	26 672	27 427	72 573	99 245	21
40	26 739	27 496	72 504	99 243	**20**
41	26 806	27 566	72 434	99 241	19
42	26 873	27 635	72 365	99 238	18
43	26 940	27 704	72 296	99 236	17
44	27 007	27 773	72 227	99 233	16
45	27 073	27 842	72 158	99 231	**15**
46	27 140	27 911	72 089	99 229	14
47	27 206	27 980	72 020	99 226	13
48	27 273	28 049	71 951	99 224	12
49	27 339	28 117	71 883	99 221	11
50	27 405	28 186	71 814	99 219	**10**
51	27 471	28 254	71 746	99 217	9
52	27 537	28 323	71 677	99 214	8
53	27 602	28 391	71 609	99 212	7
54	27 668	28 459	71 541	99 209	6
55	27 734	28 527	71 473	99 207	**5**
56	27 799	28 595	71 405	99 204	4
57	27 864	28 662	71 338	99 202	3
58	27 930	28 730	71 270	99 200	2
59	27 995	28 798	71 202	99 197	1
60	28 060	28 865	71 135	99 195	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

79°

11°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	28 060	28 865	71 135̲	99 195̲	**60**
1	28 125̲	28 933	71 067	99 192	59
2	28 190	29 000	71 000	99 190	58
3	28 254	29 067	70 933	99 187	57
4	28 319	29 134	70 866	99 185̲	56
5	28 384	29 201	70 799	99 182	**55**
6	28 448	29 268	70 732	99 180	54
7	28 512	29 335̲	70 665	99 177	53
8	28 577	29 402	70 598	99 175̲	52
9	28 641	29 468	70 532	99 172	51
10	28 705̲	29 535̲	70 465	99 170	**50**
11	28 769	29 601	70 399	99 167	49
12	28 833	29 668	70 332	99 165̲	48
13	28 896	29 734	70 266	99 162	47
14	28 960	29 800	70 200	99 160	46
15	29 024	29 866	70 134	99 157	**45**
16	29 087	29 932	70 068	99 155̲	44
17	29 150	29 998	70 002	99 152	43
18	29 214	30 064	69 936	99 15̲0	42
19	29 277	30 130	69 870	99 147	41
20	29 340	30 195	69 805̲	99 145̲	**40**
21	29 403	30 261	69 739	99 142	39
22	29 466	30 326	69 674	99 140	38
23	29 529	30 391	69 609	99 137	37
24	29 591	30 457	69 543	99 135̲	36
25	29 654	30 522	69 478	99 132	**35**
26	29 716	30 587	69 413	99 130	34
27	29 779	30 652	69 348	99 127	33
28	29 841	30 717	69 283	99 124	32
29	29 903	30 782	69 218	99 122	31
30	29 966	30 846	69 154	99 119	**30**
31	30 028	30 911	69 089	99 117	29
32	30 090	30 975	69 025̲	99 114	28
33	30 151	31 040	68 960	99 112	27
34	30 213	31 104	68 896	99 109	26
35	30 275̲	31 168	68 832	99 106	**25**
36	30 336	31 233	68 767	99 104	24
37	30 398	31 297	68 703	99 101	23
38	30 459	31 361	68 639	99 099	22
39	30 521	31 425̲	68 575	99 096	21
40	30 582	31 489	68 511	99 093	**20**
41	30 643	31 552	68 448	99 091	19
42	30 704	31 616	68 384	99 088	18
43	30 765	31 679	68 321	99 086	17
44	30 826	31 743	68 257	99 083	16
45	30 887	31 806	68 194	99 080	**15**
46	30 947	31 870	68 130	99 078	14
47	31 008	31 933	68 067	99 075	13
48	31 068	31 996	68 004	99 072	12
49	31 129	32 059	67 941	99 070	11
50	31 189	32 122	67 878	99 067	**10**
51	31 25̲0	32 185	67 815̲	99 064	9
52	31 310	32 248	67 752	99 062	8
53	31 370	32 311	67 689	99 059	7
54	31 430	32 373	67 627	99 056	6
55	31 490	32 436	67 564	99 054	**5**
56	31 549	32 498	67 502	99 051	4
57	31 609	32 561	67 439	99 048	3
58	31 669	32 623	67 377	99 046	2
59	31 728	32 685	67 315̲	99 043	1
60	31 788	32 747	67 253	99 040	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

78°

12°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	31 788	32 747	67 253	99 040	**60**
1	31 847	32 810	67 190	99 038	59
2	31 907	32 872	67 128	99 035	58
3	31 966	32 933	67 067	99 032	57
4	32 025̲	32 995	67 005̲	99 030	56
5	32 084	33 057	66 943	99 027	**55**
6	32 143	33 119	66 881	99 024	54
7	32 202	33 180	66 820	99 022	53
8	32 261	33 242	66 758	99 019	52
9	32 319	33 303	66 697	99 016	51
10	32 378	33 365̲	66 635	99 013	**50**
11	32 437	33 426	66 574	99 011	49
12	32 495	33 487	66 513	99 008	48
13	32 553	33 548	66 452	99 005	47
14	32 612	33 609	66 391	99 002	46
15	32 670	33 670	66 330	99 000	**45**
16	32 728	33 731	66 269	98 997	44
17	32 786	33 792	66 208	98 994	43
18	32 844	33 853	66 147	98 991	42
19	32 902	33 913	66 087	98 989	41
20	32 960	33 974	66 026	98 986	**40**
21	33 018	34 034	65 966	98 983	39
22	33 075	34 095̲	65 905	98 980	38
23	33 133	34 155	65 845̲	98 978	37
24	33 190	34 215	65 785̲	98 975̲	36
25	33 248	34 276	65 724	98 972	**35**
26	33 305	34 336	65 664	98 969	34
27	33 362	34 396	65 604	98 967	33
28	33 420	34 456	65 544	98 964	32
29	33 477	34 516	65 484	98 961	31
30	33 534	34 576	65 424	98 958	**30**
31	33 591	34 635	65 365̲	98 955	29
32	33 647	34 695̲	65 305	98 953	28
33	33 704	34 755̲	65 245	98 95̲0	27
34	33 761	34 814	65 186	98 947	26
35	33 818	34 874	65 126	98 944	**25**
36	33 874	34 933	65 067	98 941	24
37	33 931	34 992	65 008	98 938	23
38	33 987	35 051	64 949	98 936	22
39	34 043	35 111	64 889	98 933	21
40	34 100	35 170	64 830	98 930	**20**
41	34 156	35 229	64 771	98 927	19
42	34 212	35 288	64 712	98 924	18
43	34 268	35 347	64 653	98 921	17
44	34 324	35 405	64 595̲	98 919	16
45	34 380	35 464	64 536	98 916	**15**
46	34 436	35 523	64 477	98 913	14
47	34 491	35 581	64 419	98 910	13
48	34 547	35 640	64 360	98 907	12
49	34 602	35 698	64 302	98 904	11
50	34 658	35 757	64 243	98 901	**10**
51	34 713	35 815̲	64 185	98 898	9
52	34 769	35 873	64 127	98 896	8
53	34 824	35 931	64 069	98 893	7
54	34 879	35 989	64 011	98 890	6
55	34 934	36 047	63 953	98 887	**5**
56	34 989	36 105	63 895̲	98 884	4
57	35 044	36 163	63 837	98 881	3
58	35 099	36 221	63 779	98 878	2
59	35 154	36 279	63 721	98 875	1
60	35 209	36 336	63 664	98 872	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

77°

13°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	35 209	36 336	63 664	98 872	**60**
1	35 263	36 394	63 606	98 869	59
2	35 318	36 452	63 548	98 867	58
3	35 373	36 509	63 491	98 864	57
4	35 427	36 566	63 434	98 861	56
5	35 481	36 624	63 376	98 858	**55**
6	35 536	36 681	63 319	98 855	54
7	35 590	36 738	63 262	98 852	53
8	35 644	36 795	63 205	98 849	52
9	35 698	36 852	63 148	98 846	51
10	35 752	36 909	63 091	98 843	**50**
11	35 806	36 966	63 034	98 840	49
12	35 860	37 023	62 977	98 837	48
13	35 914	37 080	62 920	98 834	47
14	35 968	37 137	62 863	98 831	46
15	36 022	37 193	62 807	98 828	**45**
16	36 075	37 250	62 750	98 825	44
17	36 129	37 306	62 694	98 822	43
18	36 182	37 363	62 637	98 819	42
19	36 236	37 419	62 581	98 816	41
20	36 289	37 476	62 524	98 813	**40**
21	36 342	37 532	62 468	98 810	39
22	36 395	37 588	62 412	98 807	38
23	36 449	37 644	62 356	98 804	37
24	36 502	37 700	62 300	98 801	36
25	36 555	37 756	62 244	98 798	**35**
26	36 608	37 812	62 188	98 795	34
27	36 660	37 868	62 132	98 792	33
28	36 713	37 924	62 076	98 789	32
29	36 766	37 980	62 020	98 786	31
30	36 819	38 035	61 965	98 783	**30**
31	36 871	38 091	61 909	98 780	29
32	36 924	38 147	61 853	98 777	28
33	36 976	38 202	61 798	98 774	27
34	37 028	38 257	61 743	98 771	26
35	37 081	38 313	61 687	98 768	**25**
36	37 133	38 368	61 632	98 765	24
37	37 185	38 423	61 577	98 762	23
38	37 237	38 479	61 521	98 759	22
39	37 289	38 534	61 466	98 756	21
40	37 341	38 589	61 411	98 753	**20**
41	37 393	38 644	61 356	98 750	19
42	37 445	38 699	61 301	98 746	18
43	37 497	38 754	61 246	98 743	17
44	37 549	38 808	61 192	98 740	16
45	37 600	38 863	61 137	98 737	**15**
46	37 652	38 918	61 082	98 734	14
47	37 703	38 972	61 028	98 731	13
48	37 755	39 027	60 973	98 728	12
49	37 806	39 082	60 918	98 725	11
50	37 858	39 136	60 864	98 722	**10**
51	37 909	39 190	60 810	98 719	9
52	37 960	39 245	60 755	98 715	8
53	38 011	39 299	60 701	98 712	7
54	38 062	39 353	60 647	98 709	6
55	38 113	39 407	60 593	98 706	**5**
56	38 164	39 461	60 539	98 703	4
57	38 215	39 515	60 485	98 700	3
58	38 266	39 569	60 431	98 697	2
59	38 317	39 623	60 377	98 694	1
60	38 368	39 677	60 323	98 690	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

76°

14°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	38 368	39 677	60 323	98 690	**60**
1	38 418	39 731	60 269	98 687	59
2	38 469	39 785	60 215	98 684	58
3	38 519	39 838	60 162	98 681	57
4	38 570	39 892	60 108	98 678	56
5	38 620	39 945	60 055	98 675	**55**
6	38 670	39 999	60 001	98 671	54
7	38 721	40 052	59 948	98 668	53
8	38 771	40 106	59 894	98 665	52
9	38 821	40 159	59 841	98 662	51
10	38 871	40 212	59 788	98 659	**50**
11	38 921	40 266	59 734	98 656	49
12	38 971	40 319	59 681	98 652	48
13	39 021	40 372	59 628	98 649	47
14	39 071	40 425	59 575	98 646	46
15	39 121	40 478	59 522	98 643	**45**
16	39 170	40 531	59 469	98 640	44
17	39 220	40 584	59 416	98 636	43
18	39 270	40 636	59 364	98 633	42
19	39 319	40 689	59 311	98 630	41
20	39 369	40 742	59 258	98 627	**40**
21	39 418	40 795	59 205	98 623	39
22	39 467	40 847	59 153	98 620	38
23	39 517	40 900	59 100	98 617	37
24	39 566	40 952	59 048	98 614	36
25	39 615	41 005	58 995	98 610	**35**
26	39 664	41 057	58 943	98 607	34
27	39 713	41 109	58 891	98 604	33
28	39 762	41 161	58 839	98 601	32
29	39 811	41 214	58 786	98 597	31
30	39 860	41 266	58 734	98 594	**30**
31	39 909	41 318	58 682	98 591	29
32	39 958	41 370	58 630	98 588	28
33	40 006	41 422	58 578	98 584	27
34	40 055	41 474	58 526	98 581	26
35	40 103	41 526	58 474	98 578	**25**
36	40 152	41 578	58 422	98 574	24
37	40 200	41 629	58 371	98 571	23
38	40 249	41 681	58 319	98 568	22
39	40 297	41 733	58 267	98 565	21
40	40 346	41 784	58 216	98 561	**20**
41	40 394	41 836	58 164	98 558	19
42	40 442	41 887	58 113	98 555	18
43	40 490	41 939	58 061	98 551	17
44	40 538	41 990	58 010	98 548	16
45	40 586	42 041	57 959	98 545	**15**
46	40 634	42 093	57 907	98 541	14
47	40 682	42 144	57 856	98 538	13
48	40 730	42 195	57 805	98 535	12
49	40 778	42 246	57 754	98 531	11
50	40 825	42 297	57 703	98 528	**10**
51	40 873	42 348	57 652	98 525	9
52	40 921	42 399	57 601	98 521	8
53	40 968	42 450	57 550	98 518	7
54	41 016	42 501	57 499	98 515	6
55	41 063	42 552	57 448	98 511	**5**
56	41 111	42 603	57 397	98 508	4
57	41 158	42 653	57 347	98 505	3
58	41 205	42 704	57 296	98 501	2
59	41 252	42 755	57 245	98 498	1
60	41 300	42 805	57 195	98 494	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

75°

15°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	41 300	42 805	57 19$\underline{5}$	98 494	**60**
1	41 347	42 856	57 144	98 491	59
2	41 394	42 906	57 094	98 488	58
3	41 441	42 957	57 043	98 484	57
4	41 488	43 007	56 993	98 481	56
5	41 53$\underline{5}$	43 057	56 943	98 477	**55**
6	41 582	43 108	56 892	98 474	54
7	41 628	43 158	56 842	98 471	53
8	41 675	43 208	56 792	98 467	52
9	41 722	43 258	56 742	98 464	51
10	41 768	43 308	56 692	98 460	**50**
11	41 81$\underline{5}$	43 358	56 642	98 457	49
12	41 861	43 408	56 592	98 453	48
13	41 908	43 458	56 542	98 450	47
14	41 954	43 508	56 492	98 447	46
15	42 001	43 558	56 442	98 443	**45**
16	42 047	43 607	56 393	98 440	44
17	42 093	43 657	56 343	98 436	43
18	42 140	43 707	56 293	98 433	42
19	42 186	43 756	56 244	98 429	41
20	42 232	43 806	56 194	98 426	**40**
21	42 278	43 855	56 14$\underline{5}$	98 422	39
22	42 324	43 90$\underline{5}$	56 095	98 419	38
23	42 370	43 954	56 046	98 415	37
24	42 416	44 004	55 996	98 412	36
25	42 461	44 053	55 947	98 409	**35**
26	42 507	44 102	55 898	98 405	34
27	42 553	44 151	55 849	98 402	33
28	42 599	44 201	55 799	98 398	32
29	42 644	44 2$\underline{5}$0	55 750	98 39$\underline{5}$	31
30	42 690	44 299	55 701	98 391	**30**
31	42 735	44 348	55 652	98 388	29
32	42 781	44 397	55 603	98 384	28
33	42 826	44 446	55 554	98 381	27
34	42 872	44 49$\underline{5}$	55 505	98 377	26
35	42 917	44 544	55 456	98 373	**25**
36	42 962	44 592	55 408	98 370	24
37	43 008	44 641	55 359	98 366	23
38	43 053	44 690	55 310	98 363	22
39	43 098	44 738	55 262	98 359	21
40	43 143	44 787	55 213	98 356	**20**
41	43 188	44 836	55 164	98 352	19
42	43 233	44 884	55 116	98 349	18
43	43 278	44 933	55 067	98 345	17
44	43 323	44 981	55 019	98 342	16
45	43 367	45 029	54 971	98 338	**15**
46	43 412	45 078	54 922	98 334	14
47	43 457	45 126	54 874	98 331	13
48	43 502	45 174	54 826	98 327	12
49	43 546	45 222	54 778	98 324	11
50	43 591	45 271	54 729	98 320	**10**
51	43 635	45 319	54 681	98 317	9
52	43 680	45 367	54 633	98 313	8
53	43 724	45 41$\underline{5}$	54 585	98 309	7
54	43 769	45 463	54 537	98 306	6
55	43 813	45 511	54 489	98 302	**5**
56	43 857	45 559	54 441	98 299	4
57	43 901	45 606	54 394	98 295	3
58	43 946	45 654	54 346	98 291	2
59	43 990	45 702	54 298	98 288	1
60	44 034	45 7$\underline{5}$0	54 250	98 284	**0**
′	9 log cos	9 log cot	10 log tan	9 log sin	′

74°

16°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	44 034	45 7$\underline{5}$0	54 250	98 284	**60**
1	44 078	45 797	54 203	98 281	59
2	44 122	45 84$\underline{5}$	54 155	98 277	58
3	44 166	45 892	54 108	98 273	57
4	44 210	45 940	54 060	98 270	56
5	44 253	45 987	54 013	98 266	**55**
6	44 297	46 03$\underline{5}$	53 965	98 262	54
7	44 341	46 082	53 918	98 259	53
8	44 38$\underline{5}$	46 130	53 870	98 255	52
9	44 428	46 177	53 823	98 251	51
10	44 472	46 224	53 776	98 248	**50**
11	44 516	46 271	53 729	98 244	49
12	44 559	46 319	53 681	98 240	48
13	44 602	46 366	53 634	98 237	47
14	44 646	46 413	53 587	98 233	46
15	44 689	46 460	53 540	98 229	**45**
16	44 733	46 507	53 493	98 226	44
17	44 776	46 554	53 446	98 222	43
18	44 819	46 601	53 399	98 218	42
19	44 862	46 648	53 352	98 21$\underline{5}$	41
20	44 905	46 694	53 306	98 211	**40**
21	44 948	46 741	53 259	98 207	39
22	44 992	46 788	53 212	98 204	38
23	45 03$\underline{5}$	46 83$\underline{5}$	53 165	98 200	37
24	45 077	46 881	53 119	98 196	36
25	45 120	46 928	53 072	98 192	**35**
26	45 163	46 97$\underline{5}$	53 025	98 189	34
27	45 206	47 021	52 979	98 18$\underline{5}$	33
28	45 249	47 068	52 932	98 181	32
29	45 292	47 114	52 886	98 177	31
30	45 334	47 160	52 840	98 174	**30**
31	45 377	47 207	52 793	98 170	29
32	45 419	47 253	52 747	98 166	28
33	45 462	47 299	52 701	98 162	27
34	45 504	47 346	52 654	98 159	26
35	45 547	47 392	52 608	98 15$\underline{5}$	**25**
36	45 589	47 438	52 562	98 151	24
37	45 632	47 484	52 516	98 147	23
38	45 674	47 530	52 470	98 144	22
39	45 716	47 576	52 424	98 140	21
40	45 758	47 622	52 378	98 136	**20**
41	45 801	47 668	52 332	98 132	19
42	45 843	47 714	52 286	98 129	18
43	45 88$\underline{5}$	47 760	52 240	98 12$\underline{5}$	17
44	45 927	47 806	52 194	98 121	16
45	45 969	47 852	52 148	98 117	**15**
46	46 011	47 897	52 103	98 113	14
47	46 053	47 943	52 057	98 110	13
48	46 09$\underline{5}$	47 989	52 011	98 106	12
49	46 136	48 03$\underline{5}$	51 965	98 102	11
50	46 178	48 080	51 920	98 098	**10**
51	46 220	48 126	51 874	98 094	9
52	46 262	48 171	51 829	98 090	8
53	46 303	48 217	51 783	98 087	7
54	46 34$\underline{5}$	48 262	51 738	98 083	6
55	46 386	48 307	51 693	98 079	**5**
56	46 428	48 353	51 647	98 075	4
57	46 469	48 398	51 602	98 071	3
58	46 511	48 443	51 557	98 067	2
59	46 552	48 489	51 511	98 063	1
60	46 594	48 534	51 466	98 060	**0**
′	9 log cos	9 log cot	10 log tan	9 log sin	′

73°

17°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	46 594	48 534	51 466	98 060	**60**
1	46 635	48 579	51 421	98 056	59
2	46 676	48 624	51 376	98 052	58
3	46 717	48 669	51 331	98 048	57
4	46 758	48 714	51 286	98 044	56
5	46 800	48 759	51 241	98 040	**55**
6	46 841	48 804	51 196	98 036	54
7	46 882	48 849	51 151	98 032	53
8	46 923	48 894	51 106	98 029	52
9	46 964	48 939	51 061	98 025	51
10	47 005	48 984	51 016	98 021	**50**
11	47 045	49 029	50 971	98 017	49
12	47 086	49 073	50 927	98 013	48
13	47 127	49 118	50 882	98 009	47
14	47 168	49 163	50 837	98 005	46
15	47 209	49 207	50 793	98 001	**45**
16	47 249	49 252	50 748	97 997	44
17	47 290	49 296	50 704	97 993	43
18	47 330	49 341	50 659	97 989	42
19	47 371	49 385	50 615	97 986	41
20	47 411	49 430	50 570	97 982	**40**
21	47 452	49 474	50 526	97 978	39
22	47 492	49 519	50 481	97 974	38
23	47 533	49 563	50 437	97 970	37
24	47 573	49 607	50 393	97 966	36
25	47 613	49 652	50 348	97 962	**35**
26	47 654	49 696	50 304	97 958	34
27	47 694	49 740	50 260	97 954	33
28	47 734	49 784	50 216	97 950	32
29	47 774	49 828	50 172	97 946	31
30	47 814	49 872	50 128	97 942	**30**
31	47 854	49 916	50 084	97 938	29
32	47 894	49 960	50 040	97 934	28
33	47 934	50 004	49 996	97 930	27
34	47 974	50 048	49 952	97 926	26
35	48 014	50 092	49 908	97 922	**25**
36	48 054	50 136	49 864	97 918	24
37	48 094	50 180	49 820	97 914	23
38	48 133	50 223	49 777	97 910	22
39	48 173	50 267	49 733	97 906	21
40	48 213	50 311	49 689	97 902	**20**
41	48 252	50 355	49 645	97 898	19
42	48 292	50 398	49 602	97 894	18
43	48 332	50 442	49 558	97 890	17
44	48 371	50 485	49 515	97 886	16
45	48 411	50 529	49 471	97 882	**15**
46	48 450	50 572	49 428	97 878	14
47	48 490	50 616	49 384	97 874	13
48	48 529	50 659	49 341	97 870	12
49	48 568	50 703	49 297	97 866	11
50	48 607	50 746	49 254	97 861	**10**
51	48 647	50 789	49 211	97 857	9
52	48 686	50 833	49 167	97 853	8
53	48 725	50 876	49 124	97 849	7
54	48 764	50 919	49 081	97 845	6
55	48 803	50 962	49 038	97 841	**5**
56	48 842	51 005	48 995	97 837	4
57	48 881	51 048	48 952	97 833	3
58	48 920	51 092	48 908	97 829	2
59	48 959	51 135	48 865	97 825	1
60	48 998	51 178	48 822	97 821	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

72°

18°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	48 998	51 178	48 822	97 821	**60**
1	49 037	51 221	48 779	97 817	59
2	49 076	51 264	48 736	97 812	58
3	49 115	51 306	48 694	97 808	57
4	49 153	51 349	48 651	97 804	56
5	49 192	51 392	48 608	97 800	**55**
6	49 231	51 435	48 565	97 796	54
7	49 269	51 478	48 522	97 792	53
8	49 308	51 520	48 480	97 788	52
9	49 347	51 563	48 437	97 784	51
10	49 385	51 606	48 394	97 779	**50**
11	49 424	51 648	48 352	97 775	49
12	49 462	51 691	48 309	97 771	48
13	49 500	51 734	48 266	97 767	47
14	49 539	51 776	48 224	97 763	46
15	49 577	51 819	48 181	97 759	**45**
16	49 615	51 861	48 139	97 754	44
17	49 654	51 903	48 097	97 750	43
18	49 692	51 946	48 054	97 746	42
19	49 730	51 988	48 012	97 742	41
20	49 768	52 031	47 969	97 738	**40**
21	49 806	52 073	47 927	97 734	39
22	49 844	52 115	47 885	97 729	38
23	49 882	52 157	47 843	97 725	37
24	49 920	52 200	47 800	97 721	36
25	49 958	52 242	47 758	97 717	**35**
26	49 996	52 284	47 716	97 713	34
27	50 034	52 326	47 674	97 708	33
28	50 072	52 368	47 632	97 704	32
29	50 110	52 410	47 590	97 700	31
30	50 148	52 452	47 548	97 696	**30**
31	50 185	52 494	47 506	97 691	29
32	50 223	52 536	47 464	97 687	28
33	50 261	52 578	47 422	97 683	27
34	50 298	52 620	47 380	97 679	26
35	50 336	52 661	47 339	97 674	**25**
36	50 374	52 703	47 297	97 670	24
37	50 411	52 745	47 255	97 666	23
38	50 449	52 787	47 213	97 662	22
39	50 486	52 829	47 171	97 657	21
40	50 523	52 870	47 130	97 653	**20**
41	50 561	52 912	47 088	97 649	19
42	50 598	52 953	47 047	97 645	18
43	50 635	52 995	47 005	97 640	17
44	50 673	53 037	46 963	97 636	16
45	50 710	53 078	46 922	97 632	**15**
46	50 747	53 120	46 880	97 628	14
47	50 784	53 161	46 839	97 623	13
48	50 821	53 202	46 798	97 619	12
49	50 858	53 244	46 756	97 615	11
50	50 896	53 285	46 715	97 610	**10**
51	50 933	53 327	46 673	97 606	9
52	50 970	53 368	46 632	97 602	8
53	51 007	53 409	46 591	97 597	7
54	51 043	53 450	46 550	97 593	6
55	51 080	53 492	46 508	97 589	**5**
56	51 117	53 533	46 467	97 584	4
57	51 154	53 574	46 426	97 580	3
58	51 191	53 615	46 385	97 576	2
59	51 227	53 656	46 344	97 571	1
60	51 264	53 697	46 303	97 567	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

71°

19°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	51 264	53 697	46 303	97 567	**60**
1	51 301	53 738	46 262	97 563	59
2	51 338	53 779	46 221	97 558	58
3	51 374	53 820	46 180	97 554	57
4	51 411	53 861	46 139	97 55̲0	56
5	51 447	53 902	46 098	97 545	**55**
6	51 484	53 943	46 057	97 541	54
7	51 520	53 984	46 016	97 536	53
8	51 557	54 025̲	45 975	97 532	52
9	51 593	54 065	45 935̲	97 528	51
10	51 629	54 106	45 894	97 523	**50**
11	51 666	54 147	45 853	97 519	49
12	51 702	54 187	45 813	97 515̲	48
13	51 738	54 228	45 772	97 510	47
14	51 774	54 269	45 731	97 506	46
15	51 811	54 309	45 691	97 501	**45**
16	51 847	54 35̲0	45 650	97 497	44
17	51 883	54 390	45 610	97 492	43
18	51 919	54 431	45 569	97 488	42
19	51 955	54 471	45 529	97 484	41
20	51 991	54 512	45 488	97 479	**40**
21	52 027	54 552	45 448	97 475̲	39
22	52 063	54 593	45 407	97 470	38
23	52 099	54 633	45 367	97 466	37
24	52 135̲	54 673	45 327	97 461	36
25	52 171	54 714	45 286	97 457	**35**
26	52 207	54 754	45 246	97 453	34
27	52 242	54 794	45 206	97 448	33
28	52 278	54 835̲	45 165	97 444	32
29	52 314	54 875̲	45 125	97 439	31
30	52 35̲0	54 915̲	45 085	97 435̲	**30**
31	52 385	54 955	45 045̲	97 430	29
32	52 421	54 995	45 005̲	97 426	28
33	52 456	55 035	44 965̲	97 421	27
34	52 492	55 075	44 925̲	97 417	26
35	52 527	55 115	44 885̲	97 412	**25**
36	52 563	55 155	44 845̲	97 408	24
37	52 598	55 195	44 805̲	97 403	23
38	52 634	55 235	44 765̲	97 399	22
39	52 669	55 275	44 725̲	97 394	21
40	52 705̲	55 315̲	44 685	97 390	**20**
41	52 740	55 355̲	44 645	97 385	19
42	52 775	55 395̲	44 605	97 381	18
43	52 811	55 434	44 566	97 376	17
44	52 846	55 474	44 526	97 372	16
45	52 881	55 514	44 486	97 367	**15**
46	52 916	55 554	44 446	97 363	14
47	52 951	55 593	44 407	97 358	13
48	52 986	55 633	44 367	97 353	12
49	53 021	55 673	44 327	97 349	11
50	53 056	55 712	44 288	97 344	**10**
51	53 092	55 752	44 248	97 340	9
52	53 126	55 791	44 209	97 335	8
53	53 161	55 831	44 169	97 331	7
54	53 196	55 870	44 130	97 326	6
55	53 231	55 910	44 090	97 322	**5**
56	53 266	55 949	44 051	97 317	4
57	53 301	55 989	44 011	97 312	3
58	53 336	56 028	43 972	97 308	2
59	53 370	56 067	43 933	97 303	1
60	53 405	56 107	43 893	97 299	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

70°

20°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	53 405	56 107	43 893	97 299	**60**
1	53 440	56 146	43 854	97 294	59
2	53 475̲	56 185	43 815̲	97 289	58
3	53 509	56 224	43 776	97 285̲	57
4	53 544	56 264	43 736	97 280	56
5	53 578	56 303	43 697	97 276	**55**
6	53 613	56 342	43 658	97 271	54
7	53 647	56 381	43 619	97 266	53
8	53 682	56 420	43 580	97 262	52
9	53 716	56 459	43 541	97 257	51
10	53 751	56 498	43 502	97 252	**50**
11	53 785	56 537	43 463	97 248	49
12	53 819	56 576	43 424	97 243	48
13	53 854	56 615	43 385̲	97 238	47
14	53 888	56 654	43 346	97 234	46
15	53 922	56 693	43 307	97 229	**45**
16	53 957	56 732	43 268	97 224	44
17	53 991	56 771	43 229	97 220	43
18	54 025̲	56 810	43 190	97 215	42
19	54 059	56 849	43 151	97 210	41
20	54 093	56 887	43 113	97 206	**40**
21	54 127	56 926	43 074	97 201	39
22	54 161	56 965̲	43 035	97 196	38
23	54 195	57 004	42 996	97 192	37
24	54 229	57 042	42 958	97 187	36
25	54 263	57 081	42 919	97 182	**35**
26	54 297	57 120	42 880	97 178	34
27	54 331	57 158	42 842	97 173	33
28	54 365̲	57 197	42 803	97 168	32
29	54 399	57 235	42 765̲	97 163	31
30	54 433	57 274	42 726	97 159	**30**
31	54 466	57 312	42 688	97 154	29
32	54 500	57 351	42 649	97 149	28
33	54 534	57 389	42 611	97 145̲	27
34	54 567	57 428	42 572	97 140	26
35	54 601	57 466	42 534	97 135	**25**
36	54 635̲	57 504	42 496	97 130	24
37	54 668	57 543	42 457	97 126	23
38	54 702	57 581	42 419	97 121	22
39	54 735	57 619	42 381	97 116	21
40	54 769	57 658	42 342	97 111	**20**
41	54 802	57 696	42 304	97 107	19
42	54 836	57 734	42 266	97 102	18
43	54 869	57 772	42 228	97 097	17
44	54 903	57 810	42 190	97 092	16
45	54 936	57 849	42 151	97 087	**15**
46	54 969	57 887	42 113	97 083	14
47	55 003	57 925̲	42 075	97 078	13
48	55 036	57 963	42 037	97 073	12
49	55 069	58 001	41 999	97 068	11
50	55 102	58 039	41 961	97 063	**10**
51	55 136	58 077	41 923	97 059	9
52	55 169	58 115̲	41 885	97 054	8
53	55 202	58 153	41 847	97 049	7
54	55 235̲	58 191	41 809	97 044	6
55	55 268	58 229	41 771	97 039	**5**
56	55 301	58 267	41 733	97 035̲	4
57	55 334	58 304	41 696	97 030	3
58	55 367	58 342	41 658	97 025̲	2
59	55 400	58 380	41 620	97 020	1
60	55 433	58 418	41 582	97 015	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

69°

21°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	55 433	58 418	41 582	97 015	**60**
1	55 466	58 455	41 545	97 010	59
2	55 499	58 493	41 507	97 005	58
3	55 532	58 531	41 469	97 001	57
4	55 564	58 569	41 431	96 996	56
5	55 597	58 606	41 394	96 991	**55**
6	55 630	58 644	41 356	96 986	54
7	55 663	58 681	41 319	96 981	53
8	55 695	58 719	41 281	96 976	52
9	55 728	58 757	41 243	96 971	51
10	55 761	58 794	41 206	96 966	**50**
11	55 793	58 832	41 168	96 962	49
12	55 826	58 869	41 131	96 957	48
13	55 858	58 907	41 093	96 952	47
14	55 891	58 944	41 056	96 947	46
15	55 923	58 981	41 019	96 942	**45**
16	55 956	59 019	40 981	96 937	44
17	55 988	59 056	40 944	96 932	43
18	56 021	59 094	40 906	96 927	42
19	56 053	59 131	40 869	96 922	41
20	56 085	59 168	40 832	96 917	**40**
21	56 118	59 205	40 795	96 912	39
22	56 150	59 243	40 757	96 907	38
23	56 182	59 280	40 720	96 903	37
24	56 215	59 317	40 683	96 898	36
25	56 247	59 354	40 646	96 893	**35**
26	56 279	59 391	40 609	96 888	34
27	56 311	59 429	40 571	96 883	33
28	56 343	59 466	40 534	96 878	32
29	56 375	59 503	40 497	96 873	31
30	56 408	59 540	40 460	96 868	**30**
31	56 440	59 577	40 423	96 863	29
32	56 472	59 614	40 386	96 858	28
33	56 504	59 651	40 349	96 853	27
34	56 536	59 688	40 312	96 848	26
35	56 568	59 725	40 275	96 843	**25**
36	56 599	59 762	40 238	96 838	24
37	56 631	59 799	40 201	96 833	23
38	56 663	59 835	40 165	96 828	22
39	56 695	59 872	40 128	96 823	21
40	56 727	59 909	40 091	96 818	**20**
41	56 759	59 946	40 054	96 813	19
42	56 790	59 983	40 017	96 808	18
43	56 822	60 019	39 981	96 803	17
44	56 854	60 056	39 944	96 798	16
45	56 886	60 093	39 907	96 793	**15**
46	56 917	60 130	39 870	96 788	14
47	56 949	60 166	39 834	96 783	13
48	56 980	60 203	39 797	96 778	12
49	57 012	60 240	39 760	96 772	11
50	57 044	60 276	39 724	96 767	**10**
51	57 075	60 313	39 687	96 762	9
52	57 107	60 349	39 651	96 757	8
53	57 138	60 386	39 614	96 752	7
54	57 169	60 422	39 578	96 747	6
55	57 201	60 459	39 541	96 742	**5**
56	57 232	60 495	39 505	96 737	4
57	57 264	60 532	39 468	96 732	3
58	57 295	60 568	39 432	96 727	2
59	57 326	60 605	39 395	96 722	1
60	57 358	60 641	39 359	96 717	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

68°

22°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	57 358	60 641	39 359	96 717	**60**
1	57 389	60 677	39 323	96 711	59
2	57 420	60 714	39 286	96 706	58
3	57 451	60 750	39 250	96 701	57
4	57 482	60 786	39 214	96 696	56
5	57 514	60 823	39 177	96 691	**55**
6	57 545	60 859	39 141	96 686	54
7	57 576	60 895	39 105	96 681	53
8	57 607	60 931	39 069	96 676	52
9	57 638	60 967	39 033	96 670	51
10	57 669	61 004	38 996	96 665	**50**
11	57 700	61 040	38 960	96 660	49
12	57 731	61 076	38 924	96 655	48
13	57 762	61 112	38 888	96 650	47
14	57 793	61 148	38 852	96 645	46
15	57 824	61 184	38 816	96 640	**45**
16	57 855	61 220	38 780	96 634	44
17	57 885	61 256	38 744	96 629	43
18	57 916	61 292	38 708	96 624	42
19	57 947	61 328	38 672	96 619	41
20	57 978	61 364	38 636	96 614	**40**
21	58 008	61 400	38 600	96 608	39
22	58 039	61 436	38 564	96 603	38
23	58 070	61 472	38 528	96 598	37
24	58 101	61 508	38 492	96 593	36
25	58 131	61 544	38 456	96 588	**35**
26	58 162	61 579	38 421	96 582	34
27	58 192	61 615	38 385	96 577	33
28	58 223	61 651	38 349	96 572	32
29	58 253	61 687	38 313	96 567	31
30	58 284	61 722	38 278	96 562	**30**
31	58 314	61 758	38 242	96 556	29
32	58 345	61 794	38 206	96 551	28
33	58 375	61 830	38 170	96 546	27
34	58 406	61 865	38 135	96 541	26
35	58 436	61 901	38 099	96 535	**25**
36	58 467	61 936	38 064	96 530	24
37	58 497	61 972	38 028	96 525	23
38	58 527	62 008	37 992	96 520	22
39	58 557	62 043	37 957	96 514	21
40	58 588	62 079	37 921	96 509	**20**
41	58 618	62 114	37 886	96 504	19
42	58 648	62 150	37 850	96 498	18
43	58 678	62 185	37 815	96 493	17
44	58 709	62 221	37 779	96 488	16
45	58 739	62 256	37 744	96 483	**15**
46	58 769	62 292	37 708	96 477	14
47	58 799	62 327	37 673	96 472	13
48	58 829	62 362	37 638	96 467	12
49	58 859	62 398	37 602	96 461	11
50	58 889	62 433	37 567	96 456	**10**
51	58 919	62 468	37 532	96 451	9
52	58 949	62 504	37 496	96 445	8
53	58 979	62 539	37 461	96 440	7
54	59 009	62 574	37 426	96 435	6
55	59 039	62 609	37 391	96 429	**5**
56	59 069	62 645	37 355	96 424	4
57	59 098	62 680	37 320	96 419	3
58	59 128	62 715	37 285	96 413	2
59	59 158	62 750	37 250	96 408	1
60	59 188	62 785	37 215	96 403	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

67°

23°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	59 188	62 785	37 215̲	96 403	**60**
1	59 218	62 820	37 180	96 397	59
2	59 247	62 855	37 145̲	96 392	58
3	59 277	62 890	37 110	96 387	57
4	59 307	62 926	37 074	96 381	56
5	59 336	62 961	37 039	96 376	**55**
6	59 366	62 996	37 004	96 370	54
7	59 396	63 031	36 969	96 365̲	53
8	59 425	63 066	36 934	96 360	52
9	59 455̲	63 101	36 899	96 354	51
10	59 484	63 135	36 865̲	96 349	**50**
11	59 514	63 170	36 830	96 343	49
12	59 543	63 205	36 795̲	96 338	48
13	59 573	63 240	36 760	96 333	47
14	59 602	63 275	36 725̲	96 327	46
15	59 632	63 310	36 690	96 322	**45**
16	59 661	63 345̲	36 655	96 316	44
17	59 690	63 379	36 621	96 311	43
18	59 720	63 414	36 586	96 305	42
19	59 749	63 449	36 551	96 300	41
20	59 778	63 484	36 516	96 294	**40**
21	59 808	63 519	36 481	96 289	39
22	59 837	63 553	36 447	96 284	38
23	59 866	63 588	36 412	96 278	37
24	59 895	63 623	36 377	96 273	36
25	59 924	63 657	36 343	96 267	**35**
26	59 954	63 692	36 308	96 262	34
27	59 983	63 726	36 274	96 256	33
28	60 012	63 761	36 239	96 251	32
29	60 041	63 796	36 204	96 245	31
30	60 070	63 830	36 170	96 240	**30**
31	60 099	63 865̲	36 135	96 234	29
32	60 128	63 899	36 101	96 229	28
33	60 157	63 934	36 066	96 223	27
34	60 186	63 968	36 032	96 218	26
35	60 215̲	64 003	35 997	96 212	**25**
36	60 244	64 037	35 963	96 207	24
37	60 273	64 072	35 928	96 201	23
38	60 302	64 106	35 894	96 196	22
39	60 331	64 140	35 860	96 190	21
40	60 359	64 175̲	35 825	96 185̲	**20**
41	60 388	64 209	35 791	96 179	19
42	60 417	64 243	35 757	96 174	18
43	60 446	64 278	35 722	96 168	17
44	60 474	64 312	35 688	96 162	16
45	60 503	64 346	35 654	96 157	**15**
46	60 532	64 381	35 619	96 151	14
47	60 561	64 415̲	35 585	96 146	13
48	60 589	64 449	35 551	96 140	12
49	60 618	64 483	35 517	96 135̲	11
50	60 646	64 517	35 483	96 129	**10**
51	60 675	64 552	35 448	96 123	9
52	60 704	64 586	35 414	96 118	8
53	60 732	64 620	35 380	96 112	7
54	60 761	64 654	35 346	96 107	6
55	60 789	64 688	35 312	96 101	**5**
56	60 818	64 722	35 278	96 095	4
57	60 846	64 756	35 244	96 090	3
58	60 875̲	64 790	35 210	96 084	2
59	60 903	64 824	35 176	96 079	1
60	60 931	64 858	35 142	96 073	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

66°

24°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	60 931	64 858	35 142	96 073	**60**
1	60 960	64 892	35 108	96 067	59
2	60 988	64 926	35 074	96 062	58
3	61 016	64 960	35 040	96 056	57
4	61 045̲	64 994	35 006	96 050	56
5	61 073	65 028	34 972	96 045	**55**
6	61 101	65 062	34 938	96 039	54
7	61 129	65 096	34 904	96 034	53
8	61 158	65 130	34 870	96 028	52
9	61 186	65 164	34 836	96 022	51
10	61 214	65 197	34 803	96 017	**50**
11	61 242	65 231	34 769	96 011	49
12	61 270	65 265	34 735̲	96 005	48
13	61 298	65 299	34 701	96 000	47
14	61 326	65 333	34 667	95 994	46
15	61 354	65 366	34 634	95 988	**45**
16	61 382	65 400	34 600	95 982	44
17	61 411	65 434	34 566	95 977	43
18	61 438	65 467	34 533	95 971	42
19	61 466	65 501	34 499	95 965	41
20	61 494	65 535̲	34 465	95 960	**40**
21	61 522	65 568	34 432	95 954	39
22	61 550	65 602	34 398	95 948	38
23	61 578	65 636	34 364	95 942	37
24	61 606	65 669	34 331	95 937	36
25	61 634	65 703	34 297	95 931	**35**
26	61 662	65 736	34 264	95 925	34
27	61 689	65 770	34 230	95 920	33
28	61 717	65 803	34 197	95 914	32
29	61 745̲	65 837	34 163	95 908	31
30	61 773	65 870	34 130	95 902	**30**
31	61 800	65 904	34 096	95 897	29
32	61 828	65 937	34 063	95 891	28
33	61 856	65 971	34 029	95 885̲	27
34	61 883	66 004	33 996	95 879	26
35	61 911	66 038	33 962	95 873	**25**
36	61 939	66 071	33 929	95 868	24
37	61 966	66 104	33 896	95 862	23
38	61 994	66 138	33 862	95 856	22
39	62 021	66 171	33 829	95 850	21
40	62 049	66 204	33 796	95 844	**20**
41	62 076	66 238	33 762	95 839	19
42	62 104	66 271	33 729	95 833	18
43	62 131	66 304	33 696	95 827	17
44	62 159	66 337	33 663	95 821	16
45	62 186	66 371	33 629	95 815	**15**
46	62 214	66 404	33 596	95 810	14
47	62 241	66 437	33 563	95 804	13
48	62 268	66 470	33 530	95 798	12
49	62 296	66 503	33 497	95 792	11
50	62 323	66 537	33 463	95 786	**10**
51	62 350	66 570	33 430	95 780	9
52	62 377	66 603	33 397	95 775̲	8
53	62 405̲	66 636	33 364	95 769	7
54	62 432	66 669	33 331	95 763	6
55	62 459	66 702	33 298	95 757	**5**
56	62 486	66 735	33 265̲	95 751	4
57	62 513	66 768	33 232	95 745	3
58	62 541	66 801	33 199	95 739	2
59	62 568	66 834	33 166	95 733	1
60	62 595̲	66 867	33 133	95 728	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

65°

25°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	62 595	66 867	33 133	95 728	**60**
1	62 622	66 900	33 100	95 722	59
2	62 649	66 933	33 067	95 716	58
3	62 676	66 966	33 034	95 710	57
4	62 703	66 999	33 001	95 704	56
5	62 730	67 032	32 968	95 698	**55**
6	62 757	67 065	32 935	95 692	54
7	62 784	67 098	32 902	95 686	53
8	62 811	67 131	32 869	95 680	52
9	62 838	67 163	32 837	95 674	51
10	62 865	67 196	32 804	95 668	**50**
11	62 892	67 229	32 771	95 663	49
12	62 918	67 262	32 738	95 657	48
13	62 945	67 295	32 705	95 651	47
14	62 972	67 327	32 673	95 645	46
15	62 999	67 360	32 640	95 639	**45**
16	63 026	67 393	32 607	95 633	44
17	63 052	67 426	32 574	95 627	43
18	63 079	67 458	32 542	95 621	42
19	63 106	67 491	32 509	95 615	41
20	63 133	67 524	32 476	95 609	**40**
21	63 159	67 556	32 444	95 603	39
22	63 186	67 589	32 411	95 597	38
23	63 213	67 622	32 378	95 591	37
24	63 239	67 654	32 346	95 585	36
25	63 266	67 687	32 313	95 579	**35**
26	63 292	67 719	32 281	95 573	34
27	63 319	67 752	32 248	95 567	33
28	63 345	67 785	32 215	95 561	32
29	63 372	67 817	32 183	95 555	31
30	63 398	67 850	32 150	95 549	**30**
31	63 425	67 882	32 118	95 543	29
32	63 451	67 915	32 085	95 537	28
33	63 478	67 947	32 053	95 531	27
34	63 504	67 980	32 020	95 525	26
35	63 531	68 012	31 988	95 519	**25**
36	63 557	68 044	31 956	95 513	24
37	63 583	68 077	31 923	95 507	23
38	63 610	68 109	31 891	95 500	22
39	63 636	68 142	31 858	95 494	21
40	63 662	68 174	31 826	95 488	**20**
41	63 689	68 206	31 794	95 482	19
42	63 715	68 239	31 761	95 476	18
43	63 741	68 271	31 729	95 470	17
44	63 767	68 303	31 697	95 464	16
45	63 794	68 336	31 664	95 458	**15**
46	63 820	68 368	31 632	95 452	14
47	63 846	68 400	31 600	95 446	13
48	63 872	68 432	31 568	95 440	12
49	63 898	68 465	31 535	95 434	11
50	63 924	68 497	31 503	95 427	**10**
51	63 950	68 529	31 471	95 421	9
52	63 976	68 561	31 439	95 415	8
53	64 002	68 593	31 407	95 409	7
54	64 028	68 626	31 374	95 403	6
55	64 054	68 658	31 342	95 397	**5**
56	64 080	68 690	31 310	95 391	4
57	64 106	68 722	31 278	95 384	3
58	64 132	68 754	31 246	95 378	2
59	64 158	68 786	31 214	95 372	1
60	64 184	68 818	31 182	95 366	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

64°

26°

′	log sin	log tan	log cot	log cos	′
	9	**9**	**10**	**9**	
0	64 184	68 818	31 182	95 366	**60**
1	64 210	68 850	31 150	95 360	59
2	64 236	68 882	31 118	95 354	58
3	64 262	68 914	31 086	95 348	57
4	64 288	68 946	31 054	95 341	56
5	64 313	68 978	31 022	95 335	**55**
6	64 339	69 010	30 990	95 329	54
7	64 365	69 042	30 958	95 323	53
8	64 391	69 074	30 926	95 317	52
9	64 417	69 106	30 894	95 310	51
10	64 442	69 138	30 862	95 304	**50**
11	64 468	69 170	30 830	95 298	49
12	64 494	69 202	30 798	95 292	48
13	64 519	69 234	30 766	95 286	47
14	64 545	69 266	30 734	95 279	46
15	64 571	69 298	30 702	95 273	**45**
16	64 596	69 329	30 671	95 267	44
17	64 622	69 361	30 639	95 261	43
18	64 647	69 393	30 607	95 254	42
19	64 673	69 425	30 575	95 248	41
20	64 698	69 457	30 543	95 242	**40**
21	64 724	69 488	30 512	95 236	39
22	64 749	69 520	30 480	95 229	38
23	64 775	69 552	30 448	95 223	37
24	64 800	69 584	30 416	95 217	36
25	64 826	69 615	30 385	95 211	**35**
26	64 851	69 647	30 353	95 204	34
27	64 877	69 679	30 321	95 198	33
28	64 902	69 710	30 290	95 192	32
29	64 927	69 742	30 258	95 185	31
30	64 953	69 774	30 226	95 179	**30**
31	64 978	69 805	30 195	95 173	29
32	65 003	69 837	30 163	95 167	28
33	65 029	69 868	30 132	95 160	27
34	65 054	69 900	30 100	95 154	26
35	65 079	69 932	30 068	95 148	**25**
36	65 104	69 963	30 037	95 141	24
37	65 130	69 995	30 005	95 135	23
38	65 155	70 026	29 974	95 129	22
39	65 180	70 058	29 942	95 122	21
40	65 205	70 089	29 911	95 116	**20**
41	65 230	70 121	29 879	95 110	19
42	65 255	70 152	29 848	95 103	18
43	65 281	70 184	29 816	95 097	17
44	65 306	70 215	29 785	95 090	16
45	65 331	70 247	29 753	95 084	**15**
46	65 356	70 278	29 722	95 078	14
47	65 381	70 309	29 691	95 071	13
48	65 406	70 341	29 659	95 065	12
49	65 431	70 372	29 628	95 059	11
50	65 456	70 404	29 596	95 052	**10**
51	65 481	70 435	29 565	95 046	9
52	65 506	70 466	29 534	95 039	8
53	65 531	70 498	29 502	95 033	7
54	65 556	70 529	29 471	95 027	6
55	65 580	70 560	29 440	95 020	**5**
56	65 605	70 592	29 408	95 014	4
57	65 630	70 623	29 377	95 007	3
58	65 655	70 654	29 346	95 001	2
59	65 680	70 685	29 315	94 995	1
60	65 705	70 717	29 283	94 988	**0**
	9	**9**	**10**	**9**	
′	log cos	log cot	log tan	log sin	′

63°

27°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	65 705	70 717	29 283	94 988	**60**
1	65 729	70 748	29 252	94 982	59
2	65 754	70 779	29 221	94 975	58
3	65 779	70 810	29 190	94 969	57
4	65 804	70 841	29 159	94 962	56
5	65 828	70 873	29 127	94 956	**55**
6	65 853	70 904	29 096	94 949	54
7	65 878	70 935	29 065	94 943	53
8	65 902	70 966	29 034	94 936	52
9	65 927	70 997	29 003	94 930	51
10	65 952	71 028	28 972	94 923	**50**
11	65 976	71 059	28 941	94 917	49
12	66 001	71 090	28 910	94 911	48
13	66 025	71 121	28 879	94 904	47
14	66 050	71 153	28 847	94 898	46
15	66 075	71 184	28 816	94 891	**45**
16	66 099	71 215	28 785	94 885	44
17	66 124	71 246	28 754	94 878	43
18	66 148	71 277	28 723	94 871	42
19	66 173	71 308	28 692	94 865	41
20	66 197	71 339	28 661	94 858	**40**
21	66 221	71 370	28 630	94 852	39
22	66 246	71 401	28 599	94 845	38
23	66 270	71 431	28 569	94 839	37
24	66 295	71 462	28 538	94 832	36
25	66 319	71 493	28 507	94 826	**35**
26	66 343	71 524	28 476	94 819	34
27	66 368	71 555	28 445	94 813	33
28	66 392	71 586	28 414	94 806	32
29	66 416	71 617	28 383	94 799	31
30	66 441	71 648	28 352	94 793	**30**
31	66 465	71 679	28 321	94 786	29
32	66 489	71 709	28 291	94 780	28
33	66 513	71 740	28 260	94 773	27
34	66 537	71 771	28 229	94 767	26
35	66 562	71 802	28 198	94 760	**25**
36	66 586	71 833	28 167	94 753	24
37	66 610	71 863	28 137	94 747	23
38	66 634	71 894	28 106	94 740	22
39	66 658	71 925	28 075	94 734	21
40	66 682	71 955	28 045	94 727	**20**
41	66 706	71 986	28 014	94 720	19
42	66 731	72 017	27 983	94 714	18
43	66 755	72 048	27 952	94 707	17
44	66 779	72 078	27 922	94 700	16
45	66 803	72 109	27 891	94 694	**15**
46	66 827	72 140	27 860	94 687	14
47	66 851	72 170	27 830	94 680	13
48	66 875	72 201	27 799	94 674	12
49	66 899	72 231	27 769	94 667	11
50	66 922	72 262	27 738	94 660	**10**
51	66 946	72 293	27 707	94 654	9
52	66 970	72 323	27 677	94 647	8
53	66 994	72 354	27 646	94 640	7
54	67 018	72 384	27 616	94 634	6
55	67 042	72 415	27 585	94 627	**5**
56	67 066	72 445	27 555	94 620	4
57	67 090	72 476	27 524	94 614	3
58	67 113	72 506	27 494	94 607	2
59	67 137	72 537	27 463	94 600	1
60	67 161	72 567	27 433	94 593	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

62°

28°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	67 161	72 567	27 433	94 593	**60**
1	67 185	72 598	27 402	94 587	59
2	67 208	72 628	27 372	94 580	58
3	67 232	72 659	27 341	94 573	57
4	67 256	72 689	27 311	94 567	56
5	67 280	72 720	27 280	94 560	**55**
6	67 303	72 750	27 250	94 553	54
7	67 327	72 780	27 220	94 546	53
8	67 350	72 811	27 189	94 540	52
9	67 374	72 841	27 159	94 533	51
10	67 398	72 872	27 128	94 526	**50**
11	67 421	72 902	27 098	94 519	49
12	67 445	72 932	27 068	94 513	48
13	67 468	72 963	27 037	94 506	47
14	67 492	72 993	27 007	94 499	46
15	67 515	73 023	26 977	94 492	**45**
16	67 539	73 054	26 946	94 485	44
17	67 562	73 084	26 916	94 479	43
18	67 586	73 114	26 886	94 472	42
19	67 609	73 144	26 856	94 465	41
20	67 633	73 175	26 825	94 458	**40**
21	67 656	73 205	26 795	94 451	39
22	67 680	73 235	26 765	94 445	38
23	67 703	73 265	26 735	94 438	37
24	67 726	73 295	26 705	94 431	36
25	67 750	73 326	26 674	94 424	**35**
26	67 773	73 356	26 644	94 417	34
27	67 796	73 386	26 614	94 410	33
28	67 820	73 416	26 584	94 404	32
29	67 843	73 446	26 554	94 397	31
30	67 866	73 476	26 524	94 390	**30**
31	67 890	73 507	26 493	94 383	29
32	67 913	73 537	26 463	94 376	28
33	67 936	73 567	26 433	94 369	27
34	67 959	73 597	26 403	94 362	26
35	67 982	73 627	26 373	94 355	**25**
36	68 006	73 657	26 343	94 349	24
37	68 029	73 687	26 313	94 342	23
38	68 052	73 717	26 283	94 335	22
39	68 075	73 747	26 253	94 328	21
40	68 098	73 777	26 223	94 321	**20**
41	68 121	73 807	26 193	94 314	19
42	68 144	73 837	26 163	94 307	18
43	68 167	73 867	26 133	94 300	17
44	68 190	73 897	26 103	94 293	16
45	68 213	73 927	26 073	94 286	**15**
46	68 237	73 957	26 043	94 279	14
47	68 260	73 987	26 013	94 273	13
48	68 283	74 017	25 983	94 266	12
49	68 305	74 047	25 953	94 259	11
50	68 328	74 077	25 923	94 252	**10**
51	68 351	74 107	25 893	94 245	9
52	68 374	74 137	25 863	94 238	8
53	68 397	74 166	25 834	94 231	7
54	68 420	74 196	25 804	94 224	6
55	68 443	74 226	25 774	94 217	**5**
56	68 466	74 256	25 744	94 210	4
57	68 489	74 286	25 714	94 203	3
58	68 512	74 316	25 684	94 196	2
59	68 534	74 345	25 655	94 189	1
60	68 557	74 375	25 625	94 182	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

61°

29°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	68 557	74 375	25 625	94 182	**60**
1	68 580	74 405	25 595	94 175	59
2	68 603	74 435	25 565	94 168	58
3	68 625	74 465	25 535	94 161	57
4	68 648	74 494	25 506	94 154	56
5	68 671	74 524	25 476	94 147	**55**
6	68 694	74 554	25 446	94 140	54
7	68 716	74 583	25 417	94 133	53
8	68 739	74 613	25 387	94 126	52
9	68 762	74 643	25 357	94 119	51
10	68 784	74 673	25 327	94 112	**50**
11	68 807	74 702	25 298	94 105	49
12	68 829	74 732	25 268	94 098	48
13	68 852	74 762	25 238	94 090	47
14	68 875	74 791	25 209	94 083	46
15	68 897	74 821	25 179	94 076	**45**
16	68 920	74 851	25 149	94 069	44
17	68 942	74 880	25 120	94 062	43
18	68 965	74 910	25 090	94 055	42
19	68 987	74 939	25 061	94 048	41
20	69 010	74 969	25 031	94 041	**40**
21	69 032	74 998	25 002	94 034	39
22	69 055	75 028	24 972	94 027	38
23	69 077	75 058	24 942	94 020	37
24	69 100	75 087	24 913	94 012	36
25	69 122	75 117	24 883	94 005	**35**
26	69 144	75 146	24 854	93 998	34
27	69 167	75 176	24 824	93 991	33
28	69 189	75 205	24 795	93 984	32
29	69 212	75 235	24 765	93 977	31
30	69 234	75 264	24 736	93 970	**30**
31	69 256	75 294	24 706	93 963	29
32	69 279	75 323	24 677	93 955	28
33	69 301	75 353	24 647	93 948	27
34	69 323	75 382	24 618	93 941	26
35	69 345	75 411	24 589	93 934	**25**
36	69 368	75 441	24 559	93 927	24
37	69 390	75 470	24 530	93 920	23
38	69 412	75 500	24 500	93 912	22
39	69 434	75 529	24 471	93 905	21
40	69 456	75 558	24 442	93 898	**20**
41	69 479	75 588	24 412	93 891	19
42	69 501	75 617	24 383	93 884	18
43	69 523	75 647	24 353	93 876	17
44	69 545	75 676	24 324	93 869	16
45	69 567	75 705	24 295	93 862	**15**
46	69 589	75 735	24 265	93 855	14
47	69 611	75 764	24 236	93 847	13
48	69 633	75 793	24 207	93 840	12
49	69 655	75 822	24 178	93 833	11
50	69 677	75 852	24 148	93 826	**10**
51	69 699	75 881	24 119	93 819	9
52	69 721	75 910	24 090	93 811	8
53	69 743	75 939	24 061	93 804	7
54	69 765	75 969	24 031	93 797	6
55	69 787	75 998	24 002	93 789	**5**
56	69 809	76 027	23 973	93 782	4
57	69 831	76 056	23 944	93 775	3
58	69 853	76 086	23 914	93 768	2
59	69 875	76 115	23 885	93 760	1
60	69 897	76 144	23 856	93 753	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

60°

30°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	69 897	76 144	23 856	93 753	**60**
1	69 919	76 173	23 827	93 746	59
2	69 941	76 202	23 798	93 738	58
3	69 963	76 231	23 769	93 731	57
4	69 984	76 261	23 739	93 724	56
5	70 006	76 290	23 710	93 717	**55**
6	70 028	76 319	23 681	93 709	54
7	70 050	76 348	23 652	93 702	53
8	70 072	76 377	23 623	93 695	52
9	70 093	76 406	23 594	93 687	51
10	70 115	76 435	23 565	93 680	**50**
11	70 137	76 464	23 536	93 673	49
12	70 159	76 493	23 507	93 665	48
13	70 180	76 522	23 478	93 658	47
14	70 202	76 551	23 449	93 650	46
15	70 224	76 580	23 420	93 643	**45**
16	70 245	76 609	23 391	93 636	44
17	70 267	76 639	23 361	93 628	43
18	70 288	76 668	23 332	93 621	42
19	70 310	76 697	23 303	93 614	41
20	70 332	76 725	23 275	93 606	**40**
21	70 353	76 754	23 246	93 599	39
22	70 375	76 783	23 217	93 591	38
23	70 396	76 812	23 188	93 584	37
24	70 418	76 841	23 159	93 577	36
25	70 439	76 870	23 130	93 569	**35**
26	70 461	76 899	23 101	93 562	34
27	70 482	76 928	23 072	93 554	33
28	70 504	76 957	23 043	93 547	32
29	70 525	76 986	23 014	93 539	31
30	70 547	77 015	22 985	93 532	**30**
31	70 568	77 044	22 956	93 525	29
32	70 590	77 073	22 927	93 517	28
33	70 611	77 101	22 899	93 510	27
34	70 633	77 130	22 870	93 502	26
35	70 654	77 159	22 841	93 495	**25**
36	70 675	77 188	22 812	93 487	24
37	70 697	77 217	22 783	93 480	23
38	70 718	77 246	22 754	93 472	22
39	70 739	77 274	22 726	93 465	21
40	70 761	77 303	22 697	93 457	**20**
41	70 782	77 332	22 668	93 450	19
42	70 803	77 361	22 639	93 442	18
43	70 824	77 390	22 610	93 435	17
44	70 846	77 418	22 582	93 427	16
45	70 867	77 447	22 553	93 420	**15**
46	70 888	77 476	22 524	93 412	14
47	70 909	77 505	22 495	93 405	13
48	70 931	77 533	22 467	93 397	12
49	70 952	77 562	22 438	93 390	11
50	70 973	77 591	22 409	93 382	**10**
51	70 994	77 619	22 381	93 375	9
52	71 015	77 648	22 352	93 367	8
53	71 036	77 677	22 323	93 360	7
54	71 058	77 706	22 294	93 352	6
55	71 079	77 734	22 266	93 344	**5**
56	71 100	77 763	22 237	93 337	4
57	71 121	77 791	22 209	93 329	3
58	71 142	77 820	22 180	93 322	2
59	71 163	77 849	22 151	93 314	1
60	71 184	77 877	22 123	93 307	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

59°

31°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	71 184	77 877	22 123	93 307	**60**
1	71 205	77 906	22 094	93 299	59
2	71 226	77 935	22 065	93 291	58
3	71 247	77 963	22 037	93 284	57
4	71 268	77 992	22 008	93 276	56
5	71 289	78 020	21 980	93 269	**55**
6	71 310	78 049	21 951	93 261	54
7	71 331	78 077	21 923	93 253	53
8	71 352	78 106	21 894	93 246	52
9	71 373	78 135	21 865	93 238	51
10	71 393	78 163	21 837	93 230	**50**
11	71 414	78 192	21 808	93 223	49
12	71 435	78 220	21 780	93 215	48
13	71 456	78 249	21 751	93 207	47
14	71 477	78 277	21 723	93 200	46
15	71 498	78 306	21 694	93 192	**45**
16	71 519	78 334	21 666	93 184	44
17	71 539	78 363	21 637	93 177	43
18	71 560	78 391	21 609	93 169	42
19	71 581	78 419	21 581	93 161	41
20	71 602	78 448	21 552	93 154	**40**
21	71 622	78 476	21 524	93 146	39
22	71 643	78 505	21 495	93 138	38
23	71 664	78 533	21 467	93 131	37
24	71 685	78 562	21 438	93 123	36
25	71 705	78 590	21 410	93 115	**35**
26	71 726	78 618	21 382	93 108	34
27	71 747	78 647	21 353	93 100	33
28	71 767	78 675	21 325	93 092	32
29	71 788	78 704	21 296	93 084	31
30	71 809	78 732	21 268	93 077	**30**
31	71 829	78 760	21 240	93 069	29
32	71 850	78 789	21 211	93 061	28
33	71 870	78 817	21 183	93 053	27
34	71 891	78 845	21 155	93 046	26
35	71 911	78 874	21 126	93 038	**25**
36	71 932	78 902	21 098	93 030	24
37	71 952	78 930	21 070	93 022	23
38	71 973	78 959	21 041	93 014	22
39	71 994	78 987	21 013	93 007	21
40	72 014	79 015	20 985	92 999	**20**
41	72 034	79 043	20 957	92 991	19
42	72 055	79 072	20 928	92 983	18
43	72 075	79 100	20 900	92 976	17
44	72 096	79 128	20 872	92 968	16
45	72 116	79 156	20 844	92 960	**15**
46	72 137	79 185	20 815	92 952	14
47	72 157	79 213	20 787	92 944	13
48	72 177	79 241	20 759	92 936	12
49	72 198	79 269	20 731	92 929	11
50	72 218	79 297	20 703	92 921	**10**
51	72 238	79 326	20 674	92 913	9
52	72 259	79 354	20 646	92 905	8
53	72 279	79 382	20 618	92 897	7
54	72 299	79 410	20 590	92 889	6
55	72 320	79 438	20 562	92 881	**5**
56	72 340	79 466	20 534	92 874	4
57	72 360	79 495	20 505	92 866	3
58	72 381	79 523	20 477	92 858	2
59	72 401	79 551	20 449	92 850	1
60	72 421	79 579	20 421	92 842	**0**
′	9 log cos	9 log cot	10 log tan	9 log sin	′

58°

32°

′	log sin 9	log tan 9	log cot 10	log cos 9	′
0	72 421	79 579	20 421	92 842	**60**
1	72 441	79 607	20 393	92 834	59
2	72 461	79 635	20 365	92 826	58
3	72 482	79 663	20 337	92 818	57
4	72 502	79 691	20 309	92 810	56
5	72 522	79 719	20 281	92 803	**55**
6	72 542	79 747	20 253	92 795	54
7	72 562	79 776	20 224	92 787	53
8	72 582	79 804	20 196	92 779	52
9	72 602	79 832	20 168	92 771	51
10	72 622	79 860	20 140	92 763	**50**
11	72 643	79 888	20 112	92 755	49
12	72 663	79 916	20 084	92 747	48
13	72 683	79 944	20 056	92 739	47
14	72 703	79 972	20 028	92 731	46
15	72 723	80 000	20 000	92 723	**45**
16	72 743	80 028	19 972	92 715	44
17	72 763	80 056	19 944	92 707	43
18	72 783	80 084	19 916	92 699	42
19	72 803	80 112	19 888	92 691	41
20	72 823	80 140	19 860	92 683	**40**
21	72 843	80 168	19 832	92 675	39
22	72 863	80 195	19 805	92 667	38
23	72 883	80 223	19 777	92 659	37
24	72 902	80 251	19 749	92 651	36
25	72 922	80 279	19 721	92 643	**35**
26	72 942	80 307	19 693	92 635	34
27	72 962	80 335	19 665	92 627	33
28	72 982	80 363	19 637	92 619	32
29	73 002	80 391	19 609	92 611	31
30	73 022	80 419	19 581	92 603	**30**
31	73 041	80 447	19 553	92 595	29
32	73 061	80 474	19 526	92 587	28
33	73 081	80 502	19 498	92 579	27
34	73 101	80 530	19 470	92 571	26
35	73 121	80 558	19 442	92 563	**25**
36	73 140	80 586	19 414	92 555	24
37	73 160	80 614	19 386	92 546	23
38	73 180	80 642	19 358	92 538	22
39	73 200	80 669	19 331	92 530	21
40	73 219	80 697	19 303	92 522	**20**
41	73 239	80 725	19 275	92 514	19
42	73 259	80 753	19 247	92 506	18
43	73 278	80 781	19 219	92 498	17
44	73 298	80 808	19 192	92 490	16
45	73 318	80 836	19 164	92 482	**15**
46	73 337	80 864	19 136	92 473	14
47	73 357	80 892	19 108	92 465	13
48	73 377	80 919	19 081	92 457	12
49	73 396	80 947	19 053	92 449	11
50	73 416	80 975	19 025	92 441	**10**
51	73 435	81 003	18 997	92 433	9
52	73 455	81 030	18 970	92 425	8
53	73 474	81 058	18 942	92 416	7
54	73 494	81 086	18 914	92 408	6
55	73 513	81 113	18 887	92 400	**5**
56	73 533	81 141	18 859	92 392	4
57	73 552	81 169	18 831	92 384	3
58	73 572	81 196	18 804	92 376	2
59	73 591	81 224	18 776	92 367	1
60	73 611	81 252	18 748	92 359	**0**
′	9 log cos	9 log cot	10 log tan	9 log sin	′

57°

33°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	73 611	81 252	18 748	92 359	**60**
1	73 630	81 279	18 721	92 351	59
2	73 650	81 307	18 693	92 343	58
3	73 669	81 335	18 665	92 335	57
4	73 689	81 362	18 638	92 326	56
5	73 708	81 390	18 610	92 318	**55**
6	73 727	81 418	18 582	92 310	54
7	73 747	81 445	18 555	92 302	53
8	73 766	81 473	18 527	92 293	52
9	73 785	81 500	18 500	92 285	51
10	73 805	81 528	18 472	92 277	**50**
11	73 824	81 556	18 444	92 269	49
12	73 843	81 583	18 417	92 260	48
13	73 863	81 611	18 389	92 252	47
14	73 882	81 638	18 362	92 244	46
15	73 901	81 666	18 334	92 235	**45**
16	73 921	81 693	18 307	92 227	44
17	73 940	81 721	18 279	92 219	43
18	73 959	81 748	18 252	92 211	42
19	73 978	81 776	18 224	92 202	41
20	73 997	81 803	18 197	92 194	**40**
21	74 017	81 831	18 169	92 186	39
22	74 036	81 858	18 142	92 177	38
23	74 055	81 886	18 114	92 169	37
24	74 074	81 913	18 087	92 161	36
25	74 093	81 941	18 059	92 152	**35**
26	74 113	81 968	18 032	92 144	34
27	74 132	81 996	18 004	92 136	33
28	74 151	82 023	17 977	92 127	32
29	74 170	82 051	17 949	92 119	31
30	74 189	82 078	17 922	92 111	**30**
31	74 208	82 106	17 894	92 102	29
32	74 227	82 133	17 867	92 094	28
33	74 246	82 161	17 839	92 086	27
34	74 265	82 188	17 812	92 077	26
35	74 284	82 215	17 785	92 069	**25**
36	74 303	82 243	17 757	92 060	24
37	74 322	82 270	17 730	92 052	23
38	74 341	82 298	17 702	92 044	22
39	74 360	82 325	17 675	92 035	21
40	74 379	82 352	17 648	92 027	**20**
41	74 398	82 380	17 620	92 018	19
42	74 417	82 407	17 593	92 010	18
43	74 436	82 435	17 565	92 002	17
44	74 455	82 462	17 538	91 993	16
45	74 474	82 489	17 511	91 985	**15**
46	74 493	82 517	17 483	91 976	14
47	74 512	82 544	17 456	91 968	13
48	74 531	82 571	17 429	91 959	12
49	74 549	82 599	17 401	91 951	11
50	74 568	82 626	17 374	91 942	**10**
51	74 587	82 653	17 347	91 934	9
52	74 606	82 681	17 319	91 925	8
53	74 625	82 708	17 292	91 917	7
54	74 644	82 735	17 265	91 908	6
55	74 662	82 762	17 238	91 900	**5**
56	74 681	82 790	17 210	91 891	4
57	74 700	82 817	17 183	91 883	3
58	74 719	82 844	17 156	91 874	2
59	74 737	82 871	17 129	91 866	1
60	74 756	82 899	17 101	91 857	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

56°

34°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	74 756	82 899	17 101	91 857	**60**
1	74 775	82 926	17 074	91 849	59
2	74 794	82 953	17 047	91 840	58
3	74 812	82 980	17 020	91 832	57
4	74 831	83 008	16 992	91 823	56
5	74 850	83 035	16 965	91 815	**55**
6	74 868	83 062	16 938	91 806	54
7	74 887	83 089	16 911	91 798	53
8	74 906	83 117	16 883	91 789	52
9	74 924	83 144	16 856	91 781	51
10	74 943	83 171	16 829	91 772	**50**
11	74 961	83 198	16 802	91 763	49
12	74 980	83 225	16 775	91 755	48
13	74 999	83 252	16 748	91 746	47
14	75 017	83 280	16 720	91 738	46
15	75 036	83 307	16 693	91 729	**45**
16	75 054	83 334	16 666	91 720	44
17	75 073	83 361	16 639	91 712	43
18	75 091	83 388	16 612	91 703	42
19	75 110	83 415	16 585	91 695	41
20	75 128	83 442	16 558	91 686	**40**
21	75 147	83 470	16 530	91 677	39
22	75 165	83 497	16 503	91 669	38
23	75 184	83 524	16 476	91 660	37
24	75 202	83 551	16 449	91 651	36
25	75 221	83 578	16 422	91 643	**35**
26	75 239	83 605	16 395	91 634	34
27	75 258	83 632	16 368	91 625	33
28	75 276	83 659	16 341	91 617	32
29	75 294	83 686	16 314	91 608	31
30	75 313	83 713	16 287	91 599	**30**
31	75 331	83 740	16 260	91 591	29
32	75 350	83 768	16 232	91 582	28
33	75 368	83 795	16 205	91 573	27
34	75 386	83 822	16 178	91 565	26
35	75 405	83 849	16 151	91 556	**25**
36	75 423	83 876	16 124	91 547	24
37	75 441	83 903	16 097	91 538	23
38	75 459	83 930	16 070	91 530	22
39	75 478	83 957	16 043	91 521	21
40	75 496	83 984	16 016	91 512	**20**
41	75 514	84 011	15 989	91 504	19
42	75 533	84 038	15 962	91 495	18
43	75 551	84 065	15 935	91 486	17
44	75 569	84 092	15 908	91 477	16
45	75 587	84 119	15 881	91 469	**15**
46	75 605	84 146	15 854	91 460	14
47	75 624	84 173	15 827	91 451	13
48	75 642	84 200	15 800	91 442	12
49	75 660	84 227	15 773	91 433	11
50	75 678	84 254	15 746	91 425	**10**
51	75 696	84 280	15 720	91 416	9
52	75 714	84 307	15 693	91 407	8
53	75 733	84 334	15 666	91 398	7
54	75 751	84 361	15 639	91 389	6
55	75 769	84 388	15 612	91 381	**5**
56	75 787	84 415	15 585	91 372	4
57	75 805	84 442	15 558	91 363	3
58	75 823	84 469	15 531	91 354	2
59	75 841	84 496	15 504	91 345	1
60	75 859	84 523	15 477	91 336	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

55°

35°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	75 859	84 523	15 477	91 336	60
1	75 877	84 550	15 450	91 328	59
2	75 895	84 576	15 424	91 319	58
3	75 913	84 603	15 397	91 310	57
4	75 931	84 630	15 370	91 301	56
5	75 949	84 657	15 343	91 292	55
6	75 967	84 684	15 316	91 283	54
7	75 985	84 711	15 289	91 274	53
8	76 003	84 738	15 262	91 266	52
9	76 021	84 764	15 236	91 257	51
10	76 039	84 791	15 209	91 248	50
11	76 057	84 818	15 182	91 239	49
12	76 075	84 845	15 155	91 230	48
13	76 093	84 872	15 128	91 221	47
14	76 111	84 899	15 101	91 212	46
15	76 129	84 925	15 075	91 203	45
16	76 146	84 952	15 048	91 194	44
17	76 164	84 979	15 021	91 185	43
18	76 182	85 006	14 994	91 176	42
19	76 200	85 033	14 967	91 167	41
20	76 218	85 059	14 941	91 158	40
21	76 236	85 086	14 914	91 149	39
22	76 253	85 113	14 887	91 141	38
23	76 271	85 140	14 860	91 132	37
24	76 289	85 166	14 834	91 123	36
25	76 307	85 193	14 807	91 114	35
26	76 324	85 220	14 780	91 105	34
27	76 342	85 247	14 753	91 096	33
28	76 360	85 273	14 727	91 087	32
29	76 378	85 300	14 700	91 078	31
30	76 395	85 327	14 673	91 069	30
31	76 413	85 354	14 646	91 060	29
32	76 431	85 380	14 620	91 051	28
33	76 448	85 407	14 593	91 042	27
34	76 466	85 434	14 566	91 033	26
35	76 484	85 460	14 540	91 023	25
36	76 501	85 487	14 513	91 014	24
37	76 519	85 514	14 486	91 005	23
38	76 537	85 540	14 460	90 996	22
39	76 554	85 567	14 433	90 987	21
40	76 572	85 594	14 406	90 978	20
41	76 590	85 620	14 380	90 969	19
42	76 607	85 647	14 353	90 960	18
43	76 625	85 674	14 326	90 951	17
44	76 642	85 700	14 300	90 942	16
45	76 660	85 727	14 273	90 933	15
46	76 677	85 754	14 246	90 924	14
47	76 695	85 780	14 220	90 915	13
48	76 712	85 807	14 193	90 906	12
49	76 730	85 834	14 166	90 896	11
50	76 747	85 860	14 140	90 887	10
51	76 765	85 887	14 113	90 878	9
52	76 782	85 913	14 087	90 869	8
53	76 800	85 940	14 060	90 860	7
54	76 817	85 967	14 033	90 851	6
55	76 835	85 993	14 007	90 842	5
56	76 852	86 020	13 980	90 832	4
57	76 870	86 046	13 954	90 823	3
58	76 887	86 073	13 927	90 814	2
59	76 904	86 100	13 900	90 805	1
60	76 922	86 126	13 874	90 796	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

54°

36°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	76 922	86 126	13 874	90 796	60
1	76 939	86 153	13 847	90 787	59
2	76 957	86 179	13 821	90 777	58
3	76 974	86 206	13 794	90 768	57
4	76 991	86 232	13 768	90 759	56
5	77 009	86 259	13 741	90 750	55
6	77 026	86 285	13 715	90 741	54
7	77 043	86 312	13 688	90 731	53
8	77 061	86 338	13 662	90 722	52
9	77 078	86 365	13 635	90 713	51
10	77 095	86 392	13 608	90 704	50
11	77 112	86 418	13 582	90 694	49
12	77 130	86 445	13 555	90 685	48
13	77 147	86 471	13 529	90 676	47
14	77 164	86 498	13 502	90 667	46
15	77 181	86 524	13 476	90 657	45
16	77 199	86 551	13 449	90 648	44
17	77 216	86 577	13 423	90 639	43
18	77 233	86 603	13 397	90 630	42
19	77 250	86 630	13 370	90 620	41
20	77 268	86 656	13 344	90 611	40
21	77 285	86 683	13 317	90 602	39
22	77 302	86 709	13 291	90 592	38
23	77 319	86 736	13 264	90 583	37
24	77 336	86 762	13 238	90 574	36
25	77 353	86 789	13 211	90 565	35
26	77 370	86 815	13 185	90 555	34
27	77 387	86 842	13 158	90 546	33
28	77 405	86 868	13 132	90 537	32
29	77 422	86 894	13 106	90 527	31
30	77 439	86 921	13 079	90 518	30
31	77 456	86 947	13 053	90 509	29
32	77 473	86 974	13 026	90 499	28
33	77 490	87 000	13 000	90 490	27
34	77 507	87 027	12 973	90 480	26
35	77 524	87 053	12 947	90 471	25
36	77 541	87 079	12 921	90 462	24
37	77 558	87 106	12 894	90 452	23
38	77 575	87 132	12 868	90 443	22
39	77 592	87 158	12 842	90 434	21
40	77 609	87 185	12 815	90 424	20
41	77 626	87 211	12 789	90 415	19
42	77 643	87 238	12 762	90 405	18
43	77 660	87 264	12 736	90 396	17
44	77 677	87 290	12 710	90 386	16
45	77 694	87 317	12 683	90 377	15
46	77 711	87 343	12 657	90 368	14
47	77 728	87 369	12 631	90 358	13
48	77 744	87 396	12 604	90 349	12
49	77 761	87 422	12 578	90 339	11
50	77 778	87 448	12 552	90 330	10
51	77 795	87 475	12 525	90 320	9
52	77 812	87 501	12 499	90 311	8
53	77 829	87 527	12 473	90 301	7
54	77 846	87 554	12 446	90 292	6
55	77 862	87 580	12 420	90 282	5
56	77 879	87 606	12 394	90 273	4
57	77 896	87 633	12 367	90 263	3
58	77 913	87 659	12 341	90 254	2
59	77 930	87 685	12 315	90 244	1
60	77 946	87 711	12 289	90 235	0
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

53°

37°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	77 946	87 711	12 289	90 235	**60**
1	77 963	87 738	12 262	90 225	59
2	77 980	87 764	12 236	90 216	58
3	77 997	87 790	12 210	90 206	57
4	78 013	87 817	12 183	90 197	56
5	78 030	87 843	12 157	90 187	**55**
6	78 047	87 869	12 131	90 178	54
7	78 063	87 895	12 105	90 168	53
8	78 080	87 922	12 078	90 159	52
9	78 097	87 948	12 052	90 149	51
10	78 113	87 974	12 026	90 139	**50**
11	78 130	88 000	12 000	90 130	49
12	78 147	88 027	11 973	90 120	48
13	78 163	88 053	11 947	90 111	47
14	78 180	88 079	11 921	90 101	46
15	78 197	88 105	11 895	90 091	**45**
16	78 213	88 131	11 869	90 082	44
17	78 230	88 158	11 842	90 072	43
18	78 246	88 184	11 816	90 063	42
19	78 263	88 210	11 790	90 053	41
20	78 280	88 236	11 764	90 043	**40**
21	78 296	88 262	11 738	90 034	39
22	78 313	88 289	11 711	90 024	38
23	78 329	88 315	11 685	90 014	37
24	78 346	88 341	11 659	90 005	36
25	78 362	88 367	11 633	89 995	**35**
26	78 379	88 393	11 607	89 985	34
27	78 395	88 420	11 580	89 976	33
28	78 412	88 446	11 554	89 966	32
29	78 428	88 472	11 528	89 956	31
30	78 445	88 498	11 502	89 947	**30**
31	78 461	88 524	11 476	89 937	29
32	78 478	88 550	11 450	89 927	28
33	78 494	88 577	11 423	89 918	27
34	78 510	88 603	11 397	89 908	26
35	78 527	88 629	11 371	89 898	**25**
36	78 543	88 655	11 345	89 888	24
37	78 560	88 681	11 319	89 879	23
38	78 576	88 707	11 293	89 869	22
39	78 592	88 733	11 267	89 859	21
40	78 609	88 759	11 241	89 849	**20**
41	78 625	88 786	11 214	89 840	19
42	78 642	88 812	11 188	89 830	18
43	78 658	88 838	11 162	89 820	17
44	78 674	88 864	11 136	89 810	16
45	78 691	88 890	11 110	89 801	**15**
46	78 707	88 916	11 084	89 791	14
47	78 723	88 942	11 058	89 781	13
48	78 739	88 968	11 032	89 771	12
49	78 756	88 994	11 006	89 761	11
50	78 772	89 020	10 980	89 752	**10**
51	78 788	89 046	10 954	89 742	9
52	78 805	89 073	10 927	89 732	8
53	78 821	89 099	10 901	89 722	7
54	78 837	89 125	10 875	89 712	6
55	78 853	89 151	10 849	89 702	**5**
56	78 869	89 177	10 823	89 693	4
57	78 886	89 203	10 797	89 683	3
58	78 902	89 229	10 771	89 673	2
59	78 918	89 255	10 745	89 663	1
60	78 934	89 281	10 719	89 653	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

52°

38°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	78 934	89 281	10 719	89 653	**60**
1	78 950	89 307	10 693	89 643	59
2	78 967	89 333	10 667	89 633	58
3	78 983	89 359	10 641	89 624	57
4	78 999	89 385	10 615	89 614	56
5	79 015	89 411	10 589	89 604	**55**
6	79 031	89 437	10 563	89 594	54
7	79 047	89 463	10 537	89 584	53
8	79 063	89 489	10 511	89 574	52
9	79 079	89 515	10 485	89 564	51
10	79 095	89 541	10 459	89 554	**50**
11	79 111	89 567	10 433	89 544	49
12	79 128	89 593	10 407	89 534	48
13	79 144	89 619	10 381	89 524	47
14	79 160	89 645	10 355	89 514	46
15	79 176	89 671	10 329	89 504	**45**
16	79 192	89 697	10 303	89 495	44
17	79 208	89 723	10 277	89 485	43
18	79 224	89 749	10 251	89 475	42
19	79 240	89 775	10 225	89 465	41
20	79 256	89 801	10 199	89 455	**40**
21	79 272	89 827	10 173	89 445	39
22	79 288	89 853	10 147	89 435	38
23	79 304	89 879	10 121	89 425	37
24	79 319	89 905	10 095	89 415	36
25	79 335	89 931	10 069	89 405	**35**
26	79 351	89 957	10 043	89 395	34
27	79 367	89 983	10 017	89 385	33
28	79 383	90 009	09 991	89 375	32
29	79 399	90 035	09 965	89 364	31
30	79 415	90 061	09 939	89 354	**30**
31	79 431	90 086	09 914	89 344	29
32	79 447	90 112	09 888	89 334	28
33	79 463	90 138	09 862	89 324	27
34	79 478	90 164	09 836	89 314	26
35	79 494	90 190	09 810	89 304	**25**
36	79 510	90 216	09 784	89 294	24
37	79 526	90 242	09 758	89 284	23
38	79 542	90 268	09 732	89 274	22
39	79 558	90 294	09 706	89 264	21
40	79 573	90 320	09 680	89 254	**20**
41	79 589	90 346	09 654	89 244	19
42	79 605	90 371	09 629	89 233	18
43	79 621	90 397	09 603	89 223	17
44	79 636	90 423	09 577	89 213	16
45	79 652	90 449	09 551	89 203	**15**
46	79 668	90 475	09 525	89 193	14
47	79 684	90 501	09 499	89 183	13
48	79 699	90 527	09 473	89 173	12
49	79 715	90 553	09 447	89 162	11
50	79 731	90 578	09 422	89 152	**10**
51	79 746	90 604	09 396	89 142	9
52	79 762	90 630	09 370	89 132	8
53	79 778	90 656	09 344	89 122	7
54	79 793	90 682	09 318	89 112	6
55	79 809	90 708	09 292	89 101	**5**
56	79 825	90 734	09 266	89 091	4
57	79 840	90 759	09 241	89 081	3
58	79 856	90 785	09 215	89 071	2
59	79 872	90 811	09 189	89 060	1
60	79 887	90 837	09 163	89 050	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

51°

39°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	79 887	90 837	09 163	89 050	**60**
1	79 903	90 863	09 137	89 040	59
2	79 918	90 889	09 111	89 030	58
3	79 934	90 914	09 086	89 020	57
4	79 950	90 940	09 060	89 009	56
5	79 965	90 966	09 034	88 999	**55**
6	79 981	90 992	09 008	88 989	54
7	79 996	91 018	08 982	88 978	53
8	80 012	91 043	08 957	88 968	52
9	80 027	91 069	08 931	88 958	51
10	80 043	91 095	08 905	88 948	**50**
11	80 058	91 121	08 879	88 937	49
12	80 074	91 147	08 853	88 927	48
13	80 089	91 172	08 828	88 917	47
14	80 105	91 198	08 802	88 906	46
15	80 120	91 224	08 776	88 896	**45**
16	80 136	91 250	08 750	88 886	44
17	80 151	91 276	08 724	88 875	43
18	80 166	91 301	08 699	88 865	42
19	80 182	91 327	08 673	88 855	41
20	80 197	91 353	08 647	88 844	**40**
21	80 213	91 379	08 621	88 834	39
22	80 228	91 404	08 596	88 824	38
23	80 244	91 430	08 570	88 813	37
24	80 259	91 456	08 544	88 803	36
25	80 274	91 482	08 518	88 793	**35**
26	80 290	91 507	08 493	88 782	34
27	80 305	91 533	08 467	88 772	33
28	80 320	91 559	08 441	88 761	32
29	80 336	91 585	08 415	88 751	31
30	80 351	91 610	08 390	88 741	**30**
31	80 366	91 636	08 364	88 730	29
32	80 382	91 662	08 338	88 720	28
33	80 397	91 688	08 312	88 709	27
34	80 412	91 713	08 287	88 699	26
35	80 428	91 739	08 261	88 688	**25**
36	80 443	91 765	08 235	88 678	24
37	80 458	91 791	08 209	88 668	23
38	80 473	91 816	08 184	88 657	22
39	80 489	91 842	08 158	88 647	21
40	80 504	91 868	08 132	88 636	**20**
41	80 519	91 893	08 107	88 626	19
42	80 534	91 919	08 081	88 615	18
43	80 550	91 945	08 055	88 605	17
44	80 565	91 971	08 029	88 594	16
45	80 580	91 996	08 004	88 584	**15**
46	80 595	92 022	07 978	88 573	14
47	80 610	92 048	07 952	88 563	13
48	80 625	92 073	07 927	88 552	12
49	80 641	92 099	07 901	88 542	11
50	80 656	92 125	07 875	88 531	**10**
51	80 671	92 150	07 850	88 521	9
52	80 686	92 176	07 824	88 510	8
53	80 701	92 202	07 798	88 499	7
54	80 716	92 227	07 773	88 489	6
55	80 731	92 253	07 747	88 478	**5**
56	80 746	92 279	07 721	88 468	4
57	80 762	92 304	07 696	88 457	3
58	80 777	92 330	07 670	88 447	2
59	80 792	92 356	07 644	88 436	1
60	80 807	92 381	07 619	88 425	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

50°

40°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	80 807	92 381	07 619	88 425	**60**
1	80 822	92 407	07 593	88 415	59
2	80 837	92 433	07 567	88 404	58
3	80 852	92 458	07 542	88 394	57
4	80 867	92 484	07 516	88 383	56
5	80 882	92 510	07 490	88 372	**55**
6	80 897	92 535	07 465	88 362	54
7	80 912	92 561	07 439	88 351	53
8	80 927	92 587	07 413	88 340	52
9	80 942	92 612	07 388	88 330	51
10	80 957	92 638	07 362	88 319	**50**
11	80 972	92 663	07 337	88 308	49
12	80 987	92 689	07 311	88 298	48
13	81 002	92 715	07 285	88 287	47
14	81 017	92 740	07 260	88 276	46
15	81 032	92 766	07 234	88 266	**45**
16	81 047	92 792	07 208	88 255	44
17	81 061	92 817	07 183	88 244	43
18	81 076	92 843	07 157	88 234	42
19	81 091	92 868	07 132	88 223	41
20	81 106	92 894	07 106	88 212	**40**
21	81 121	92 920	07 080	88 201	39
22	81 136	92 945	07 055	88 191	38
23	81 151	92 971	07 029	88 180	37
24	81 166	92 996	07 004	88 169	36
25	81 180	93 022	06 978	88 158	**35**
26	81 195	93 048	06 952	88 148	34
27	81 210	93 073	06 927	88 137	33
28	81 225	93 099	06 901	88 126	32
29	81 240	93 124	06 876	88 115	31
30	81 254	93 150	06 850	88 105	**30**
31	81 269	93 175	06 825	88 094	29
32	81 284	93 201	06 799	88 083	28
33	81 299	93 227	06 773	88 072	27
34	81 314	93 252	06 748	88 061	26
35	81 328	93 278	06 722	88 051	**25**
36	81 343	93 303	06 697	88 040	24
37	81 358	93 329	06 671	88 029	23
38	81 372	93 354	06 646	88 018	22
39	81 387	93 380	06 620	88 007	21
40	81 402	93 406	06 594	87 996	**20**
41	81 417	93 431	06 569	87 985	19
42	81 431	93 457	06 543	87 975	18
43	81 446	93 482	06 518	87 964	17
44	81 461	93 508	06 492	87 953	16
45	81 475	93 533	06 467	87 942	**15**
46	81 490	93 559	06 441	87 931	14
47	81 505	93 584	06 416	87 920	13
48	81 519	93 610	06 390	87 909	12
49	81 534	93 636	06 364	87 898	11
50	81 549	93 661	06 339	87 887	**10**
51	81 563	93 687	06 313	87 877	9
52	81 578	93 712	06 288	87 866	8
53	81 592	93 738	06 262	87 855	7
54	81 607	93 763	06 237	87 844	6
55	81 622	93 789	06 211	87 833	**5**
56	81 636	93 814	06 186	87 822	4
57	81 651	93 840	06 160	87 811	3
58	81 665	93 865	06 135	87 800	2
59	81 680	93 891	06 109	87 789	1
60	81 694	93 916	06 084	87 778	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

49°

41°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	81 694	93 916	06 084	87 778	**60**
1	81 709	93 942	06 058	87 767	59
2	81 723	93 967	06 033	87 756	58
3	81 738	93 993	06 007	87 745	57
4	81 752	94 018	05 982	87 734	56
5	81 767	94 044	05 956	87 723	**55**
6	81 781	94 069	05 931	87 712	54
7	81 796	94 095	05 905	87 701	53
8	81 810	94 120	05 880	87 690	52
9	81 825	94 146	05 854	87 679	51
10	81 839	94 171	05 829	87 668	**50**
11	81 854	94 197	05 803	87 657	49
12	81 868	94 222	05 778	87 646	48
13	81 882	94 248	05 752	87 635	47
14	81 897	94 273	05 727	87 624	46
15	81 911	94 299	05 701	87 613	**45**
16	81 926	94 324	05 676	87 601	44
17	81 940	94 350	05 650	87 590	43
18	81 955	94 375	05 625	87 579	42
19	81 969	94 401	05 599	87 568	41
20	81 983	94 426	05 574	87 557	**40**
21	81 998	94 452	05 548	87 546	39
22	82 012	94 477	05 523	87 535	38
23	82 026	94 503	05 497	87 524	37
24	82 041	94 528	05 472	87 513	36
25	82 055	94 554	05 446	87 501	**35**
26	82 069	94 579	05 421	87 490	34
27	82 084	94 604	05 396	87 479	33
28	82 098	94 630	05 370	87 468	32
29	82 112	94 655	05 345	87 457	31
30	82 126	94 681	05 319	87 446	**30**
31	82 141	94 706	05 294	87 434	29
32	82 155	94 732	05 268	87 423	28
33	82 169	94 757	05 243	87 412	27
34	82 184	94 783	05 217	87 401	26
35	82 198	94 808	05 192	87 390	**25**
36	82 212	94 834	05 166	87 378	24
37	82 226	94 859	05 141	87 367	23
38	82 240	94 884	05 116	87 356	22
39	82 255	94 910	05 090	87 345	21
40	82 269	94 935	05 065	87 334	**20**
41	82 283	94 961	05 039	87 322	19
42	82 297	94 986	05 014	87 311	18
43	82 311	95 012	04 988	87 300	17
44	82 326	95 037	04 963	87 288	16
45	82 340	95 062	04 938	87 277	**15**
46	82 354	95 088	04 912	87 266	14
47	82 368	95 113	04 887	87 255	13
48	82 382	95 139	04 861	87 243	12
49	82 396	95 164	04 836	87 232	11
50	82 410	95 190	04 810	87 221	**10**
51	82 424	95 215	04 785	87 209	9
52	82 439	95 240	04 760	87 198	8
53	82 453	95 266	04 734	87 187	7
54	82 467	95 291	04 709	87 175	6
55	82 481	95 317	04 683	87 164	**5**
56	82 495	95 342	04 658	87 153	4
57	82 509	95 368	04 632	87 141	3
58	82 523	95 393	04 607	87 130	2
59	82 537	95 418	04 582	87 119	1
60	82 551	95 444	04 556	87 107	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

48°

42°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	82 551	95 444	04 556	87 107	**60**
1	82 565	95 469	04 531	87 096	59
2	82 579	95 495	04 505	87 085	58
3	82 593	95 520	04 480	87 073	57
4	82 607	95 545	04 455	87 062	56
5	82 621	95 571	04 429	87 050	**55**
6	82 635	95 596	04 404	87 039	54
7	82 649	95 622	04 378	87 028	53
8	82 663	95 647	04 353	87 016	52
9	82 677	95 672	04 328	87 005	51
10	82 691	95 698	04 302	86 993	**50**
11	82 705	95 723	04 277	86 982	49
12	82 719	95 748	04 252	86 970	48
13	82 733	95 774	04 226	86 959	47
14	82 747	95 799	04 201	86 947	46
15	82 761	95 825	04 175	86 936	**45**
16	82 775	95 850	04 150	86 924	44
17	82 788	95 875	04 125	86 913	43
18	82 802	95 901	04 099	86 902	42
19	82 816	95 926	04 074	86 890	41
20	82 830	95 952	04 048	86 879	**40**
21	82 844	95 977	04 023	86 867	39
22	82 858	96 002	03 998	86 855	38
23	82 872	96 028	03 972	86 844	37
24	82 885	96 053	03 947	86 832	36
25	82 899	96 078	03 922	86 821	**35**
26	82 913	96 104	03 896	86 809	34
27	82 927	96 129	03 871	86 798	33
28	82 941	96 155	03 845	86 786	32
29	82 955	96 180	03 820	86 775	31
30	82 968	96 205	03 795	86 763	**30**
31	82 982	96 231	03 769	86 752	29
32	82 996	96 256	03 744	86 740	28
33	83 010	96 281	03 719	86 728	27
34	83 023	96 307	03 693	86 717	26
35	83 037	96 332	03 668	86 705	**25**
36	83 051	96 357	03 643	86 694	24
37	83 065	96 383	03 617	86 682	23
38	83 078	96 408	03 592	86 670	22
39	83 092	96 433	03 567	86 659	21
40	83 106	96 459	03 541	86 647	**20**
41	83 120	96 484	03 516	86 635	19
42	83 133	96 510	03 490	86 624	18
43	83 147	96 535	03 465	86 612	17
44	83 161	96 560	03 440	86 600	16
45	83 174	96 586	03 414	86 589	**15**
46	83 188	96 611	03 389	86 577	14
47	83 202	96 636	03 364	86 565	13
48	83 215	96 662	03 338	86 554	12
49	83 229	96 687	03 313	86 542	11
50	83 242	96 712	03 288	86 530	**10**
51	83 256	96 738	03 262	86 518	9
52	83 270	96 763	03 237	86 507	8
53	83 283	96 788	03 212	86 495	7
54	83 297	96 814	03 186	86 483	6
55	83 310	96 839	03 161	86 472	**5**
56	83 324	96 864	03 136	86 460	4
57	83 338	96 890	03 110	86 448	3
58	83 351	96 915	03 085	86 436	2
59	83 365	96 940	03 060	86 425	1
60	83 378	96 966	03 034	86 413	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

47°

43°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	83 378	96 966	03 034	86 413	**60**
1	83 392	96 991	03 009	86 401	59
2	83 405	97 016	02 984	86 389	58
3	83 419	97 042	02 958	86 377	57
4	83 432	97 067	02 933	86 366	56
5	83 446	97 092	02 908	86 354	**55**
6	83 459	97 118	02 882	86 342	54
7	83 473	97 143	02 857	86 330	53
8	83 486	97 168	02 832	86 318	52
9	83 500	97 193	02 807	86 306	51
10	83 513	97 219	02 781	86 295	**50**
11	83 527	97 244	02 756	86 283	49
12	83 540	97 269	02 731	86 271	48
13	83 554	97 295	02 705	86 259	47
14	83 567	97 320	02 680	86 247	46
15	83 581	97 345	02 655	86 235	**45**
16	83 594	97 371	02 629	86 223	44
17	83 608	97 396	02 604	86 211	43
18	83 621	97 421	02 579	86 200	42
19	83 634	97 447	02 553	86 188	41
20	83 648	97 472	02 528	86 176	**40**
21	83 661	97 497	02 503	86 164	39
22	83 674	97 523	02 477	86 152	38
23	83 688	97 548	02 452	86 140	37
24	83 701	97 573	02 427	86 128	36
25	83 715	97 598	02 402	86 116	**35**
26	83 728	97 624	02 376	86 104	34
27	83 741	97 649	02 351	86 092	33
28	83 755	97 674	02 326	86 080	32
29	83 768	97 700	02 300	86 068	31
30	83 781	97 725	02 275	86 056	**30**
31	83 795	97 750	02 250	86 044	29
32	83 808	97 776	02 224	86 032	28
33	83 821	97 801	02 199	86 020	27
34	83 834	97 826	02 174	86 008	26
35	83 848	97 851	02 149	85 996	**25**
36	83 861	97 877	02 123	85 984	24
37	83 874	97 902	02 098	85 972	23
38	83 887	97 927	02 073	85 960	22
39	83 901	97 953	02 047	85 948	21
40	83 914	97 978	02 022	85 936	**20**
41	83 927	98 003	01 997	85 924	19
42	83 940	98 029	01 971	85 912	18
43	83 954	98 054	01 946	85 900	17
44	83 967	98 079	01 921	85 888	16
45	83 980	98 104	01 896	85 876	**15**
46	83 993	98 130	01 870	85 864	14
47	84 006	98 155	01 845	85 851	13
48	84 020	98 180	01 820	85 839	12
49	84 033	98 206	01 794	85 827	11
50	84 046	98 231	01 769	85 815	**10**
51	84 059	98 256	01 744	85 803	9
52	84 072	98 281	01 719	85 791	8
53	84 085	98 307	01 693	85 779	7
54	84 098	98 332	01 668	85 766	6
55	84 112	98 357	01 643	85 754	**5**
56	84 125	98 383	01 617	85 742	4
57	84 138	98 408	01 592	85 730	3
58	84 151	98 433	01 567	85 718	2
59	84 164	98 458	01 542	85 706	1
60	84 177	98 484	01 516	85 693	**0**
	9	9	10	9	
′	log cos	log cot	log tan	log sin	′

46°

44°

′	log sin	log tan	log cot	log cos	′
	9	9	10	9	
0	84 177	98 484	01 516	85 693	**60**
1	84 190	98 509	01 491	85 681	59
2	84 203	98 534	01 466	85 669	58
3	84 216	98 560	01 440	85 657	57
4	84 229	98 585	01 415	85 645	56
5	84 242	98 610	01 390	85 632	**55**
6	84 255	98 635	01 365	85 620	54
7	84 269	98 661	01 339	85 608	53
8	84 282	98 686	01 314	85 596	52
9	84 295	98 711	01 289	85 583	51
10	84 308	98 737	01 263	85 571	**50**
11	84 321	98 762	01 238	85 559	49
12	84 334	98 787	01 213	85 547	48
13	84 347	98 812	01 188	85 534	47
14	84 360	98 838	01 162	85 522	46
15	84 373	98 863	01 137	85 510	**45**
16	84 385	98 888	01 112	85 497	44
17	84 398	98 913	01 087	85 485	43
18	84 411	98 939	01 061	85 473	42
19	84 424	98 964	01 036	85 460	41
20	84 437	98 989	01 011	85 448	**40**
21	84 450	99 015	00 985	85 436	39
22	84 463	99 040	00 960	85 423	38
23	84 476	99 065	00 935	85 411	37
24	84 489	99 090	00 910	85 399	36
25	84 502	99 116	00 884	85 386	**35**
26	84 515	99 141	00 859	85 374	34
27	84 528	99 166	00 834	85 361	33
28	84 540	99 191	00 809	85 349	32
29	84 553	99 217	00 783	85 337	31
30	84 566	99 242	00 758	85 324	**30**
31	84 579	99 267	00 733	85 312	29
32	84 592	99 293	00 707	85 299	28
33	84 605	99 318	00 682	85 287	27
34	84 618	99 343	00 657	85 274	26
35	84 630	99 368	00 632	85 262	**25**
36	84 643	99 394	00 606	85 250	24
37	84 656	99 419	00 581	85 237	23
38	84 669	99 444	00 556	85 225	22
39	84 682	99 469	00 531	85 212	21
40	84 694	99 495	00 505	85 200	**20**
41	84 707	99 520	00 480	85 187	19
42	84 720	99 545	00 455	85 175	18
43	84 733	99 570	00 430	85 162	17
44	84 745	99 596	00 404	85 150	16
45	84 758	99 621	00 379	85 137	**15**
46	84 771	99 646	00 354	85 125	14
47	84 784	99 672	00 328	85 112	13
48	84 796	99 697	00 303	85 100	12
49	84 809	99 722	00 278	85 087	11
50	84 822	99 747	00 253	85 074	**10**
51	84 835	99 773	00 227	85 062	9
52	84 847	99 798	00 202	85 049	8
53	84 860	99 823	00 177	85 037	7
54	84 873	99 848	00 152	85 024	6
55	84 885	99 874	00 126	85 012	**5**
56	84 898	99 899	00 101	84 999	4
57	84 911	99 924	00 076	84 986	3
58	84 923	99 949	00 051	84 974	2
59	84 936	99 975	00 025	84 961	1
60	84 949	00 000	00 000	84 949	**0**
	9	10	10	9	
′	log cos	log cot	log tan	log sin	′

45°

TABLE III.

For Determining with Greater Accuracy than can be done by Means of Table II:

1. *log sin*, *log tan*, and *log cot*, when the angle is between 0° and 2°;
2. *log cos*, *log tan*, and *log cot*, when the angle is between 88° and 90°;
3. The value of the angle when the logarithm of the function does *not* lie between the limits **8. 54 684** and **11. 45 316.**

FORMULAS FOR THE USE OF THE NUMBERS S AND T

when the angle x is between 0° and 2°:

$$\log \sin x = \log x + S.$$
$$\log \tan x = \log x + T.$$

Values of S and T.

x	S	log sin *x*	*x*	T	log tan *x*	*x*	T	log tan *x*
0		–	0		–	5 146		8. 39 713
	4. 68 557			4. 68 557			4. 68 567	
2 409		8. 06 740	200		6. 98 660	5 424		8. 41 999
	4. 68 556			4. 68 558			4. 68 568	
3 417		8. 21 920	1 726		7. 92 263	5 689		8. 44 072
	4. 68 555			4. 68 559			4. 68 569	
3 823		8. 26 795	2 432		8. 07 156	5 941		8. 45 955
	4. 68 555			4. 68 560			4. 68 570	
4 190		8. 30 776	2 976		8. 15 924	6 184		8. 47 697
	4. 68 554			4. 68 561			4. 68 571	
4 840		8. 37 038	3 434		8. 22 142	6 417		8. 49 305
	4. 68 553			4. 68 562			4. 68 572	
5 414		8. 41 904	3 838		8. 26 973	6 642		8. 50 802
	4. 68 552			4. 68 563			4. 68 573	
5 932		8. 45 872	4 204		8. 30 930	6 859		8. 52 200
	4. 68 551			4. 68 564			4. 68 574	
6 408		8. 49 223	4 540		8. 34 270	7 070		8. 53 516
	4. 68 550			4. 68 56$\underline{5}$			4. 68 57$\underline{5}$	
6 633		8. 50 721	4 699		8. 35 766	7 173		8. 54 145
	4. 68 55$\underline{0}$			4. 68 565			4. 68 575	
6 851		8. 52 125	4 853		8. 37 167	7 274		8. 54 753
	4. 68 549			4. 68 566				
7 267		8. 54 684	5 146		8. 39 713			
x	S	log sin *x*	*x*	T	log tan *x*	*x*	T	log tan *x*

TABLE IV. — NATURAL FUNCTIONS.

′	0°		1°		2°		3°		4°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0000	1.000	0175	9998	0349	9994	0523	9986	0698	9976	**60**
1	0003	1.000	0177	9998	0352	9994	0526	9986	0700	9975	59
2	0006	1.000	0180	9998	0355	9994	0529	9986	0703	9975	58
3	0009	1.000	0183	9998	0358	9994	0532	9986	0706	9975	57
4	0012	1.000	0186	9998	0361	9993	0535	9986	0709	9975	56
5	0015	1.000	0189	9998	0364	9993	0538	9986	0712	9975	**55**
6	0017	1.000	0192	9998	0366	9993	0541	9985	0715	9974	54
7	0020	1.000	0195	9998	0369	9993	0544	9985	0718	9974	53
8	0023	1.000	0198	9998	0372	9993	0547	9985	0721	9974	52
9	0026	1.000	0201	9998	0375	9993	0550	9985	0724	9974	51
10	0029	1.000	0204	9998	0378	9993	0552	9985	0727	9974	**50**
11	0032	1.000	0207	9998	0381	9993	0555	9985	0729	9973	49
12	0035	1.000	0209	9998	0384	9993	0558	9984	0732	9973	48
13	0038	1.000	0212	9998	0387	9993	0561	9984	0735	9973	47
14	0041	1.000	0215	9998	0390	9992	0564	9984	0738	9973	46
15	0044	1.000	0218	9998	0393	9992	0567	9984	0741	9973	**45**
16	0047	1.000	0221	9998	0396	9992	0570	9984	0744	9972	44
17	0049	1.000	0224	9997	0398	9992	0573	9984	0747	9972	43
18	0052	1.000	0227	9997	0401	9992	0576	9983	0750	9972	42
19	0055	1.000	0230	9997	0404	9992	0579	9983	0753	9972	41
20	0058	1.000	0233	9997	0407	9992	0581	9983	0756	9971	**40**
21	0061	1.000	0236	9997	0410	9992	0584	9983	0758	9971	39
22	0064	1.000	0239	9997	0413	9991	0587	9983	0761	9971	38
23	0067	1.000	0241	9997	0416	9991	0590	9983	0764	9971	37
24	0070	1.000	0244	9997	0419	9991	0593	9982	0767	9971	36
25	0073	1.000	0247	9997	0422	9991	0596	9982	0770	9970	**35**
26	0076	1.000	0250	9997	0425	9991	0599	9982	0773	9970	34
27	0079	1.000	0253	9997	0427	9991	0602	9982	0776	9970	33
28	0081	1.000	0256	9997	0430	9991	0605	9982	0779	9970	32
29	0084	1.000	0259	9997	0433	9991	0608	9982	0782	9969	31
30	0087	1.000	0262	9997	0436	9990	0610	9981	0785	9969	**30**
31	0090	1.000	0265	9996	0439	9990	0613	9981	0787	9969	29
32	0093	1.000	0268	9996	0442	9990	0616	9981	0790	9969	28
33	0096	1.000	0270	9996	0445	9990	0619	9981	0793	9968	27
34	0099	1.000	0273	9996	0448	9990	0622	9981	0796	9968	26
35	0102	9999	0276	9996	0451	9990	0625	9980	0799	9968	**25**
36	0105	9999	0279	9996	0454	9990	0628	9980	0802	9968	24
37	0108	9999	0282	9996	0457	9990	0631	9980	0805	9968	23
38	0111	9999	0285	9996	0459	9989	0634	9980	0808	9967	22
39	0113	9999	0288	9996	0462	9989	0637	9980	0811	9967	21
40	0116	9999	0291	9996	0465	9989	0640	9980	0814	9967	**20**
41	0119	9999	0294	9996	0468	9989	0642	9979	0816	9967	19
42	0122	9999	0297	9996	0471	9989	0645	9979	0819	9966	18
43	0125	9999	0300	9996	0474	9989	0648	9979	0822	9966	17
44	0128	9999	0302	9995	0477	9989	0651	9979	0825	9966	16
45	0131	9999	0305	9995	0480	9988	0654	9979	0828	9966	**15**
46	0134	9999	0308	9995	0483	9988	0657	9978	0831	9965	14
47	0137	9999	0311	9995	0486	9988	0660	9978	0834	9965	13
48	0140	9999	0314	9995	0488	9988	0663	9978	0837	9965	12
49	0143	9999	0317	9995	0491	9988	0666	9978	0840	9965	11
50	0145	9999	0320	9995	0494	9988	0669	9978	0843	9964	**10**
51	0148	9999	0323	9995	0497	9988	0671	9977	0845	9964	9
52	0151	9999	0326	9995	0500	9987	0674	9977	0848	9964	8
53	0154	9999	0329	9995	0503	9987	0677	9977	0851	9964	7
54	0157	9999	0332	9995	0506	9987	0680	9977	0854	9963	6
55	0160	9999	0334	9994	0509	9987	0683	9977	0857	9963	**5**
56	0163	9999	0337	9994	0512	9987	0686	9976	0860	9963	4
57	0166	9999	0340	9994	0515	9987	0689	9976	0863	9963	3
58	0169	9999	0343	9994	0518	9987	0692	9976	0866	9962	2
59	0172	9999	0346	9994	0520	9986	0695	9976	0869	9962	1
60	0175	9999	0349	9994	0523	9986	0698	9976	0872	9962	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0872	9962	1045	9945	1219	9925	1392	9903	1564	9877	**60**
1	0874	9962	1048	9945	1222	9925	1395	9902	1567	9876	59
2	0877	9961	1051	9945	1224	9925	1397	9902	1570	9876	58
3	0880	9961	1054	9944	1227	9924	1400	9901	1573	9876	57
4	0883	9961	1057	9944	1230	9924	1403	9901	1576	9875	56
5	0886	9961	1060	9944	1233	9924	1406	9901	1579	9875	**55**
6	0889	9960	1063	9943	1236	9923	1409	9900	1582	9874	54
7	0892	9960	1066	9943	1239	9923	1412	9900	1584	9874	53
8	0895	9960	1068	9943	1241	9923	1415	9899	1587	9873	52
9	0898	9960	1071	9942	1245	9922	1418	9899	1590	9873	51
10	0901	9959	1074	9942	1248	9922	1421	9899	1593	9872	**50**
11	0903	9959	1077	9942	1250	9922	1423	9898	1596	9872	49
12	0906	9959	1080	9942	1253	9921	1426	9898	1599	9871	48
13	0909	9959	1083	9941	1256	9921	1429	9897	1602	9871	47
14	0912	9958	1086	9941	1259	9920	1432	9897	1605	9870	46
15	0915	9958	1089	9941	1262	9920	1435	9897	1607	9870	**45**
16	0918	9958	1092	9940	1265	9920	1438	9896	1610	9869	44
17	0921	9958	1094	9940	1268	9919	1441	9896	1613	9869	43
18	0924	9957	1097	9940	1271	9919	1444	9895	1616	9869	42
19	0927	9957	1100	9939	1274	9919	1446	9895	1619	9868	41
20	0929	9957	1103	9939	1276	9918	1449	9894	1622	9868	**40**
21	0932	9956	1106	9939	1279	9918	1452	9894	1625	9867	39
22	0935	9956	1109	9938	1282	9917	1455	9894	1628	9867	38
23	0938	9956	1112	9938	1285	9917	1458	9893	1630	9866	37
24	0941	9956	1115	9938	1288	9917	1461	9893	1633	9866	36
25	0944	9955	1118	9937	1291	9916	1464	9892	1636	9865	**35**
26	0947	9955	1120	9937	1294	9916	1467	9892	1639	9865	34
27	0950	9955	1123	9937	1297	9916	1469	9891	1642	9864	33
28	0953	9955	1126	9935	1299	9915	1472	9891	1645	9864	32
29	0956	9954	1129	9936	1302	9915	1475	9891	1648	9863	31
30	0958	9954	1132	9936	1305	9914	1478	9890	1650	9863	**30**
31	0961	9954	1135	9935	1308	9914	1481	9890	1653	9862	29
32	0964	9953	1138	9935	1311	9914	1484	9889	1656	9862	28
33	0967	9953	1141	9935	1314	9913	1487	9889	1659	9861	27
34	0970	9953	1144	9934	1317	9913	1490	9888	1662	9861	26
35	0973	9953	1146	9934	1320	9913	1492	9888	1665	9860	**25**
36	0976	9952	1149	9934	1323	9912	1495	9888	1668	9860	24
37	0979	9952	1152	9933	1325	9912	1498	9887	1671	9859	23
38	0982	9952	1155	9933	1328	9911	1501	9887	1673	9859	22
39	0985	9951	1158	9933	1331	9911	1504	9886	1676	9859	21
40	0987	9951	1161	9932	1334	9911	1507	9886	1679	9858	**20**
41	0990	9951	1164	9932	1337	9910	1510	9885	1682	9858	19
42	0993	9951	1167	9932	1340	9910	1513	9885	1685	9857	18
43	0996	9950	1170	9931	1343	9909	1515	9884	1688	9857	17
44	0999	9950	1172	9931	1346	9909	1518	9884	1691	9856	16
45	1002	9950	1175	9931	1349	9909	1521	9884	1693	9856	**15**
46	1005	9949	1178	9930	1351	9908	1524	9883	1696	9855	14
47	1008	9949	1181	9930	1354	9908	1527	9883	1699	9855	13
48	1011	9949	1184	9930	1357	9907	1530	9882	1702	9854	12
49	1013	9949	1187	9929	1360	9907	1533	9882	1705	9854	11
50	1016	9948	1190	9929	1363	9907	1536	9881	1708	9853	**10**
51	1019	9948	1193	9929	1366	9906	1538	9881	1711	9853	9
52	1022	9948	1196	9928	1369	9906	1541	9880	1714	9852	8
53	1025	9947	1198	9928	1372	9905	1544	9880	1716	9852	7
54	1028	9947	1201	9928	1374	9905	1547	9880	1719	9851	6
55	1031	9947	1204	9927	1377	9905	1550	9879	1722	9851	**5**
56	1034	9946	1207	9927	1380	9904	1553	9879	1725	9850	4
57	1037	9946	1210	9927	1383	9904	1556	9878	1728	9850	3
58	1039	9946	1213	9926	1386	9903	1559	9878	1731	9849	2
59	1042	9946	1216	9926	1389	9903	1561	9877	1734	9849	1
60	1045	9945	1219	9925	1392	9903	1564	9877	1736	9848	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	1736	9848	1908	9816	2079	9781	2250	9744	2419	9703	**60**
1	1739	9848	1911	9816	2082	9781	2252	9743	2422	9702	59
2	1742	9847	1914	9815	2085	9780	2255	9742	2425	9702	58
3	1745	9847	1917	9815	2088	9780	2258	9742	2428	9701	57
4	1748	9846	1920	9814	2090	9779	2261	9741	2431	9700	56
5	1751	9846	1922	9813	2093	9778	2264	9740	2433	9699	**55**
6	1754	9845	1925	9813	2096	9778	2267	9740	2436	9699	54
7	1757	9845	1928	9812	2099	9777	2269	9739	2439	9698	53
8	1759	9844	1931	9812	2102	9777	2272	9738	2442	9697	52
9	1762	9843	1934	9811	2105	9776	2275	9738	2445	9697	51
10	1765	9843	1937	9811	2108	9775	2278	9737	2447	9696	**50**
11	1768	9842	1939	9810	2110	9775	2281	9736	2450	9695	49
12	1771	9842	1942	9810	2113	9774	2284	9736	2453	9694	48
13	1774	9841	1945	9809	2116	9774	2286	9735	2456	9694	47
14	1777	9841	1948	9808	2119	9773	2289	9734	2459	9693	46
15	1779	9840	1951	9808	2122	9772	2292	9734	2462	9692	**45**
16	1782	9840	1954	9807	2125	9772	2295	9733	2464	9692	44
17	1785	9839	1957	9807	2127	9771	2298	9732	2467	9691	43
18	1788	9839	1959	9806	2130	9770	2300	9732	2470	9690	42
19	1791	9838	1962	9806	2133	9770	2303	9731	2473	9689	41
20	1794	9838	1965	9805	2136	9769	2306	9730	2476	9689	**40**
21	1797	9837	1968	9804	2139	9769	2309	9730	2478	9688	39
22	1799	9837	1971	9804	2142	9768	2312	9729	2481	9687	38
23	1802	9836	1974	9803	2145	9767	2315	9728	2484	9687	37
24	1805	9836	1977	9803	2147	9767	2317	9728	2487	9686	36
25	1808	9835	1979	9802	2150	9766	2320	9727	2490	9685	**35**
26	1811	9835	1982	9802	2153	9765	2323	9726	2493	9684	34
27	1814	9834	1985	9801	2156	9765	2326	9726	2495	9684	33
28	1817	9834	1988	9800	2159	9764	2329	9725	2498	9683	32
29	1819	9833	1991	9800	2162	9764	2332	9724	2501	9682	31
30	1822	9833	1994	9799	2164	9763	2334	9724	2504	9681	**30**
31	1825	9832	1997	9799	2167	9762	2337	9723	2507	9681	29
32	1828	9831	1999	9798	2170	9762	2340	9722	2509	9680	28
33	1831	9831	2002	9798	2173	9761	2343	9722	2512	9679	27
34	1834	9830	2005	9797	2176	9760	2346	9721	2515	9679	26
35	1837	9830	2008	9796	2179	9760	2349	9720	2518	9678	**25**
36	1840	9829	2011	9796	2181	9759	2351	9720	2521	9677	24
37	1842	9829	2014	9795	2184	9759	2354	9719	2524	9676	23
38	1845	9828	2016	9795	2187	9758	2357	9718	2526	9676	22
39	1848	9828	2019	9794	2190	9757	2360	9718	2529	9675	21
40	1851	9827	2022	9793	2193	9757	2363	9717	2532	9674	**20**
41	1854	9827	2025	9793	2196	9756	2366	9716	2535	9673	19
42	1857	9826	2028	9792	2198	9755	2368	9715	2538	9673	18
43	1860	9826	2031	9792	2201	9755	2371	9715	2540	9672	17
44	1862	9825	2034	9791	2204	9754	2374	9714	2543	9671	16
45	1865	9825	2036	9790	2207	9753	2377	9713	2546	9670	**15**
46	1868	9824	2039	9790	2210	9753	2380	9713	2549	9670	14
47	1871	9823	2042	9789	2213	9752	2383	9712	2552	9669	13
48	1874	9823	2045	9789	2215	9751	2385	9711	2554	9668	12
49	1877	9822	2048	9788	2218	9751	2388	9711	2557	9667	11
50	1880	9822	2051	9787	2221	9750	2391	9710	2560	9667	**10**
51	1882	9821	2054	9787	2224	9750	2394	9709	2563	9666	9
52	1885	9821	2056	9786	2227	9749	2397	9709	2566	9665	8
53	1888	9820	2059	9786	2230	9748	2399	9708	2569	9665	7
54	1891	9820	2062	9785	2233	9748	2402	9707	2571	9664	6
55	1894	9819	2065	9784	2235	9747	2405	9706	2574	9663	**5**
56	1897	9818	2068	9784	2238	9746	2408	9706	2577	9662	4
57	1900	9818	2071	9783	2241	9746	2411	9705	2580	9662	3
58	1902	9817	2073	9783	2244	9745	2414	9704	2583	9661	2
59	1905	9817	2076	9782	2247	9744	2416	9704	2585	9660	1
60	1908	9816	2079	9781	2250	9744	2419	9703	2588	9659	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	2588	9659	2756	9613	2924	9563	3090	9511	3256	9455	**60**
1	2591	9659	2759	9612	2926	9562	3093	9510	3258	9454	59
2	2594	9658	2762	9611	2929	9561	3096	9509	3261	9453	58
3	2597	9657	2165	9610	2932	9560	3098	9508	3264	9452	57
4	2599	9656	2768	9609	2935	9560	3101	9507	3267	9451	56
5	2602	9655	2770	9609	2938	9559	3104	9506	3269	9450	**55**
6	2605	9655	2773	9608	2940	9558	3107	9505	3272	9449	54
7	2608	9654	2776	9607	2943	9557	3110	9504	3275	9449	53
8	2611	9653	2779	9606	2946	9556	3112	9503	3278	9448	52
9	2613	9652	2782	9605	2949	9555	3115	9502	3280	9447	51
10	2616	9652	2784	9605	2952	9555	3118	9502	3283	9446	**50**
11	2619	9651	2787	9604	2954	9554	3121	9501	3286	9445	49
12	2622	9650	2790	9603	2957	9553	3123	9500	3289	9444	48
13	2625	9649	2793	9602	2960	9552	3126	9499	3291	9443	47
14	2628	9649	2795	9601	2963	9551	3129	9498	3294	9442	46
15	2630	9648	2798	9600	2965	9550	3132	9497	3297	9441	**45**
16	2633	9647	2801	9600	2968	9549	3134	9496	3300	9440	44
17	2636	9646	2804	9599	2971	9548	3137	9495	3302	9439	43
18	2639	9646	2807	9598	2974	9548	3140	9494	3305	9438	42
19	2642	9645	2809	9597	2977	9547	3143	9493	3308	9437	41
20	2644	9644	2812	9596	2979	9546	3145	9492	3311	9436	**40**
21	2647	9643	2815	9596	2982	9545	3148	9492	3313	9435	39
22	2650	9642	2818	9595	2985	9544	3151	9491	3316	9434	38
23	2653	9642	2821	9594	2988	9543	3154	9490	3319	9433	37
24	2656	9641	2823	9593	2990	9542	3156	9489	3322	9432	36
25	2658	9640	2826	9592	2993	9542	3159	9488	3324	9431	**35**
26	2661	9639	2829	9591	2996	9541	3162	9487	3327	9430	34
27	2664	9639	2832	9591	2999	9540	3165	9486	3330	9429	33
28	2667	9638	2835	9590	3002	9539	3168	9485	3333	9428	32
29	2670	9637	2837	9589	3004	9538	3170	9484	3335	9427	31
30	2672	9636	2840	9588	3007	9537	3173	9483	3338	9426	**30**
31	2675	9636	2843	9587	3010	9536	3176	9482	3341	9425	29
32	2678	9635	2846	9587	3013	9535	3179	9481	3344	9424	28
33	2681	9634	2849	9586	3015	9535	3181	9480	3346	9423	27
34	2684	9633	2851	9585	3018	9534	3184	9480	3349	9423	26
35	2686	9632	2854	9584	3021	9533	3187	9479	3352	9422	**25**
36	2689	9632	2857	9583	3024	9532	3190	9478	3355	9421	24
37	2692	9631	2860	9582	3026	9531	3192	9477	3357	9420	23
38	2695	9630	2862	9582	3029	9530	3195	9476	3360	9419	22
39	2698	9629	2865	9581	3032	9529	3198	9475	3363	9418	21
40	2700	9628	2868	9580	3035	9528	3201	9474	3365	9417	**20**
41	2703	9628	2871	9579	3038	9527	3203	9473	3368	9416	19
42	2706	9627	2874	9578	3040	9527	3206	9472	3371	9415	18
43	2709	9626	2876	9577	3043	9526	3209	9471	3374	9414	17
44	2712	9625	2879	9577	3046	9525	3212	9470	3376	9413	16
45	2714	9625	2882	9576	3049	9524	3214	9469	3379	9412	**15**
46	2717	9624	2885	9575	3051	9523	3217	9468	3382	9411	14
47	2720	9623	2888	9574	3054	9522	3220	9467	3385	9410	13
48	2723	9622	2890	9573	3057	9521	3223	9466	3387	9409	12
49	2726	9621	2893	9572	3060	9520	3225	9466	3390	9408	11
50	2728	9621	2896	9572	3062	9520	3228	9465	3393	9407	**10**
51	2731	9620	2899	9571	3065	9519	3231	9464	3396	9406	9
52	2734	9619	2901	9570	3068	9518	3234	9463	3398	9405	8
53	2737	9618	2904	9569	3071	9517	3236	9462	3401	9404	7
54	2740	9617	2907	9568	3074	9516	3239	9461	3404	9403	6
55	2742	9617	2910	9567	3076	9515	3242	9460	3407	9402	**5**
56	2745	9616	2913	9566	3079	9514	3245	9459	3409	9401	4
57	2748	9615	2915	9566	3082	9513	3247	9458	3412	9400	3
58	2751	9614	2918	9565	3085	9512	3250	9457	3415	9399	2
59	2754	9613	2921	9564	3087	9511	3253	9456	3417	9398	1
60	2756	9613	2924	9563	3090	9511	3256	9455	3420	9397	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	3420	9397	3584	9336	3746	9272	3907	9205	4067	9135	**60**
1	3423	9396	3586	9335	3749	9271	3910	9204	4070	9134	59
2	3426	9395	3589	9334	3751	9270	3913	9203	4073	9133	58
3	3428	9394	3592	9333	3754	9269	3915	9202	4075	9132	57
4	3431	9393	3595	9332	3757	9267	3918	9200	4078	9131	56
5	3434	9392	3597	9331	3760	9266	3921	9199	4081	9130	**55**
6	3437	9391	3600	9330	3762	9265	3923	9198	4083	9128	54
7	3439	9390	3603	9328	3765	9264	3926	9197	4086	9127	53
8	3442	9389	3605	9327	3768	9263	3929	9196	4089	9126	52
9	3445	9388	3608	9326	3770	9262	3931	9195	4091	9125	51
10	3448	9387	3611	9325	3773	9261	3934	9194	4094	9124	**50**
11	3450	9386	3614	9324	3776	9260	3937	9192	4097	9122	49
12	3453	9385	3616	9323	3778	9259	3939	9191	4099	9121	48
13	3456	9384	3619	9322	3781	9258	3942	9190	4102	9120	47
14	3458	9383	3622	9321	3784	9257	3945	9189	4105	9119	46
15	3461	9382	3624	9320	3786	9255	3947	9188	4107	9118	**45**
16	3464	9381	3627	9319	3789	9254	3950	9187	4110	9116	44
17	3467	9380	3630	9318	3792	9253	3953	9186	4112	9115	43
18	3469	9379	3633	9317	3795	9252	3955	9184	4115	9114	42
19	3472	9378	3635	9316	3797	9251	3958	9183	4118	9113	41
20	3475	9377	3638	9315	3800	9250	3961	9182	4120	9112	**40**
21	3478	9376	3641	9314	3803	9249	3963	9181	4123	9110	39
22	3480	9375	3643	9313	3805	9248	3966	9180	4126	9109	38
23	3483	9374	3646	9312	3808	9247	3969	9179	4128	9108	37
24	3486	9373	3649	9311	3811	9245	3971	9178	4131	9107	36
25	3488	9372	3651	9309	3813	9244	3974	9176	4134	9106	**35**
26	3491	9371	3654	9308	3816	9243	3977	9175	4136	9104	34
27	3494	9370	3657	9307	3819	9242	3979	9174	4139	9103	33
28	3497	9369	3660	9306	3821	9241	3982	9173	4142	9102	32
29	3499	9368	3662	9305	3824	9240	3985	9172	4144	9101	31
30	3502	9367	3665	9304	3827	9239	3987	9171	4147	9100	**30**
31	3505	9366	3668	9303	3830	9238	3990	9169	4150	9098	29
32	3508	9365	3670	9302	3832	9237	3993	9168	4152	9097	28
33	3510	9364	3673	9301	3835	9235	3995	9167	4155	9096	27
34	3513	9363	3676	9300	3838	9234	3998	9166	4158	9095	26
35	3516	9362	3679	9299	3840	9233	4001	9165	4160	9094	**25**
36	3518	9361	3681	9298	3843	9232	4003	9164	4163	9092	24
37	3521	9360	3684	9297	3846	9231	4006	9162	4165	9091	23
38	3524	9359	3687	9296	3848	9230	4009	9161	4168	9090	22
39	3527	9358	3689	9295	3851	9229	4011	9160	4171	9088	21
40	3529	9356	3692	9293	3854	9228	4014	9159	4173	9088	**20**
41	3532	9355	3695	9292	3856	9227	4017	9158	4176	9086	19
42	3535	9354	3697	9291	3859	9225	4019	9157	4179	9085	18
43	3537	9353	3700	9290	3862	9224	4022	9155	4181	9084	17
44	3540	9352	3703	9289	3864	9223	4025	9154	4184	9083	16
45	3543	9351	3706	9288	3867	9222	4027	9153	4187	9081	**15**
46	3546	9350	3708	9287	3870	9221	4030	9152	4189	9080	14
47	3548	9349	3711	9286	3872	9220	4033	9151	4192	9079	13
48	3551	9348	3714	9285	3875	9219	4035	9150	4195	9078	12
49	3554	9347	3716	9284	3878	9218	4038	9148	4197	9077	11
50	3557	9346	3719	9283	3881	9216	4041	9147	4200	9075	**10**
51	3559	9345	3722	9282	3883	9215	4043	9146	4202	9074	9
52	3562	9344	3724	9281	3886	9214	4046	9145	4205	9073	8
53	3565	9343	3727	9279	3889	9213	4049	9144	4208	9072	7
54	3567	9342	3730	9278	3891	9212	4051	9143	4210	9070	6
55	3570	9341	3733	9277	3894	9211	4054	9141	4213	9069	**5**
56	3573	9340	3735	9276	3897	9210	4057	9140	4216	9068	4
57	3576	9339	3738	9275	3899	9208	4059	9139	4218	9067	3
58	3578	9338	3741	9274	3902	9207	4062	9138	4221	9066	2
59	3581	9337	3743	9273	3905	9206	4065	9137	4224	9064	1
60	3584	9336	3746	9272	3907	9205	4067	9135	4226	9063	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
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1	4229	9062	4386	8987	4542	8909	4697	8828	4851	8745	59
2	4231	9061	4389	8985	4545	8907	4700	8827	4853	8743	58
3	4234	9059	4392	8984	4548	8906	4702	8825	4856	8742	57
4	4237	9058	4394	8983	4550	8905	4705	8824	4858	8741	56
5	4239	9057	4397	8982	4553	8903	4708	8823	4861	8739	**55**
6	4242	9056	4399	8980	4555	8902	4710	8821	4863	8738	54
7	4245	9054	4402	8979	4558	8901	4713	8820	4866	8736	53
8	4247	9053	4405	8978	4561	8899	4715	8819	4868	8735	52
9	4250	9052	4407	8976	4563	8898	4718	8817	4871	8733	51
10	4253	9051	4410	8975	4566	8897	4720	8816	4874	8732	**50**
11	4255	9050	4412	8974	4568	8895	4723	8814	4876	8731	49
12	4258	9048	4415	8973	4571	8894	4726	8813	4879	8729	48
13	4260	9047	4418	8971	4574	8893	4728	8812	4881	8728	47
14	4263	9046	4420	8970	4576	8892	4731	8810	4884	8726	46
15	4266	9045	4423	8969	4579	8890	4733	8809	4886	8725	**45**
16	4268	9043	4425	8967	4581	8889	4736	8808	4889	8724	44
17	4271	9042	4428	8966	4584	8888	4738	8806	4891	8722	43
18	4274	9041	4431	8965	4586	8886	4741	8805	4894	8721	42
19	4276	9040	4433	8964	4589	8885	4743	8803	4896	8719	41
20	4279	9038	4436	8962	4592	8884	4746	8802	4899	8718	**40**
21	4281	9037	4439	8961	4594	8882	4749	8801	4901	8716	39
22	4284	9036	4441	8960	4597	8881	4751	8799	4904	8715	38
23	4287	9035	4444	8958	4599	8879	4754	8798	4907	8714	37
24	4289	9033	4446	8957	4602	8878	4756	8796	4909	8712	36
25	4292	9032	4449	8956	4605	8877	4759	8795	4912	8711	**35**
26	4295	9031	4452	8955	4607	8875	4761	8794	4914	8709	34
27	4297	9030	4454	8953	4610	8874	4764	8792	4917	8708	33
28	4300	9028	4457	8952	4612	8873	4766	8791	4919	8706	32
29	4302	9027	4459	8951	4615	8871	4769	8790	4922	8705	31
30	4305	9026	4462	8949	4617	8870	4772	8788	4924	8704	**30**
31	4308	9025	4465	8948	4620	8869	4774	8787	4927	8702	29
32	4310	9023	4467	8947	4623	8867	4777	8785	4929	8701	28
33	4313	9022	4470	8945	4625	8866	4779	8784	4932	8699	27
34	4316	9021	4472	8944	4628	8865	4782	8783	4934	8698	26
35	4318	9020	4475	8943	4630	8863	4784	8781	4937	8696	**25**
36	4321	9018	4478	8942	4633	8862	4787	8780	4939	8695	24
37	4323	9017	4480	8940	4636	8861	4789	8778	4942	8694	23
38	4326	9016	4483	8939	4638	8859	4792	8777	4944	8692	22
39	4329	9015	4485	8938	4641	8858	4795	8776	4947	8691	21
40	4331	9013	4488	8936	4643	8857	4797	8774	4950	8689	**20**
41	4334	9012	4491	8935	4646	8855	4800	8773	4952	8688	19
42	4337	9011	4493	8934	4648	8854	4802	8771	4955	8686	18
43	4339	9010	4496	8932	4651	8853	4805	8770	4957	8685	17
44	4342	9008	4498	8931	4654	8851	4807	8769	4960	8683	16
45	4344	9007	4501	8930	4656	8850	4810	8767	4962	8682	**15**
46	4347	9006	4504	8928	4659	8849	4812	8766	4965	8681	14
47	4350	9004	4506	8927	4661	8847	4815	8764	4967	8679	13
48	4352	9003	4509	8926	4664	8846	4818	8763	4970	8678	12
49	4355	9002	4511	8925	4666	8844	4820	8762	4972	8676	11
50	4358	9001	4514	8923	4669	8843	4823	8760	4975	8675	**10**
51	4360	8999	4517	8922	4672	8842	4825	8759	4977	8673	9
52	4363	8998	4519	8921	4674	8840	4828	8757	4980	8672	8
53	4365	8997	4522	8919	4677	8839	4830	8756	4982	8670	7
54	4368	8996	4524	8918	4679	8838	4833	8755	4985	8669	6
55	4371	8994	4527	8917	4682	8836	4835	8753	4987	8668	**5**
56	4373	8993	4530	8915	4684	8835	4838	8752	4990	8666	4
57	4376	8992	4532	8914	4687	8834	4840	8750	4992	8665	3
58	4378	8990	4535	8913	4690	8832	4843	8749	4995	8663	2
59	4381	8989	4537	8911	4692	8831	4846	8748	4997	8662	1
60	4384	8988	4540	8910	4695	8829	4848	8746	5000	8660	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
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1	5003	8659	5153	8570	5302	8479	5449	8385	5594	8289	59
2	5005	8657	5155	8569	5304	8477	5451	8384	5597	8287	58
3	5008	8656	5158	8567	5307	8476	5454	8382	5599	8285	57
4	5010	8654	5160	8566	5309	8474	5456	8380	5602	8284	56
5	5013	8653	5163	8564	5312	8473	5459	8379	5604	8282	**55**
6	5015	8652	5165	8563	5314	8471	5461	8377	5606	8281	54
7	5018	8650	5168	8561	5316	8470	5463	8376	5609	8279	53
8	5020	8649	5170	8560	5319	8468	5466	8374	5611	8277	52
9	5023	8647	5173	8558	5321	8467	5468	8372	5614	8276	51
10	5025	8646	5175	8557	5324	8465	5471	8371	5616	8274	**50**
11	5028	8644	5178	8555	5326	8463	5473	8369	5618	8272	49
12	5030	8643	5180	8554	5329	8462	5476	8368	5621	8271	48
13	5033	8641	5183	8552	5331	8460	5478	8366	5623	8269	47
14	5035	8640	5185	8551	5334	8459	5480	8364	5626	8268	46
15	5038	8638	5188	8549	5336	8457	5483	8363	5628	8266	**45**
16	5040	8637	5190	8548	5339	8456	5485	8361	5630	8264	44
17	5043	8635	5193	8546	5341	8454	5488	8360	5633	8263	43
18	5045	8634	5195	8545	5344	8453	5490	8358	5635	8261	42
19	5048	8632	5198	8543	5346	8451	5493	8356	5638	8259	41
20	5050	8631	5200	8542	5348	8450	5495	8355	5640	8258	**40**
21	5053	8630	5203	8540	5351	8448	5498	8353	5642	8256	39
22	5055	8628	5205	8539	5353	8446	5500	8352	5645	8254	38
23	5058	8627	5208	8537	5356	8445	5502	8350	5647	8253	37
24	5060	8625	5210	8536	5358	8443	5505	8348	5650	8251	36
25	5063	8624	5213	8534	5361	8442	5507	8347	5652	8249	**35**
26	5065	8622	5215	8532	5363	8440	5510	8345	5654	8248	34
27	5068	8621	5218	8531	5366	8439	5512	8344	5657	8246	33
28	5070	8619	5220	8529	5368	8437	5515	8342	5659	8245	32
29	5073	8618	5223	8528	5371	8435	5517	8340	5662	8243	31
30	5075	8616	5225	8526	5373	8434	5519	8339	5664	8241	**30**
31	5078	8615	5227	8525	5375	8432	5522	8337	5666	8240	29
32	5080	8613	5230	8523	5378	8431	5524	8336	5669	8238	28
33	5083	8612	5232	8522	5380	8429	5527	8334	5671	8236	27
34	5085	8610	5235	8520	5383	8428	5529	8332	5674	8235	26
35	5088	8609	5237	8519	5385	8426	5531	8331	5676	8233	**25**
36	5090	8607	5240	8517	5388	8425	5534	8329	5678	8231	24
37	5093	8606	5242	8516	5390	8423	5536	8328	5681	8230	23
38	5095	8604	5245	8514	5393	8421	5539	8326	5683	8228	22
39	5098	8603	5247	8513	5395	8420	5541	8324	5686	8226	21
40	5100	8601	5250	8511	5398	8418	5544	8323	5688	8225	**20**
41	5103	8600	5252	8510	5400	8417	5546	8321	5690	8223	19
42	5105	8599	5255	8508	5402	8415	5548	8320	5693	8221	18
43	5108	8597	5257	8507	5405	8414	5551	8318	5695	8220	17
44	5110	8596	5260	8505	5407	8412	5553	8316	5698	8218	16
45	5113	8594	5262	8504	5410	8410	5556	8315	5700	8216	**15**
46	5115	8593	5265	8502	5412	8409	5558	8313	5702	8215	14
47	5118	8591	5267	8500	5415	8407	5561	8311	5705	8213	13
48	5120	8590	5270	8499	5417	8406	5563	8310	5707	8211	12
49	5123	8588	5272	8497	5420	8404	5565	8308	5710	8210	11
50	5125	8587	5275	8496	5422	8403	5568	8307	5712	8208	**10**
51	5128	8585	5277	8494	5424	8401	5570	8305	5714	8207	9
52	5130	8584	5279	8493	5427	8399	5573	8303	5717	8205	8
53	5133	8582	5282	8491	5429	8398	5575	8302	5719	8203	7
54	5135	8581	5284	8490	5432	8396	5577	8300	5721	8202	6
55	5138	8579	5287	8488	5434	8395	5580	8299	5724	8200	**5**
56	5140	8578	5289	8487	5437	8393	5582	8297	5726	8198	4
57	5143	8576	5292	8485	5439	8391	5585	8295	5729	8197	3
58	5145	8575	5294	8484	5442	8390	5587	8294	5731	8195	2
59	5148	8573	5297	8482	5444	8388	5590	8292	5733	8193	1
60	5150	8572	5299	8480	5446	8387	5592	8290	5736	8192	**0**
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′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
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1	5738	8190	5880	8088	6020	7985	6159	7878	6295	7770	59
2	5741	8188	5883	8087	6023	7983	6161	7877	6298	7768	58
3	5743	8187	5885	8085	6025	7981	6163	7875	6300	7766	57
4	5745	8185	5887	8083	6027	7979	6166	7873	6302	7764	56
5	5748	8183	5890	8082	6030	7978	6168	7871	6305	7762	**55**
6	5750	8181	5892	8080	6032	7976	6170	7869	6307	7760	54
7	5752	8180	5894	8078	6034	7974	6173	7868	6309	7759	53
8	5755	8178	5897	8076	6037	7972	6175	7866	6311	7757	52
9	5757	8176	5899	8075	6039	7971	6177	7864	6314	7755	51
10	5760	8175	5901	8073	6041	7969	6180	7862	6316	7753	**50**
11	5762	8173	5904	8071	6044	7967	6182	7860	6318	7751	49
12	5764	8171	5906	8070	6046	7965	6184	7859	6320	7749	48
13	5767	8170	5908	8068	6048	7964	6186	7857	6323	7748	47
14	5769	8168	5911	8066	6051	7962	6189	7855	6325	7746	46
15	5771	8166	5913	8064	6053	7960	6191	7853	6327	7744	**45**
16	5774	8165	5915	8063	6055	7958	6193	7851	6329	7742	44
17	5776	8163	5918	8061	6058	7956	6196	7850	6332	7740	43
18	5779	8161	5920	8059	6060	7955	6198	7848	6334	7738	42
19	5781	8160	5922	8058	6062	7953	6200	7346	6336	7737	41
20	5783	8158	5925	8056	6065	7951	6202	7844	6338	7735	**40**
21	5786	8156	5927	8054	6067	7950	6205	7842	6341	7733	39
22	5788	8155	5930	8052	6069	7948	6207	7841	6343	7731	38
23	5790	8153	5932	8051	6071	7946	6209	7839	6345	7729	37
24	5793	8151	5934	8049	6074	7944	6211	7837	6347	7727	36
25	5795	8150	5937	8047	6076	7942	6214	7835	6350	7725	**35**
26	5798	8148	5939	8045	6078	7941	6216	7833	6352	7724	34
27	5800	8146	5941	8044	6081	7939	6218	7832	6354	7722	33
28	5802	8145	5944	8042	6083	7937	6221	7830	6356	7720	32
29	5805	8143	5946	8040	6085	7935	6223	7828	6359	7718	31
30	5807	8141	5948	8039	6088	7934	6225	7826	6361	7716	**30**
31	5809	8139	5951	8037	6090	7932	6227	7824	6363	7714	29
32	5812	8138	5953	8035	6092	7930	6230	7822	6365	7713	28
33	5814	8136	5955	8033	6095	7928	6232	7821	6368	7711	27
34	5816	8134	5958	8032	6097	7926	6234	7819	6370	7709	26
35	5819	8133	5960	8030	6099	7925	6237	7817	6372	7707	**25**
36	5821	8131	5962	8028	6101	7923	6239	7815	6374	7705	24
37	5824	8129	5965	8026	6104	7921	6241	7813	9376	7703	23
38	5826	8128	5967	8025	6106	7919	6243	7812	6379	7701	22
39	5828	8126	5969	8023	6108	7918	6246	7810	6381	7700	21
40	5831	8124	5972	8021	6111	7916	6248	7808	6383	7698	**20**
41	5833	8123	5974	8020	6113	7914	6250	7806	6385	7696	19
42	5835	8121	5976	8018	6115	7912	6252	7804	6388	7694	18
43	5838	8119	5979	8016	6118	7910	6255	7802	6390	7692	17
44	5840	8117	5981	8014	6120	7909	6257	7801	6392	7690	16
45	5842	8116	5983	8013	6122	7907	6259	7799	6394	7688	**15**
46	5845	8114	5986	8011	6124	7905	6262	7797	6397	7687	14
47	5847	8112	5988	8009	6127	7903	6264	7795	6399	7685	13
48	5850	8111	5990	8007	6129	7902	6266	7793	6401	7683	12
49	5852	8109	5993	8006	6131	7900	6268	7792	6403	7681	11
50	5854	8107	5995	8004	6134	7898	6271	7790	6406	7679	**10**
51	5857	8106	5997	8002	6136	7896	6273	7788	6408	7677	9
52	5859	8104	6000	8000	6138	7894	6275	7786	6410	7675	8
53	5861	8102	6002	7999	6141	7893	6277	7784	6412	7674	7
54	5864	8100	6004	7997	6143	7891	6280	7782	6414	7672	6
55	5866	8099	6007	7995	6145	7889	6282	7781	6417	7670	**5**
56	5868	8097	6009	7993	6147	7887	6284	7779	6419	7668	4
57	5871	8095	6011	7992	6150	7885	6286	7777	6421	7666	3
58	5873	8094	6014	7990	6152	7884	6289	7775	6423	7664	2
59	5875	8092	6016	7988	6154	7882	6291	7773	6426	7662	1
60	5878	8090	6018	7986	6157	7880	6293	7771	6428	7660	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	6428	7660	6561	7547	6691	7431	6820	7314	6947	7193	**60**
1	6430	7659	6563	7545	6693	7430	6822	7312	6949	7191	59
2	6432	7657	6565	7543	6696	7428	6824	7310	6951	7189	58
3	6435	7655	6567	7541	6698	7426	6826	7308	6953	7187	57
4	6437	7653	6569	7539	6700	7424	6828	7306	6955	7185	56
5	6439	7651	6572	7538	6702	7422	6831	7304	6957	7183	**55**
6	6441	7649	6574	7536	6704	7420	6833	7302	6959	7181	54
7	6443	7647	6576	7534	6706	7418	6835	7300	6961	7179	53
8	6446	7645	6578	7532	6709	7416	6837	7298	6963	7177	52
9	6448	7644	6580	7530	6711	7414	6839	7296	6965	7175	51
10	6450	7642	6583	7528	6713	7412	6841	7294	6967	7173	**50**
11	6452	7640	6585	7526	6715	7410	6843	7292	6970	7171	49
12	6455	7638	6587	7524	6717	7408	6845	7290	6972	7169	48
13	6457	7636	6589	7522	6719	7406	6848	7288	6974	7167	47
14	6459	7634	6591	7520	6722	7404	6850	7286	6976	7165	46
15	6461	7632	6593	7518	6724	7402	6852	7284	6978	7163	**45**
16	6463	7630	6596	7516	6726	7400	6854	7282	6980	7161	44
17	6466	7629	6598	7515	6728	7398	6856	7280	6982	7159	43
18	6468	7627	6600	7513	6730	7396	6858	7278	6984	7157	42
19	6470	7625	6602	7511	6732	7394	6860	7276	6986	7155	41
20	6472	7623	6604	7509	6734	7392	6862	7274	6988	7153	**40**
21	6475	7621	6607	7507	6737	7390	6865	7272	6990	7151	39
22	6477	7619	6609	7505	6739	7388	6867	7270	6992	7149	38
23	6479	7617	6611	7503	6741	7387	6869	7268	6995	7147	37
24	6481	7615	6613	7501	6743	7385	6871	7266	6997	7145	36
25	6483	7613	6615	7499	6745	7383	6873	7264	6999	7143	**35**
26	6486	7612	6617	7497	6747	7381	6875	7262	7001	7141	34
27	6488	7610	6620	7495	6749	7379	6877	7260	7003	7139	33
28	6490	7608	6622	7493	6752	7377	6879	7258	7005	7137	32
29	6492	7606	6624	7491	6754	7375	6881	7256	7007	7135	31
30	6494	7604	6626	7490	6756	7373	6884	7254	7009	7133	**30**
31	6497	7602	6628	7488	6758	7371	6886	7252	7011	7130	29
32	6499	7600	6631	7486	6760	7369	6888	7250	7013	7128	28
33	6501	7598	6633	7484	6762	7367	6890	7248	7015	7126	27
34	6503	7596	6635	7482	6764	7365	6892	7246	7017	7124	26
35	6506	7595	6637	7480	6767	7363	6894	7244	7019	7122	**25**
36	6508	7593	6639	7478	6769	7361	6896	7242	7022	7120	24
37	6510	7591	6641	7476	6771	7359	6898	7240	7024	7118	23
38	6512	7589	6644	7474	6773	7357	6900	7238	7026	7116	22
39	6514	7587	6646	7472	6775	7355	6903	7236	7028	7114	21
40	6517	7585	6648	7470	6777	7253	6905	7234	7030	7112	**20**
41	6519	7583	6650	7468	6779	7351	6907	7232	7032	7110	19
42	6521	7581	6652	7466	6782	7349	6909	7230	7034	7108	18
43	6523	7579	6654	7464	6784	7347	6911	7228	7036	7106	17
44	6525	7578	6657	7463	6786	7345	6913	7226	7038	7104	16
45	6528	7576	6659	7461	6788	7343	6915	7224	7040	7102	**15**
46	6530	7574	6661	7459	6790	7341	6917	7222	7042	7100	14
47	6532	7572	6663	7457	6792	7339	6919	7220	7044	7098	13
48	6534	7570	6665	7455	6794	7337	6921	7218	7046	7096	12
49	6536	7568	6667	7453	6797	7335	6924	7216	7048	7094	11
50	6539	7566	6670	7451	6799	7333	6926	7214	7050	7092	**10**
51	6541	7564	6672	7449	6801	7331	6928	7212	7053	7090	9
52	6543	7562	6674	7447	6803	7329	6930	7210	7055	7088	8
53	6545	7560	6676	7445	6805	7327	6932	7208	7057	7085	7
54	6547	7559	6678	7443	6807	7325	6934	7206	7059	7083	6
55	6550	7557	6680	7441	6809	7323	6936	7203	7061	7081	**5**
56	6552	7555	6683	7439	6811	7321	6938	7201	7063	7079	4
57	6554	7553	6685	7437	6814	7319	6940	7199	7065	7077	3
58	6556	7551	6687	7435	6816	7318	6942	7197	7067	7075	2
59	6558	7549	6689	7433	6818	7316	6944	7195	7069	7073	1
60	6561	7547	6691	7431	6820	7314	6947	7193	7071	7071	**0**
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	49°		48°		47°		46°		45°		′

′	**0°**		**1°**		**2°**		**3°**		**4°**		′
	tan	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	
0	0000	Infinite	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	**60**
1	0003	3437.75	0177	56.3506	0352	28.3994	0527	18.9755	0702	14.2411	59
2	0006	1718.87	0180	55.4415	0355	28.1664	0530	18.8711	0705	14.1821	58
3	0009	1145.92	0183	54.5613	0358	27.9372	0533	18.7678	0708	14.1235	57
4	0012	859.436	0186	53.7086	0361	27.7117	0536	18.6656	0711	14.0655	56
5	0015	687.549	0189	52.8821	0364	27.4899	0539	18.5645	0714	14.0079	**55**
6	0017	572.957	0192	52.0807	0367	27.2715	0542	18.4645	0717	13.9507	54
7	0020	491.106	0195	51.3032	0370	27.0566	0544	18.3655	0720	13.8940	53
8	0023	429.718	0198	50.5485	0373	26.8450	0547	18.2677	0723	13.8378	52
9	0026	381.971	0201	49.8157	0375	26.6367	0550	18.1708	0726	13.7821	51
10	0029	343.774	0204	49.1039	0378	26.4316	0553	18.0750	0729	13.7267	**50**
11	0032	312.521	0207	48.4121	0381	26.2296	0556	17.9802	0731	13.6719	49
12	0035	286.478	0209	47.7395	0384	26.0307	0559	17.8863	0734	13.6174	48
13	0038	264.441	0212	47.0853	0387	25.8348	0562	17.7934	0737	13.5634	47
14	0041	245.552	0215	46.4489	0390	25.6418	0565	17.7015	0740	13.5098	46
15	0044	229.182	0218	45.8294	0393	25.4517	0568	17.6106	0743	13.4566	**45**
16	0047	214.858	0221	45.2261	0396	25.2644	0571	17.5205	0746	13.4039	44
17	0049	202.219	0224	44.6386	0399	25.0798	0574	17.4314	0749	13.3515	43
18	0052	190.984	0227	44.0661	0402	24.8978	0577	17.3432	0752	13.2996	42
19	0055	180.932	0230	43.5081	0405	24.7185	0580	17.2558	0755	13.2480	41
20	0058	171.885	0233	42.9641	0407	24.5418	0582	17.1693	0758	13.1969	**40**
21	0061	163.700	0236	42.4335	0410	24.3675	0585	17.0837	0761	13.1461	39
22	0064	156.259	0239	41.9158	0413	24.1957	0588	16.9990	0764	13.0958	38
23	0067	149.465	0241	41.4106	0416	24.0263	0591	16.9150	0767	13.0458	37
24	0070	143.237	0244	40.9174	0419	23.8593	0594	16.8319	0769	12.9962	36
25	0073	137.507	0247	40.4358	0422	23.6945	0597	16.7496	0772	12.9469	**35**
26	0076	132.219	0250	39.9655	0425	23.5321	0600	16.6681	0775	12.8981	34
27	0079	127.321	0253	39.5059	0428	23.3718	0603	16.5874	0778	12.8496	33
28	0081	122.774	0256	39.0568	0431	23.2137	0606	16.5075	0781	12.8014	32
29	0084	118.540	0259	38.6177	0434	23.0577	0609	16.4283	0784	12.7536	31
30	0087	114.589	0262	38.1885	0437	22.9038	0612	16.3499	0787	12.7062	**30**
31	0090	110.892	0265	37.7686	0440	22.7519	0615	16.2722	0790	12.6591	29
32	0093	107.426	0268	37.3579	0442	22.6020	0617	16.1952	0793	12.6124	28
33	0096	104.171	0271	36.9560	0445	22.4541	0620	16.1190	0796	12.5660	27
34	0099	101.107	0274	36.5627	0448	22.3081	0623	16.0435	0799	12.5199	26
35	0102	98.2179	0276	36.1776	0451	22.1640	0626	15.9687	0802	12.4742	**25**
36	0105	95.4895	0279	35.8006	0454	22.0217	0629	15.8945	0805	12.4288	24
37	0108	92.9085	0282	35.4313	0457	21.8813	0632	15.8211	0808	12.3838	23
38	0111	90.4633	0285	35.0695	0460	21.7426	0635	15.7483	0810	12.3390	22
39	0113	88.1436	0288	34.7151	0463	21.6056	0638	15.6762	0813	12.2946	21
40	0116	85.9398	0291	34.3678	0466	21.4704	0641	15.6048	0816	12.2505	**20**
41	0119	83.8435	0294	34.0273	0469	21.3369	0644	15.5340	0819	12.2067	19
42	0122	81.8470	0297	33.6935	0472	21.2049	0647	15.4638	0822	12.1632	18
43	0125	79.9434	0300	33.3662	0475	21.0747	0650	15.3943	0825	12.1201	17
44	0128	78.1263	0303	33.0452	0477	20.9460	0653	15.3254	0828	12.0772	16
45	0131	76.3900	0306	32.7303	0480	20.8188	0655	15.2571	0831	12.0346	**15**
46	0134	74.7292	0308	32.4213	0483	20.6932	0658	15.1893	0834	11.9923	14
47	0137	73.1390	0311	32.1181	0486	20.5691	0661	15.1222	0837	11.9504	13
48	0140	71.6151	0314	31.8205	0489	20.4465	0664	15.0557	0840	11.9087	12
49	0143	70.1533	0317	31.5284	0492	20.3253	0667	14.9898	0843	11.8673	11
50	0146	68.7501	0320	31.2416	0495	20.2056	0670	14.9244	0846	11.8262	**10**
51	0148	67.4019	0323	30.9599	0498	20.0872	0673	14.8596	0849	11.7853	9
52	0151	66.1055	0326	30.6833	0501	19.9702	0676	14.7954	0851	11.7448	8
53	0154	64.8580	0329	30.4116	0504	19.8546	0679	14.7317	0854	11.7045	7
54	0157	63.6567	0332	30.1446	0507	19.7403	0682	14.6685	0857	11.6645	6
55	0160	62.4992	0335	29.8823	0509	19.6273	0685	14.6059	0860	11.6248	**5**
56	0163	61.3829	0338	29.6245	0512	19.5156	0688	14.5438	0863	11.5853	4
57	0166	60.3058	0340	29.3711	0515	19.4051	0690	14.4823	0866	11.5461	3
58	0169	59.2659	0343	29.1220	0518	19.2959	0693	14.4212	0869	11.5072	2
59	0172	58.2612	0346	28.8771	0521	19.1879	0696	14.3607	0872	11.4685	1
60	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	0875	11.4301	**0**
	cot	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	
′	**89°**		**88°**		**87°**		**86°**		**85°**		′

′	5°		6°		7°		8°		9°		′
	tan	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	
0	0875	11.4301	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	**60**
1	0878	11.3919	1054	9.4878	1231	8.1248	1408	7.1004	1587	6.3019	59
2	0881	11.3540	1057	9.4614	1234	8.1054	1411	7.0855	1590	6.2901	58
3	0884	11.3163	1060	9.4352	1237	8.0860	1414	7.0706	1593	6.2783	57
4	0887	11.2789	1063	9.4090	1240	8.0667	1417	7.0558	1596	6.2666	56
5	0890	11.2417	1066	9.3831	1243	8.0476	1420	7.0410	1599	6.2549	**55**
6	0892	11.2048	1069	9.3572	1246	8.0285	1423	7.0264	1602	6.2432	54
7	0895	11.1681	1072	9.3315	1249	8.0095	1426	7.0117	1605	6.2316	53
8	0898	11.1316	1075	9.3060	1251	7.9906	1429	6.9972	1608	6.2200	52
9	0901	11.0954	1078	9.2806	1254	7.9718	1432	6.9827	1611	6.2085	51
10	0904	11.0594	1080	9.2553	1257	7.9530	1435	6.9682	1614	6.1970	**50**
11	0907	11.0237	1083	9.2302	1260	7.9344	1438	6.9538	1617	6.1856	49
12	0910	10.9882	1086	9.2052	1263	7.9158	1441	6.9395	1620	6.1742	48
13	0913	10.9529	1089	9.1803	1266	7.8973	1444	6.9252	1623	6.1628	47
14	0916	10.9178	1092	9.1555	1269	7.8789	1447	6.9110	1626	6.1515	46
15	0919	10.8829	1095	9.1309	1272	7.8606	1450	6.8969	1629	6.1402	**45**
16	0922	10.8483	1098	9.1065	1275	7.8424	1453	6.8828	1632	6.1290	44
17	0925	10.8139	1101	9.0821	1278	7.8243	1456	6.8687	1635	6.1178	43
18	0928	10.7797	1104	9.0579	1281	7.8062	1459	6.8548	1638	6.1066	42
19	0931	10.7457	1107	9.0338	1284	7.7883	1462	6.8408	1641	6.0955	41
20	0934	10.7119	1110	9.0098	1287	7.7704	1465	6.8269	1644	6.0844	**40**
21	0936	10.6783	1113	8.9860	1290	7.7525	1468	6.8131	1647	6.0734	39
22	0939	10.6450	1116	8.9623	1293	7.7348	1471	6.7994	1650	6.0624	38
23	0942	10.6118	1119	8.9387	1296	7.7171	1474	6.7856	1653	6.0514	37
24	0945	10.5789	1122	8.9152	1299	7.6996	1477	6.7720	1655	6.0405	36
25	0948	10.5462	1125	8.8919	1302	7.6821	1480	6.7584	1658	6.0296	**35**
26	0951	10.5136	1128	8.8686	1305	7.6647	1483	6.7448	1661	6.0188	34
27	0954	10.4813	1131	8.8455	1308	7.6473	1486	6.7313	1664	6.0080	33
28	0957	10.4491	1134	8.8225	1311	7.6301	1489	6.7179	1667	5.9972	32
29	0960	10.4172	1136	8.7996	1314	7.6129	1492	6.7045	1670	5.9865	31
30	0963	10.3854	1139	8.7769	1317	7.5958	1495	6.6912	1673	5.9758	**30**
31	0966	10.3538	1142	8.7542	1319	7.5787	1497	6.6779	1676	5.9651	29
32	0969	10.3224	1145	8.7317	1322	7.5618	1500	6.6646	1679	5.9545	28
33	0972	10.2913	1148	8.7093	1325	7.5449	1503	6.6514	1682	5.9439	27
34	0975	10.2602	1151	8.6870	1328	7.5281	1506	6.6383	1685	5.9333	26
35	0978	10.2294	1154	8.6648	1331	7.5113	1509	6.6252	1688	5.9228	**25**
36	0981	10.1988	1157	8.6427	1334	7.4947	1512	6.6122	1691	5.9124	24
37	0983	10.1683	1160	8.6208	1337	7.4781	1515	6.5992	1694	5.9019	23
38	0986	10.1381	1163	8.5989	1340	7.4615	1518	6.5863	1697	5.8915	22
39	0989	10.1080	1166	8.5772	1343	7.4451	1521	6.5734	1700	5.8811	21
40	0992	10.0780	1169	8.5555	1346	7.4287	1524	6.5606	1703	5.8708	**20**
41	0995	10.0483	1172	8.5340	1349	7.4124	1527	6.5478	1706	5.8605	19
42	0998	10.0187	1175	8.5126	1352	7.3962	1530	6.5350	1709	5.8502	18
43	1001	9.9893	1178	8.4913	1355	7.3800	1533	6.5223	1712	5.8400	17
44	1004	9.9601	1181	8.4701	1358	7.3639	1536	6.5097	1715	5.8298	16
45	1007	9.9310	1184	8.4490	1361	7.3479	1539	6.4971	1718	5.8197	**15**
46	1010	9.9021	1187	8.4280	1364	7.3319	1542	6.4846	1721	5.8095	14
47	1013	9.8734	1189	8.4071	1367	7.3160	1545	6.4721	1724	5.7994	13
48	1016	9.8448	1192	8.3863	1370	7.3002	1548	6.4596	1727	5.7894	12
49	1019	9.8164	1195	8.3656	1373	7.2844	1551	6.4472	1730	5.7794	11
50	1022	9.7882	1198	8.3450	1376	7.2687	1554	6.4348	1733	5.7694	**10**
51	1025	9.7601	1201	8.3245	1379	7.2531	1557	6.4225	1736	5.7594	9
52	1028	9.7322	1204	8.3041	1382	7.2375	1560	6.4103	1739	5.7495	8
53	1030	9.7044	1207	8.2838	1385	7.2220	1563	6.3980	1742	5.7396	7
54	1033	9.6768	1210	8.2636	1388	7.2066	1566	6.3859	1745	5.7297	6
55	1036	9.6499	1213	8.2434	1391	7.1912	1569	6.3737	1748	5.7199	**5**
56	1039	9.6220	1216	8.2234	1394	7.1759	1572	6.3617	1751	5.7101	4
57	1042	9.5949	1219	8.2035	1397	7.1607	1575	6.3496	1754	5.7004	3
58	1045	9.5679	1222	8.1837	1399	7.1455	1578	6.3376	1757	5.6906	2
59	1048	9.5411	1225	8.1640	1402	7.1304	1581	6.3257	1760	5.6809	1
60	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	1763	5.6713	**0**
	cot	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	**cot**	**tan**	
′	**84°**		**83°**		**82°**		**81°**		**80°**		′

′	10°		11°		12°		13°		14°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	1763	5.6713	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	**60**
1	1766	5.6617	1947	5.1366	2129	4.6979	2312	4.3257	2496	4.0058	59
2	1769	5.6521	1950	5.1286	2132	4.6912	2315	4.3200	2499	4.0009	58
3	1772	5.6425	1953	5.1207	2135	4.6845	2318	4.3143	2503	3.9959	57
4	1775	5.6330	1956	5.1128	2138	4.6779	2321	4.3086	2506	3.9910	56
5	1778	5.6234	1959	5.1049	2141	4.6712	2324	4.3029	2509	3.9861	**55**
6	1781	5.6140	1962	5.0970	2144	4.6646	2327	4.2972	2512	3.9812	54
7	1784	5.6045	1965	5.0892	2147	4.6580	2330	4.2916	2515	3.9763	53
8	1787	5.5951	1968	5.0814	2150	4.6514	2333	4.2859	2518	3.9714	52
9	1790	5.5857	1971	5.0736	2153	4.6448	2336	4.2803	2521	3.9665	51
10	1793	5.5764	1974	5.0658	2156	4.6382	2339	4.2747	2524	3.9617	**50**
11	1796	5.5671	1977	5.0581	2159	4.6317	2342	4.2691	2527	3.9568	49
12	1799	5.5578	1980	5.0504	2162	4.6252	2345	4.2635	2530	3.9520	48
13	1802	5.5485	1983	5.0427	2165	4.6187	2349	4.2580	2533	3.9471	47
14	1805	5.5393	1986	5.0350	2168	4.6122	2352	4.2524	2537	3.9423	46
15	1808	5.5301	1989	5.0273	2171	4.6057	2355	4.2468	2540	3.9375	**45**
16	1811	5.5209	1992	5.0197	2174	4.5993	2358	4.2413	2543	3.9327	44
17	1814	5.5118	1995	5.0121	2177	4.5928	2361	4.2358	2546	3.9279	43
18	1817	5.5026	1998	5.0045	2180	4.5864	2364	4.2303	2549	3.9232	42
19	1820	5.4936	2001	4.9969	2183	4.5800	2367	4.2248	2552	3.9184	41
20	1823	5.4845	2004	4.9894	2186	4.5736	2370	4.2193	2555	3.9136	**40**
21	1826	5.4755	2007	4.9819	2189	4.5673	2373	4.2139	2558	3.9089	39
22	1829	5.4665	2010	4.9744	2193	4.5609	2376	4.2084	2561	3.9042	38
23	1832	5.4575	2013	4.9669	2196	4.5546	2379	4.2030	2564	3.8995	37
24	1835	5.4486	2016	4.9594	2199	4.5483	2382	4.1976	2568	3.8947	36
25	1838	5.4397	2019	4.9520	2202	4.5420	2385	4.1922	2571	3.8900	**35**
26	1841	5.4308	2022	4.9446	2205	4.5357	2388	4.1868	2574	3.8854	34
27	1844	5.4219	2025	4.9372	2208	4.5294	2392	4.1814	2577	3.8807	33
28	1847	5.4131	2028	4.9298	2211	4.5232	2395	4.1760	2580	3.8760	32
29	1850	5.4043	2031	4.9225	2214	4.5169	2398	4.1706	2583	3.8714	31
30	1853	5.3955	2035	4.9152	2217	4.5107	2401	4.1653	2586	3.8667	**30**
31	1856	5.3868	2038	4.9078	2220	4.5045	2404	4.1600	2589	3.8621	29
32	1859	5.3781	2941	4.9006	2223	4.4983	2407	4.1547	2592	3.8575	28
33	1862	5.3694	2044	4.8933	2226	4.4922	2410	4.1493	2595	3.8528	27
34	1865	5.3607	2047	4.8860	2229	4.4860	2413	4.1441	2599	3.8482	26
35	1868	5.3521	2050	4.8788	2232	4.4799	2416	4.1388	2602	3.8436	**25**
36	1871	5.3435	2053	4.8716	2235	4.4737	2419	4.1335	2605	3.8391	24
37	1874	5.3349	2056	4.8644	2238	4.4676	2422	4.1282	2608	3.8345	23
38	1877	5.3263	2059	4.8573	2241	4.4615	2425	4.1230	2611	3.8299	22
39	1880	5.3178	2062	4.8501	2244	4.4555	2428	4.1178	2614	3.8254	21
40	1883	5.3093	2065	4.8430	2247	4.4494	2432	4.1126	2617	3.8208	**20**
41	1887	5.3008	2068	4.8359	2251	4.4434	2435	4.1074	2620	3.8163	19
42	1890	5.2924	2071	4.8288	2254	4.4374	2438	4.1022	2623	3.8118	18
43	1893	5.2839	2074	4 8218	2257	4.4313	2441	4.0970	2627	3.8073	17
44	1896	5.2755	2077	4.8147	2260	4.4253	2444	4.0918	2630	3.8028	16
45	1899	5.2672	2080	4.8077	2263	4.4194	2447	4.0867	2633	3.7983	**15**
46	1902	5.2588	2083	4.8007	2266	4.4134	2450	4.0815	2636	3.7938	14
47	1905	5.2505	2086	4.7937	2269	4.4075	2453	4.0764	2639	3.7893	13
48	1908	5.2422	2089	4.7867	2272	4.4015	2456	4.0713	2642	3.7848	12
49	1911	5.2339	2092	4.7798	2275	4.3956	2459	4.0662	2645	3.7804	11
50	1914	5.2257	2095	4.7729	2278	4.3897	2462	4.0611	2648	3.7760	**10**
51	1917	5.2174	2098	4.7659	2281	4.3838	2465	4.0560	2651	3.7715	9
52	1920	5.2092	2101	4.7591	2284	4.3779	2469	4.0509	2655	3.7671	8
53	1923	5.2011	2104	4.7522	2287	4.3721	2472	4.0459	2658	3.7627	7
54	1926	5.1929	2107	4.7453	2290	4.3662	2475	4.0408	2661	3.7583	6
55	1929	5.1848	2110	4.7385	2293	4.3604	2478	4.0358	2664	3.7539	**5**
56	1932	5.1767	2113	4.7317	2296	4.3546	2481	4.0308	2667	3.7495	4
57	1935	5.1686	2116	4.7249	2299	4.3488	2484	4.0257	2670	3.7451	3
58	1938	5.1606	2119	4.7181	2303	4.3430	2487	4.0207	2673	3.7408	2
59	1941	5.1526	2123	4.7114	2306	4.3372	2490	4.0158	2676	3.7364	1
60	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	2679	3.7321	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	2679	3.7321	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	**60**
1	2683	3.7277	2871	3.4836	3060	3.2675	3252	3.0746	3447	2.9015	59
2	2686	3.7234	2874	3.4798	3064	3.2641	3256	3.0716	3450	2.8987	58
3	2689	3.7191	2877	3.4760	3067	3.2607	3259	3.0686	3453	2.8960	57
4	2692	3.7148	2880	3.4722	3070	3.2573	3262	3.0655	3456	2.8933	56
5	2695	3.7105	2883	3.4684	3073	3.2539	3265	3.0625	3460	2.8905	**55**
6	2698	3.7062	2886	3.4646	3076	3.2506	3269	3.0595	3463	2.8878	54
7	2701	3.7019	2890	3.4608	3080	3.2472	3272	3.0565	3466	2.8851	53
8	2704	3.6976	2893	3.4570	3083	3.2438	3275	3.0535	3469	2.8824	52
9	2708	3.6933	2896	3.4533	3086	3.2405	3278	3.0505	3473	2.8797	51
10	2711	3.6891	2899	3.4495	3089	3.2371	3281	3.0475	3476	2.8770	**50**
11	2714	3.6848	2902	3.4458	3092	3.2338	3285	3.0445	3479	2.8743	49
12	2717	3.6806	2905	3.4420	3096	3.2305	3288	3.0415	3482	2.8716	48
13	2720	3.6764	2908	3.4383	3099	3.2272	3291	3.0385	3486	2.8689	47
14	2723	3.6722	2912	3.4346	3102	3.2238	3294	3.0356	3489	2.8662	46
15	2726	3.6680	2915	3.4308	3105	3.2205	3298	3.0326	3492	2.8636	**45**
16	2729	3.6638	2918	3.4271	3108	3.2172	3301	3.0296	3495	2.8609	44
17	2733	3.6596	2921	3.4234	3111	3.2139	3304	3.0267	3499	2.8582	43
18	2736	3.6554	2924	3.4197	3115	3.2106	3307	3.0237	3502	2.8556	42
19	2739	3.6512	2927	3.4160	3118	3.2073	3310	3.0208	3505	2.8529	41
20	2742	3.6470	2931	3.4124	3121	3.2041	3314	3.0178	3508	2.8502	**40**
21	2745	3.6429	2934	3.4087	3124	3.2008	3317	3.0149	3512	2.8476	39
22	2748	3.6387	2937	3.4050	3127	3.1975	3320	3.0120	3515	2.8449	38
23	2751	3.6346	2940	3.4014	3131	3.1943	3323	3.0090	3518	2.8423	37
24	2754	3.6305	2943	3.3977	3134	3.1910	3327	3.0061	3522	2.8397	36
25	2758	3.6264	2946	3.3941	3137	3.1878	3330	3.0032	3525	2.8370	**35**
26	2761	3.6222	2949	3.3904	3140	3.1845	3333	3.0003	3528	2.8344	34
27	2764	3.6181	2953	3.3868	3143	3.1813	3336	2.9974	3531	2.8318	33
28	2767	3.6140	2956	3.3832	3147	3.1780	3339	2.9945	3535	2.8291	32
29	2770	3.6100	2959	3.3796	3150	3.1748	3343	2.9916	3538	2.8265	31
30	2773	3.6059	2962	3.3759	3153	3.1716	3346	2.9887	3541	2.8239	**30**
31	2776	3.6018	2965	3.3723	3156	3.1684	3349	2.9858	3544	2.8213	29
32	2780	3.5978	2968	3.3687	3159	3.1652	3352	2.9829	3548	2.8187	28
33	2783	3.5937	2972	3.3652	3163	3.1620	3356	2.9800	3551	2.8161	27
34	2786	3.5897	2975	3.3616	3166	3.1588	3359	2.9772	3554	2.8135	26
35	2789	3.5856	2978	3.3580	3169	3.1556	3362	2.9743	3558	2.8109	**25**
36	2792	3.5816	2981	3.3544	3172	3.1524	3365	2.9714	3561	2.8083	24
37	2795	3.5776	2984	3.3509	3175	3.1492	3369	2.9686	3564	2.8057	23
38	2798	3.5736	2987	3.3473	3179	3.1460	3372	2.9657	3567	2.8032	22
39	2801	3.5696	2991	3.3438	3182	3.1429	3375	2.9629	3571	2.8006	21
40	2805	3.5656	2994	3.3402	3185	3.1397	3378	2.9600	3574	2.7980	**20**
41	2808	3.5616	2997	3.3367	3188	3.1366	3382	2.9572	3577	2.7955	19
42	2811	3.5576	3000	3.3332	3191	3.1334	3385	2.9544	3581	2.7929	18
43	2814	3.5536	3003	3.3297	3195	3.1303	3388	2.9515	3584	2.7903	17
44	2817	3.5497	3006	3.3261	3198	3.1271	3391	2.9487	3587	2.7878	16
45	2820	3.5457	3010	3.3226	3201	3.1240	3395	2.9459	3590	2.7852	**15**
46	2823	3.5418	3013	3.3191	3204	3.1209	3398	2.9431	3594	2.7827	14
47	2827	3.5379	3016	3.3156	3207	3.1178	3401	2.9403	3597	2.7801	13
48	2830	3.5339	3019	3.3122	3211	3.1146	3404	2.9375	3600	2.7776	12
49	2833	3.5300	3022	3.3087	3214	3.1115	3408	2.9347	3604	2.7751	11
50	2836	3.5261	3026	3.3052	3217	3.1084	3411	2.9319	3607	2.7725	**10**
51	2839	3.5222	3029	3.3017	3220	3.1053	3414	2.9291	3610	2.7700	9
52	2842	3.5183	3032	3.2983	3223	3.1022	3417	2.9263	3613	2.7675	8
53	2845	3.5144	3035	3.2948	3227	3.0991	3421	2.9235	3617	2.7650	7
54	2849	3.5105	3038	3.2914	3230	3.0961	3424	2.9208	3620	2.7625	6
55	2852	3.5067	3041	3.2880	3233	3.0930	3427	2.9180	3623	2.7600	**5**
56	2855	3.5028	3045	3.2845	3236	3.0899	3430	2.9152	3627	2.7575	4
57	2858	3.4989	3048	3.2811	3240	3.0868	3434	2.9125	3630	2.7550	3
58	2861	3.4951	3051	3.2777	3243	3.0838	3437	2.9097	3633	2.7525	2
59	2864	3.4912	3054	3.2743	3246	3.0807	3440	2.9070	3636	2.7500	1
60	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	3640	2.7475	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	3640	2.7475	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	**60**
1	3643	2.7450	3842	2.6028	4044	2.4730	4248	2.3539	4456	2.2443	59
2	3646	2.7425	3845	2.6006	4047	2.4709	4252	2.3520	4459	2.2425	58
3	3650	2.7400	3849	2.5983	4050	2.4689	4255	2.3501	4463	2.2408	57
4	3653	2.7376	3852	2.5961	4054	2.4668	4258	2.3483	4466	2.2390	56
5	3656	2.7351	3855	2.5938	4057	2.4648	4262	2.3464	4470	2.2373	**55**
6	3659	2.7326	3859	2.5916	4061	2.4627	4265	2.3445	4473	2.2355	54
7	3663	2.7302	3862	2.5893	4064	2.4606	4269	2.3426	4477	2.2338	53
8	3666	2.7277	3865	2.5871	4067	2.4586	4272	2.3407	4480	2.2320	52
9	3669	2.7253	3869	2.5848	4071	2.4566	4276	2.3388	4484	2.2303	51
10	3673	2.7228	3872	2.5826	4074	2.4545	4279	2.3369	4487	2.2286	**50**
11	3676	2.7204	3875	2.5804	4078	2.4525	4283	2.3351	4491	2.2268	49
12	3679	2.7179	3879	2.5782	4081	2.4504	4286	2.3332	4494	2.2251	48
13	3683	2.7155	3882	2.5759	4084	2.4484	4289	2.3313	4498	2.2234	47
14	3686	2.7130	3885	2.5737	4088	2.4464	4293	2.3294	4501	2.2216	46
15	3689	2.7106	3889	2.5715	4091	2.4443	4296	2.3276	4505	2.2199	**45**
16	3693	2.7082	3892	2.5693	4095	2.4423	4300	2.3257	4508	2.2182	44
17	3696	2.7058	3895	2.5671	4098	2.4403	4303	2.3238	4512	2.2165	43
18	3699	2.7034	3899	2.5649	4101	2.4383	4307	2.3220	4515	2.2148	42
19	3702	2.7009	3902	2.5627	4105	2.4362	4310	2.3201	4519	2.2130	41
20	3706	2.6985	3906	2.5605	4108	2.4342	4314	2.3183	4522	2.2113	**40**
21	3709	2.6961	3909	2.5533	4111	2.4322	4317	2.3164	4526	2.2096	39
22	3712	2.6937	3912	2.5561	4115	2.4302	4320	2.3146	4529	2.2079	38
23	3716	2.6913	3916	2.5539	4118	2.4282	4324	2.3127	4533	2.2062	37
24	3719	2.6889	3919	2.5517	4122	2.4262	4327	2.3109	4536	2.2045	36
25	3722	2.6865	3922	2.5495	4125	2.4242	4331	2.3090	4540	2.2028	**35**
26	3726	2.6841	3926	2.5473	4129	2.4222	4334	2.3072	4543	2.2011	34
27	3729	2.6818	3929	2.5452	4132	2.4202	4338	2.3053	4547	2.1994	33
28	3732	2.6794	3932	2.5430	4135	2.4182	4341	2.3035	4550	2.1977	32
29	3736	2.6770	3936	2.5408	4139	2.4162	4345	2.3017	4554	2.1960	31
30	3739	2.6746	3939	2.5386	4142	2.4142	4348	2.2998	4557	2.1943	**30**
31	3742	2.6723	3942	2.5365	4146	2.4122	4352	2.2980	4561	2.1926	29
32	3745	2.6699	3946	2.5343	4149	2.4102	4355	2.2962	4564	2.1909	28
33	3749	2.6675	3949	2.5322	4152	2.4083	4359	2.2944	4568	2.1892	27
34	3752	2.6652	3953	2.5300	4156	2.4063	4362	2.2925	4571	2.1876	26
35	3755	2.6628	3956	2.5279	4159	2.4043	4365	2.2907	4575	2.1859	**25**
36	3759	2.6605	3959	2.5257	4163	2.4023	4369	2.2889	4578	2.1842	24
37	3762	2.6581	3963	2.5236	4166	2.4004	4372	2.2871	4582	2.1825	23
38	3765	2.6558	3966	2.5214	4169	2.3984	4376	2.2853	4585	2.1808	22
39	3769	2.6534	3969	2.5193	4173	2.3964	4379	2.2835	4589	2.1792	21
40	3772	2.6511	3973	2.5172	4176	2.3945	4383	2.2817	4592	2.1775	**20**
41	3775	2.6488	3976	2.5150	4180	2.3925	4386	2.2799	4596	2.1758	19
42	3779	2.6464	3979	2.5129	4183	2.3906	4390	2.2781	4599	2.1742	18
43	3782	2.6441	3983	2.5108	4187	2.3886	4393	2.2763	4603	2.1725	17
44	3785	2.6418	3986	2.5086	4190	2.3867	4397	2.2745	4607	2.1708	16
45	3789	2.6395	3990	2.5065	4193	2.3847	4400	2.2727	4610	2.1692	**15**
46	3792	2.6371	3993	2.5044	4197	2.3828	4404	2.2709	4614	2.1675	14
47	3795	2.6348	3996	2.5023	4200	2.3808	4407	2.2691	4617	2.1659	13
48	3799	2.6325	4000	2.5002	4204	2.3789	4411	2.2673	4621	2.1642	12
49	3802	2.6302	4003	2.4981	4207	2.3770	4414	2.2655	4624	2.1625	11
50	3805	2.6279	4006	2.4960	4210	2.3750	4417	2.2637	4628	2.1609	**10**
51	3809	2.6256	4010	2.4939	4214	2.3731	4421	2.2620	4631	2.1592	9
52	3812	2.6233	4013	2.4918	4217	2.3712	4424	2.2602	4635	2.1576	8
53	3815	2.6210	4017	2.4897	4221	2.3693	4428	2.2584	4638	2.1560	7
54	3819	2.6187	4020	2.4876	4224	2.3673	4431	2.2566	4642	2.1543	6
55	3822	2.6165	4023	2.4855	4228	2.3654	4435	2.2549	4645	2.1527	**5**
56	3825	2.6142	4027	2.4834	4231	2.3635	4438	2.2531	4649	2.1510	4
57	3829	2.6119	4030	2.4813	4234	2.3616	4442	2.2513	4652	2.1494	3
58	3832	2.6096	4033	2.4792	4238	2.3597	4445	2.2496	4656	2.1478	2
59	3835	2.6074	4037	2.4772	4241	2.3578	4449	2.2478	4660	2.1461	1
60	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	4663	2.1445	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	69°		68°		67°		66°		65°		′